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by

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Introduction

This paper introduces variable quality into the general treatment of neoclassical economics. It also introduces subbudget decision making at all levels. The consequences of these introductions are enormous for traditional theory. Most importantly, from the perspective of comparative economics, is the realization that within the market model there exists the prospect that constraints in capitalism are not *hard*, nor exclusively determined by market prices. These developments arise from the creation of what may be called the "Hicks, Houthakker, Duesenberry model." The soft constraints in the HHD model are unlike the soft constraints articulated by János Kornai and subsequently by Gérard Roland.[1] In addition to the above, this paper argues for expansion of demand theory, and for expansion of the theory of general equilibrium. Some of these developments are not discussed in this paper (See Wadman 2009, Table of Contents). In the process the paper posits that much of neoclassical theory is a special case, and as such provides an inadequate theoretical foundation for the Anglo-Saxon version of capitalism.

The paper argues against the existence of a hard budget constraint, both for households and for firms; in fact, it argues that the constraint is influenced by subbudget decision making and by the selection of levels of quality, and as such, the constraint may become stochastic, not deterministic (See Wadman 2009, Chapters 1 and 2.). It introduces the proposition that Marginal Rates of Substitution are not equal across consumers; that Marginal Rates of Transformation are not equal across firms; that the condition MRS=MRT is not attainable; that the condition, $(MP_l / p_l) = (MP_k / p_k)$ is not the long-run equilibrium for firms; and it presents arguments against lump-sum taxation, specifically, that the theoretical results of a lump-sum tax are unattainable. The paper introduces additional conditions for attainment of Pareto optimality in welfare economics.

Ultimately, the paper argues that market forces are not impartial. This behavior surfaces in consumption space and in input space. This phenomenon arises from human decision making regarding the size of subbudgets and levels of quality, and the model explains how these issues influence the position and slope of constraints in consumption space and in input space.

The avenue to attainment of these results lies within what has always been the Achilles tendon of neoclassical economics: the homogeneity assumption, or in this case, the assumption of constant quality and the absence of subbudgets in economic decision making. Insertion of these two elements into neoclassical economics demonstrates that the theoretical foundation of neoclassical capitalism (i.e., the Anglo-Saxon version of capitalism) is itself a special case. These results evolve slowly. Initially, the changes are subtle, but build and emerge forcefully as the paper unfolds. The foundation for this analysis is built upon the works of J.R. Hicks, Hendrik Houthakker and James Duesenberry. The work of Duncan Ironmonger also plays a role. The model also builds on the author's previous work, *Variable Quality in Consumer Theory*. In order to enlarge the potential readership, the format of the paper involves the use of diagrams.

The Budget Constraint when Consumers employ Sub-Budgets within the Total Household Budget

In the traditional approach to General Equilibrium, Hicksian consumption space plays an important role. Subbudgeting is not assumed and the level of quality is assumed constant (usually subsumed within the homogeneity assumption). Consumers are assumed to confront the same market prices, e.g., p_i for the *i-th* commodity and p_j for the *j-th* commodity. The quantity of the *i-th* commodity is represented as x_i, and the quantity of the *j-th* commodity is represented as x_j. Inasmuch as household income is not subdivided into subbudgets, the consumer budget constraint is given by $M = p_i x_i + p_j x_j$, which with rearrangement becomes $x_j = (M / p_j) - (p_i / p_j) x_i$. All consumers face the same ratio of prices, (p_i / p_j), hence the slope of the constraint is the same for all consumers. A change in income, ΔM, shifts the constraint in parallel fashion, due to no change in the ratio of prices. Efficiency in exchange becomes $MRS^A = (p_i / p_j) = MRS^B$, and therefore $MRS^A = MRS^B$, where superscripts indicate different consumers.

Under the HHD model of consumption, from which is derived a four-space diagram, the arguments for efficiency in exchange are considerably weakened. The four-space diagram begins with reconstruction of Hicksian $x_i x_j$ consumption space.[2] Central to the issue at hand is the budget constraint in the reconstructed $x_i x_j$ space, under conditions of subbudgeting.

Consider the existence of a subbudget for the *i-th* commodity and, separately, a subbudget for the *j-th* commodity, i.e., assume the existence of M_i and M_j, where $M_i + M_j = M$, and M is the symbol for household income. These subbudgets arise in the Houthakkerian spaces, but their existence influences conditions in Hicksian space. The budget constraint in $x_i x_j$ space becomes $M_i + M_j = p_i x_i + p_j x_j$, which converts to

(1) $$x_j = (M_i / p_j) + (M_j / p_j) - (p_i / p_j) x_i .$$

Note that the slope of the constraint appears as in the traditional case, i.e., the ratio $-(p_i / p_j)$. (Heretofore this, or the ratio -(p_j / p_k), has determined the slope as discussed previously.) As will be seen shortly, the ratio of prices is no longer the sole component in the determination of slope.

In contrast to the traditional approach, note that in equation (1) an increase in the *i-th* subbudget, $\Delta M_i > 0$, does not influence the intercept of the *j-th* commodity, which is given by (M_j / p_j). This value, (M_j / p_j), arises in Houthakker's $v_j x_j$ consumption space and it exists beneath the traditional x_j intercept in Hicksian space, i.e., in Hicksian space the relationship is, $(M / p_j) > (M_j / p_j)$. In the traditional case, an increase in either M_i or M_j would change the intercepts of both commodities. In the HHD model, however, the intercept for the *j-th* commodity is (M_j / p_j), not the traditional (M / p_j). Similarly, the constraint may be rearranged such that:

(2) $x_i = (M_i / p_i) + (M_j / p_i) - (p_j / p_i) x_j$.

In this case, a change in M_j does not influence the intercept of the *i-th* commodity, i.e., ΔM_j does not change (M_i / p_i). As before, the value of (M_i/p_i) is derived from Houthakker's $v_i x_i$ consumption space, and in Hicksian space the relationship is, $(M / p_i) > (M_i / p_i)$.

In the new model of consumer theory, both in $v_i x_i$ space (Houthakker's quality-quantity space for the *i-th* commodity, where v_i is the quality level of the *i-th* commodity) and in $x_i x_j$ space (Hicksian consumption space), the intercept for the *i-th* commodity is $(M_i / p_i) = x_i$; and for the *j-th* commodity, in $v_j x_j$ and $x_i x_j$ spaces, the intercept is $(M_j / p_j) = x_j$. These intercepts exist in an open subspace beneath the traditional constraint in Hicksian space.[3] In establishing the slope of the constraint in $x_i x_j$ consumption space, these new intercepts are extremely important.

As regards slope, recall that the new constraint is in the open subspace, beneath the traditional constraint in $x_i x_j$ space. Further, note that the position of each intercept on its corresponding axis is influenced by the size of the corresponding subbudget, by the price of the commodity, and by the level of quality.

The Impact of Houthakker on the Constraint in Hicksian Consumption Space

In terms of calculating the slope of the constraint in the open subspace, utilize the two new intercepts, which reflect the subbudgets established by the consumer in Houthakker's $v_i x_i$ and $v_j x_j$ spaces. In $x_i x_j$ space, define the "rise" as the distance from the origin to the x_j intercept, and define the "run'" as the distance from the origin to the x_i intercept. The slope of the constraint in the $x_i x_j$ open subspace becomes: $-(M_j / p_j)/(M_i / p_i)$, which rearranges to $-(M_j / p_j)(p_i / M_i)$, or

(3) $-(M_j / M_i)(p_i / p_j)$.

Compare these results with the traditional approach, where the slope is $-(M / p_j)/(M / p_i)$, which rearranges to $-(M / p_j)(p_i / M)$, and where the household budget, *M*, cancels out, leaving the traditional $-(p_i/p_j)$. In other words, in the new model the price ratio $-(p_i/p_j)$ is one component in the calculation of constraint slope. The ratio of subbudgets, however, also is able to influence slope.

For example, assume an increase in total household income, *M*, which is allocated exclusively to the M_i subbudget, such that the original amount of funds in the M_i subbudget has increased. Given this higher distribution of funds to M_i, there is a change in the allocation of funds to the *i-th* commodity, and hence in the value of the intercept, M_i/p_i, in the open space. Assume no change in M_j and p_j, i.e., the x_j intercept is constant, and assume no change in p_i. The increase in M_i will pivot outward the constraint in $x_i x_j$ space. Note that if x_j is held constant at any point along the x_j axis, any increase (or decrease) in M_i will change the slope of the constraint. In the present case, the increase in M_i has increased the value of the x_i intercept, and hence reduced the slope of the constraint in $x_i x_j$ space.

In Figure 1 an illustration is provided of an increase in M_i. Note that the increase in M_i is shown as the outward (or rightward) shift of the constraint in Houthakker's quality-quantity space (i.e., in $v_i x_i$ space). The increase in M_i is also manifest as the outward pivot of the constraint in $x_i x_j$ space. The constraint pivots at the intercept of the *j-th* commodity.

Further note that corresponding to $\Delta M_i > 0$ there is no change in the level of quality for either the *i-th* or *j-th* commodities.

The new constraint illustrated in $x_i x_j$ space is not, however, the traditional Hicksian constraint. It is a joint-isoquality constraint.[4] The increase in M_i has reduced the magnitude of the constraint slope in $x_i x_j$ space, which is consistent with M_i in the denominator of $-(M_j / M_i)(p_i / p_j)$. In fact, in this new equation for the slope of the constraint, an increase in either numerator will increase the value of the slope; contrarily, an increase in either denominator will reduce the slope.[5]

The Issue of Movement along the Constraint and Slope of the Constraint

It should be noted that movement along the constraint, in the open space of $x_i x_j$ consumption space, is associated with a pivot of the constraint (i.e., change in slope). As stated above, if the value of x_j is held constant and a change occurs in the total value of M_i, then there will be a change in the slope of the constraint. However, a change in M_i will change the ratio of (M_j/M_i) and, therefore, change the slope of the constraint. Furthermore, if, concomitant with $\Delta M_i > 0$ there is $\Delta M_j < 0$, such that M remains constant (recall $M = M_i + M_j$), there is movement along the constraint, and simultaneously a change in slope. This phenomenon, produces an inverse relationship between x_j and x_i, as the consumer moves along the changed constraint. As indicated earlier, slope of the constraint is given by the two intercepts, M_j / p_j and M_i / p_i, and therefore change in M_i, described above, can a produce a change in slope. In effect, there is no movement along a fixed constraint, rather simultaneous movement along the constraint and change in slope of the constraint. It should also be recalled that any change by the consumer must be supported by subbudgets. This suggests that the optimum point for the consumer involves knowing the new slope and the consumer location on that new slope. A question now arises as to the use of constrained optimization in microeconomic consumer theory. A question raised in an earlier work is important to the subject at hand: are preferences reflected in subbudget decision making? (See Wadman 2009). And if so, is existence of a preference map necessary to analyze consumer behavior? Does subbudget decision making include the preference map and the constraint?

Recall that under the traditional approach p_i/p_j, establishes the slope, which may also be defined as $\Delta x_j / \Delta x_i$. Along the constraint, therefore, the following condition is met: $(\Delta x_j / \Delta x_i) = (p_i / p_j)$. In the new model, however, the slope is redefined as

(4) $(M_j/M_i)(p_i/p_j)$, or $(\Delta x_j / \Delta x_i) = (M_j / M_i)(p_i / p_j)$,

which raises the prospect that the new slope may differ from the traditional slope. Under the traditional model the slope may be described as above, or $(\Delta x_j / \Delta x_i) = (p_i / p_j)$, and therefore the possibility exists that $(p_i / p_j) \neq (M_j / M_i)(p_i / p_j)$, except where $(M_j / M_i) = 1$. More importantly to the topic at hand, decision making by the consumer will lead to a change in the constraint path from the original constraint to the new constraint and a change in slope of the new constraint. Along with the change in slope, assume that for all varieties, v, each variety is greater than its basic package, or $v_i > BP_i$ and $v_j > BP_j$. Also, assume that the prices p_i and p_j are both constant.

Subsequently, assume an increase in M which is allocated exclusively to M_i. This new allocation will change the slope. Under these conditions,

(5) $(\Delta x_j / \Delta x_i) = (M_j / M_i)(p_i / p_j) < (p_i / p_j).$

If the increase in M is allocated to M_j the slope will rise, and the condition becomes:

(6) $(\Delta x_j / \Delta x_i) = (M_j / M_i)(p_i / p_j) > (p_i / p_j).$

Of importance throughout this discussion is the prospect that the constraint in Hicksian consumption space has the potential to become unique to each consumer (i.e., stochastic), instead of the traditional deterministic approach. Also, related to this topic is the question of whether market forces will force the new constraint to approach the traditional constraint; or whether oligopolistic market structures will permit, or even foster, greater use of the new constraint?

The HHD Model and Existence of Efficiency in Exchange

Consider now the impact of these findings on the efficiency in exchange argument found in the Theory of General Equilibrium[6]. Recall from earlier consideration the condition: $MRS^A = (p_i / p_j) = MRS^B$. Under the conditions presented earlier the slope of the constraint in Hicksian space was simply, $-(p_i / p_j)$. Consider now the condition, $MRS = -(M_j / M_i)(p_i / p_j)$, where both M_j and M_i may be established by each consumer, and therefore the ratio of subbudgets may differ across consumers, albeit that the ratio of prices, $-(p_i/p_j)$, is the same for all consumers. If the ratio of sub-budgets differs across consumers, the slope of the constraint differs across consumers.

Consider Figure 2, where three different levels of income are represented. Assume the three constraints represent three different consumers, or three different income cohorts: one with high household income, another with middle household income, and the third with low household income. Symbolize these levels of income as $M^h > M^m > M^l$. In the present case, assume all three households have the same income allocated to the *j-th* commodity, i.e., (M_j / p_j) is the same for all three income groups. In other words, the conditions are: $M_i^h > M_i^m > M_i^l$.

Now, given the existence of identical joint-isoquality preference maps for all three income groups, tangencies with their corresponding constraints will result in different marginal rates of substitution for each group, or $MRS^h = [-(M_j / M_i^h)(p_i / p_j)] < MRS^m = [-(M_j / M_i^m)(p_i / p_j)] <$ $MRS^l = [-(M_j / M_i^l)(p_i / p_j)]$. Note that these conditions occur when the level of quality, for both commodities, is the same for all three income groups. Clearly, in the case presented here, efficiency in exchange does not exist across the three levels of income, albeit that the ratio of prices is the same for all three groups. These results are quite different from the traditional General Equilibrium arguments. Even without the introduction of variable quality, the arguments on behalf of impartial market forces appear to be weakening. Market forces may not produce the same results across different income cohorts.

Also, note that Figure 2 illustrates another method via which consumers influence the slope of the constraint: in this case through variation in their income.

The Impact of Variable Quality on Hicksian Space

In the alternative approach to General Equilibrium -- under conditions of variable quality and subbudgeting -- consider now the case where the ratio of subbudgets is identical for all three income groups, and (initially) the ratio of prices likewise is the same, such that the slope of the constraint is the same for all three groups. Under these conditions, examine now what happens if the level of quality (of the *i-th* commodity) becomes different for the three income groups. Assume, $v_i^h > v_i^m > v_i^l$, where the superscripts refer to levels of quality.

Inasmuch as the level of quality, v_i, is in the Houthakker price, $p_i = a_i + b_i v_i$, different levels of quality will produce different prices, i.e., v_i^h will result in p_i^h, v_i^m gives p_i^m, and v_i^l gives p_i^l. Given $v_i^h > v_i^m > v_i^l$, it follows that $p_i^h > p_i^m > p_i^l$.[7] Given the same subbudgets for M_i for all three income groups, there is, $(M_i / p_i^h) < (M_i / p_i^m) < (M_i / p_i^l)$. These conditions are the opposite of those found in Figure 2. Specifically, the steeper sloped constraint now corresponds to the higher-quality group, and so forth. In terms of MRS, the conditions are, as before, $MRS^h \neq MRS^m \neq MRS^l$, and in the present case, the specific results are: $MRS^h > MRS^m > MRS^l$, where the superscripts indicate level of quality. Note this result is the reverse of the ordering created via different-sized subbudgets. This suggests that different sized subbudgets could be used to offset different levels of quality, such that $MRS^h = MRS^m = MRS^l$. These findings hold implications for the adjustment of consumer price indices when there is change in the level of quality.[8]

The Edgeworth Box under Conditions of Sub-Budgeting

Begin this section by examining the constraint in the core under the condition that the slope for each consumer is $-(M_j / M_i)(p_i / p_j)$, and consider the impact of different slopes corresponding to different levels of subbudget income allocated to the *i-th* commodity. This is essentially the same phenomena addressed earlier in Hicksian space, i.e., different slopes at different points of tangency, and therefore, different marginal rates of substitution for different levels of income.[9]

In the concept of the core begin, as traditionally, at the intersection of two indifference curves. Assume each set of indifference curves (corresponding to two consumers) is at the same level of quality for each commodity, i.e., for consumer A the level of v_i is identical to that of v_i for consumer B, similarly for v_j, or $v_i^A = v_i^B$ and $v_j^A = v_j^B$. In other words, the two preference maps are at the same level of joint-isoquality.

In Hicksian space assume the intercept for the *j-th* commodity is identical for both consumers. Further, assume that the magnitude of the *i-th* subbudget is different between the two consumers. Assume consumer A has a high subbudget for the *i-th* commodity, which is identified $M_i^{h=A}$, and assume that consumer B has a lower subbudget, identified as $M_i^{m=B}$.

Given difference in the *i-th* subbudgets (between the two consumers), the slopes of the constraints in Hicksian space and in the Edgeworth box will differ (between the consumers), and in the Edgeworth box this difference will apply at the point of intersection of indifference curves. In the traditional case a single budget line runs through the point of intersection, and hence exchange under competitive market conditions tends to the contract line. In the present case, however, there exist two different-sloped budget lines through the point of intersection. In the traditional approach, the process of achieving the contract curve is through a "reallocation" of the two goods, e.g., x_i and x_j. In the new model, the process of reallocation involves, in addition, a reallocation of funds between subbudgets, i.e., between M_i and M_j for each consumer.

The contract curve is, as before, the locus of tangency points of the preference maps for the two consumers. Under the condition of identical joint-isoquality indifference curves, the tangencies retain their original interpretation. Now, however, the budget line may be different for each consumer and not support attainment of the contract curve, i.e., $MRS^A \neq MRS^B$. Note that $M_i^{h=A} > M_i^{m=B}$ is comparable to two consumers facing different prices. The traditional argument for efficiency in exchange, and attainment of Pareto optimality along the contract curve, is breaking down under subbudgeting by consumers.

The Edgeworth Box under Conditions of Variable Quality

Examine next the issue of exchange, where the level of quality may be different between two consumers. Assume the levels of v_i and v_j for consumer A are v_{i1} and v_{j1}, i.e., the levels of joint-isoquality are $\langle v_{i1}, v_{j1} \rangle^A$. For consumer B, assume the levels of v_i and v_j are v_{i2} and v_{j2} respectively, where $v_{i2} > v_{i1}$ and $v_{j2} > v_{j1}$. The joint-isoquality preference map for consumer B is $\langle v_{i2}, v_{j2} \rangle^B$. In summary, the preference maps for the two consumers are different, with the preference map of consumer B at a higher level of joint-isoquality than the preference map of consumer A, or $\langle v_{i1}, v_{j1} \rangle^A < \langle v_{i2}, v_{j2} \rangle^B$. Incidentally, this is only one of a number of variations away from identical joint-isoquality preference maps.

With this as background, consider now the core in the traditional Edgeworth box. The intersection of indifference curves now corresponds to two indifference curves that reflect different levels of quality. Before considering the process of exchange between these two consumers, consider the meaning of a tangency of the two different joint-isoquality indifference curves.

Tangency traditionally means $MRS^A = MRS^B$, where the rates of substitution apply to the quantity of x_i and the quantity of x_j. In other words, traditional tangency does not address rates of substitution of one level of quality for another. Substitution of this nature may be defined as intra-commodity substitution. Such substitution lies within the same commodity, e.g., v_{i2} for v_{i1} within the *i-th* commodity. Under these conditions, conceivably the value of x_i, the quantity of the *i-th* commodity, could be the same for both consumers, i.e., $x_i^A = x_i^B$, and likewise for the *j-th* commodity, or $x_j^A = x_j^B$. Here, however, due to difference in the levels of quality, the joint-isoquality indifference curves do not have a tangency. This introduces the prospect that the Edgeworth box needs to be expanded to include substitution between levels of quality, e.g., v_{i2} for v_{i1} , etc. This might be accomplished through introduction of v_i and v_j on the axes within the box diagram. (See, Wadman 2009, Chapter 5.) Analogous to the Hicks diagram for $x_i x_j$ space, introduce now the quality level of both the *i-th* and *j-th* commodities in an Edgeworth box diagram. An illustration is provided in Figure 3.

As drawn, the indifference curves are convex and two intersections exist. Note that both curves are isoquantity, where $x_i^A = x_i^B$ and $x_j^A = x_j^B$. Only quality is allowed to vary. In a sense this version of the Edgeworth box diagram has become a manifestation of Duesenberry space[10], or what could be called an Edgeworth-Duesenberry box. This diagram can now be employed to address the creation of preference maps of joint-isoquality that are identical for two consumers.

Attainment of Identical Joint-Isoquality Preference Maps

First, given variable quality, attainment of identical joint-isoquality preference maps is essential in order to have a tangency between two consumers in the traditional quantity-oriented Edgeworth box. With the Edgeworth-Duesenberry box it is possible to examine the process of exchange where quality is variable. The approach is analogous to that utilized when quantity is variable, and quality is constant. Essentially, a core concept exists in the variable-quality case. Attainment of a tangency is possible, as in the standard (variable-quantity) case. What is attained, however, at the point of tangency needs further examination.

It should be noted that along any given convex indifference curve in v_iv_j space the MRS now applies to change in the levels of quality, such that the consumer remains on the same indifference curve. The process of attaining a tangency involves voluntary (barter-type) exchange of v_i and v_j between two consumers until a tangency is attained. Once tangency has been attained, no further voluntary trades are possible. (Is this a new Pareto-optimal condition?)

Recall that the standard Edgeworth box says nothing regarding fairness of the distribution along the contract curve; however, if two consumers are to attain identical joint-isoquality preference maps, i.e., where $\langle v_i, v_j \rangle^A = \langle v_i, v_j \rangle^B$, the location of the point of tangency in the Edgeworth-Duesenberry box matters. Such a tangency must occur in the middle of the box, where $v_i^A = v_i^B$ and $v_j^A = v_j^B$, in order to attain two preference maps that have the same levels of joint-isoquality.

Under normal conditions of exchange, where self-interest is involved, what is the likelihood that a tangency will occur in the middle of the box? What will force this type of transaction? There is no reason *a priori* to believe that matching preference maps will be attained. Similarly, there is little likelihood that market-determined prices would cause convergence to the middle tangency. (Especially is this the case if levels of quality reflect "status" in society, i.e., positional goods, etc.) Rather, it appears that normal operation of the neoclassical model of General Equilibrium would result in tangencies anywhere along the contract curve, not necessarily the point of matching joint-isoquality preference maps. Without identical joint-isoquality preference maps, however, it is impossible for the General Equilibrium model to produce tangencies in the quantity-oriented traditional Edgeworth box. In other words, it is more likely that a case of general disequilibrium would arise. It is important to note, however, that this form of disequilibrium might be hidden by the "appearance of tangency," as measured exclusively by quantity of the two commodities.

Production Possibility Frontiers and the Marginal Rate of Transformation

The standard treatment of the production possibility frontier (PPF) in General Equilibrium is to show that, given the same ratio of prices, (p_i/p_j), the marginal rates of transformation (MRT) across firms are all equal to p_i/p_j, and hence equal to each other; furthermore, that since all MRS across consumers are equal to p_i/p_j, and hence equal to each other, it then follows that all MRS are equal to all MRT, or MRS=p_i/p_j=MRT. In the HHD model, however, with the introduction of variable quality and subbudgets, there exists the possibility, indeed the likelihood, that MRT are not equal across firms; and, as previously discussed, MRS likewise may not be equal across consumers; and therefore, it is extremely unlikely that MRT equals MRS.

To illustrate this phenomenon, note that Houthakker prices may be presented as $p_i = (a_{i(BP)} + a_{i(market)}) + b_i v_i$, where $a_{i(BP)}$ is the quantity price of the basic package under a repackaging approach to variable quality; $a_{i(market)}$ represents all other forces that affect the price of the *i-th* commodity, other than forces related to quality; b_i is willingness-to-pay for different levels of quality; and the v_i indicate levels of quality within the *i-th* commodity.[11]. Under Houthakker pricing, change in v_i leads to change in p_i, more specifically, $(\Delta p_i / \Delta v_i) > 0$. Now along the contour of a PPF, consider the effect of a change in p_i that is derived from three different levels of v_i. Define v_i^h as a high level of quality within the *i-th* commodity, v_i^m as a mid-level of quality, and v_i^l as a low level of quality. Given the Houthakker price system described above, the three levels of quality, v_i^h, v_i^m and v_i^l produce the corresponding prices: p_i^h, p_i^m and p_i^l. In other words, the introduction of variable quality allows for more than one price for the *i-th* commodity. The issue, however, is more complicated than multiple prices. The introduction of variable quality requires reconsideration of the production possibility frontier.

In a traditional treatment of the PPF it is implied (under the homogeneous assumption) that quality of the outputs, x_i and x_j, is not variable, i.e., that the level of quality of the *i-th* and *j-th* commodities is constant. Albeit that v_i and v_j are fixed, it is nevertheless possible to define a single PPF contour as a joint-isoquality curve, fixed at some specified levels of quality. In the HHD model, where variable quality is allowed, existence of a fixed PPF curve requires identification of the specific v_i and v_j that correspond to a joint-isoquality production possibility frontier.

To return to the case of v_i^h, v_i^m and v_i^l, consider now three different PPF that reflect the three levels of v_i. Note that v_j has remained fixed throughout this discussion, which means that the intercept for x_j has not changed. See Figure 4. This Figure is based on earlier work by Jack Hirshleifer[12], but any inverse relationship between quantity and quality will likely give the same results.

For each one of the three contours of the PPF in Figure 4 a different level of quality, v_i, corresponds. The outermost curve corresponds to v_i^l and will be identified as PPF(v_i^l). The curve representing v_i^m is identified as PPF(v_i^m). And, the innermost curve, corresponding to the highest level of quality, is identified as PPF(v_i^h). The different contours have different slopes and therefore different marginal rates of transformation. Note that the three contours pivot inward from the x_j intercept as the level of quality is increased. This is in keeping with Hirshleifer. It should also be noted that this is another example of how the consumer and/or the firm can influence the position, and slope, of the constraint, or the PPF.

For each one of the three contours, assume an increase in p_i, which is derived from an increase in $a_{i(market)}$, i.e., the same magnitude of $\Delta a_{i(market)} > 0$ applies to all three PPF. Under the maximization of revenue, where $R = p_i x_i + p_j x_j$, and the slope of R is given by the ratio p_i/p_j, the increase in p_i will increase the slope of R and produce an increase in x_i (and reduction in x_j), i.e., the objective function, R, scallops downward along one of the joint-isoquality PPF discussed above. The MRT is increasing along the PPF as the objective function scallops downward. This is true for each of the three PPF, i.e., $PPF(v_i^l)$, $PPF(v_i^m)$ and $PPF(v_i^h)$.

Under these conditions there is no single value for the ratio, p_i/p_j, but rather there are three such values -- each contingent on the level of quality. The traditional results of general equilibrium, i.e., that MRT are the same across all firms that produce x_i and x_j, does not obtain. Furthermore, the traditional result of MRT=MRS likewise does not occur.

There is a rather limited version of general equilibrium that might arguably still exist. It is a version that does not focus on the total population of consumers, or on the total population of firms. The new version is restricted to the examination of cohorts of consumers with similar characteristics (such as consuming the same level of v_i), and it focuses on cohorts of firms that are likewise restricted to similar characteristics (such as producing similar levels of v_i). In other words, the new definition of general equilibrium has essentially become an expanded version of partial equilibrium.

Creation of Identical Joint-Isoquality PPF

The restriction of equilibrium to cohorts of similar firms, however, must also address the issue of joint-isoquality PPF, and the complications that arise therein.

Similar to the earlier discussion of consumers and an Edgeworth-Duesenberry box that produces a contract curve in $v_i v_j$ consumption space, consider now such a box between two firms in output space. In the case of firms, in order to have tangencies between PPF, (where there would exist equal MRT between two firms), the levels of joint-isoquality between the two firms must be identical, i.e., as it was in the earlier discussion of MRS and consumers. An illustration of the traditional Edgeworth box that involves the PPF of two firms is provided in Figure 5.

In this diagram the PPF establish a tangency such that $MRT^1=MRT^2$, where the superscripts indicate firms 1 and 2. Also shown is the contract line between the two firms. Note that this figure reflects the traditional case of outputs x_i and x_j, or in other words, it is in quantity-of-output space. As previously, consider now the exchange between two firms when quality is variable.

As in the case of consumers, create an Edgeworth-Duesenberry box with v_i and v_j on the axes; this is a quality-quality space. An illustration is provided in Figure 6. In order to obtain identical joint-isoquality production possibility frontiers for the two firms, it is necessary to have tangency of the two PPF in the middle of the $v_i v_j$ Edgeworth-Duesenberry box. Only at that point on the contract curve is it possible to have $v_i^{\#1} = v_i^{\#2}$ and $v_j^{\#1} = v_j^{\#2}$, which establishes two joint-isoquality PPF that are identical in terms of the levels of quality of the two commodities. If it is not possible to attain identical joint-isoquality of the PPF, then a tangency cannot exist in $x_i x_j$ output space, that is, in the traditional space of the Edgeworth box. This appears to be a prerequisite for the Pareto optimality condition that MRT be equal across firms. In other words, identical joint-isoquality production possibility frontiers must exist, and may be viewed as an additional Pareto optimal condition: in that, identical joint-isoquality production possibility frontiers precede the traditional Pareto optimal condition of equal rates of MRT in quantity-of-output space.

Creation of Isoquality Supply

In line with the earlier discussion of revenue maximization under conditions of change in $a_{i(market)}$, consider now the creation of an isoquality supply curve corresponding to each of the three PPF in Figure 4. Consider Figure 7.

In this figure there is a fixed quantity of the *i-th* commodity identified as $\bar{x}_i$. Corresponding to this quantity, for each PPF the MRT, at the point $\bar{x}_i$, is different. Consequently, at the point $\bar{x}_i$ the slope of *R*, $R = p_i x_i + p_j x_j$, is different, where the slope is given by p_i/p_j. The flatter slope of *R* on $PPF(v_i^l)$ corresponds to the lower price, p_i^l, which corresponds to the lower quality, v_i^l, and so forth for the other PPF. Recall that p_j is held constant and that p_i varies due to a change in $a_{i(market)}$. For example, as $a_{i(market)}$ is increased, the supply of x_i, along each PPF, is increased.

In Figure 8 the quantity of x_i is also held constant at $\bar{x}_i$. The three dots on the three supply curves correspond to the ordering of prices: p_i^h, p_i^m and p_i^l, which correspond to the ordering: v_i^h, v_i^m and v_i^l. Given change in the quantity, $\bar{x}_i$, to some lower or higher quantity, the previous ordering does not change; therefore, through this process it is possible to trace out three supply curves. Each curve is of constant quality, with the supply curve of highest quality above the supply curve of mid-level quality, and so forth. (Parenthetically, any supply curve that intersected the three isoquality supply curves, may be classified as a variable-quality supply curve. See Wadman 2000, p. 239, for a similar result involving a variable-quality demand curve intersecting isoquality demand curves.[13])

New Aspects of Equilibrium when Quality is Variable

With the three isoquality supply curves it is now possible to introduce three isoquality demand curves[14] and consider the concept of partial equilibrium in a world of variable quality. Consider Figure 9. In this Figure three dots have been used to identify the intersection of matching levels of quality between a supply curve and a demand curve. Specifically, $S(v_i^h) = D(v_i^h)$, where v_i is identical for both curves, i.e., v_i^h is the same level of quality for both supply and demand. A similar matching of quality levels applies to $S(v_i^m) = D(v_i^m)$ and $S(v_i^l) = D(v_i^l)$. Caution should be exercised to not confuse supply and demand curves that "overlap" with an "intersection" of supply and demand. The appearance of overlap occurs when quality levels do not match; for example,

$S(v_i^m) \neq D(v_i^l)$, therefore this is not an intersection, but rather an overlap of the two curves. Arguably (see Figure 8), the higher level of quality resides above the lower quality curve. These conditions, obviously, introduce the need for a further examination of the concept of equilibrium. They also introduce three-dimensional surfaces for both demand and supply. The third dimension arises through the introduction of a v_i axis orthogonal to the origin in x_i, p_i space.

Equilibrium, in traditional models of partial equilibrium, is typically defined as $x_i^D = x_i^S$, where x_i^D represents quantity demanded of the *i-th* commodity, and x_i^S is quantity supplied of the *i-th* commodity. There now exists, however, the prospect that equilibrium also entails the condition: $v_i^D = v_i^S$, where v_i^D is the level of quality of the *i-th* commodity demanded, and v_i^S is the level of quality supplied. The HHD model introduces the prospect (indeed the likelihood) that $v_i^D \neq v_i^S$. As such, there now exists a new dimension to the concept of disequilibrium. This is, however, an old problem. It was first considered in any detail by Hans Brems[15]

Given the circumstances portrayed in Figure 9, there arises the need to examine the existence of transactions between buyers and sellers, and the conditions under which transactions might breakdown, or be reduced in number. Variability of quality across consumers and producers also opens the prospect that equilibrium be viewed as a measure of central tendency, and, as with other measures of central tendency, requires consideration of variance around the point of central tendency. At what distance from the point of central tendency do transactions fade or become statistically insignificant? In other words, how large is the variance for statistically significant transactions, etc? In order to address transactions, under conditions of variable quality, it is useful to consider the existence of quality neighborhoods, quantity neighborhoods, windows, transaction regions, offer prices and acceptable prices, and so forth.[16]

Clearly, the introduction of variable quality, and the introduction of isoquality supply and isoquality demand, along with the realization that the crossing of supply and demand curves may be an overlap of the curves and not an intersection, begs that greater attention to be given to the definition and measurement of equilibrium. This subject matter will affect econometric analyses of output markets, and possibly all markets, i.e., variable quality may be forced into the error term.

Isoquants and Cost Constraints in Input Space under Conditions of Variable Quality and Sub-Budgeting

As with consumer indifference curves and household budget constraints in consumption space, examine now isoquants and cost constraints for firms in input space. Begin with examination of the firm cost constraint.

Assume the existence of $C = p_l L + p_k K$, where C is cost, p_l is the price of labor (e.g., per hour), L is the quantity of labor time, p_k is the price of capital (e.g., per hour), and K is the quantity of capital time. Rearrangement gives $K = (C / p_k) - (p_l / p_k)L$, where the ratio, $-(p_l / p_k)$, is the traditional slope of the constraint in quantity-of-inputs space. As in the case of the consumer, construct now a new input space based on the use of subbudgets by the firm; specifically, assume that firms allocate a portion of their total budget to labor cost and (separately) to capital cost. Identify the subbudget for labor as C_l, and the subbudget for capital as C_k, where total cost is given by $C = C_l + C_k$.

Consider now the intercepts in input space. Traditionally, for the labor axis the intercept is, C / p_l, and similarly for capital the intercept is, C / p_k. In the HHD model, however, the labor intercept is C_l / p_l and the intercept for capital is C_k / p_k.[17] With these two new intercepts define the distance from the origin to the intercept on the capital axis as the "rise," and the distance from the origin to the intercept on the labor axis as the "run." The ratio of these two intercepts provides a new measure of slope of the cost constraint, or $-(C_k / p_k)/(C_l / p_l)$. Following the same procedure as previously, note that in the traditional case the slope is equal to, $-(C / p_k)/(C / p_l)$, which after cancellation of the total cost, *C*, gives $-(p_l / p_k)$ as the slope. Under subbudgeting, however, where C_k and C_l may not cancel, there is, $-(C_k / p_k)/(C_l / p_l)$, or equivalently $-(C_k / C_l)(p_l / p_k)$, which has become the slope.

Now, as before with consumers, a change by the firm in either subbudget will change the slope of the constraint. For example, assume an increase in total cost, C, which is allocated exclusively to an increase in C_k. The increase in C_k will increase the slope of the constraint, and given traditional convex isoquants, the increase in constraint slope will result in an increase in the use of capital and a reduction in the use of labor. As with consumers, the change in subbudgets may be unique for each firm and, therefore, the marginal rates of technical substitution (MRTS) may differ across firms. Significantly, the traditional general equilibrium solution of, $MRTS^1 = MRTS^2$, and so forth for all firms, may not obtain, unless the ratio of subbudgets is the same across firms.

Identical Joint-Isoquality Isoquants

Another complication, aside from the cost constraint of the firm, is the possibility that MRTS do not have tangencies in an Edgeworth box diagram. This problem arises when the joint levels of input quality differ between two firms.[18] As was discussed earlier in regards to MRS across consumers and MRT across firms, the tangency of MRTS requires that the firms have isoquants that match in terms of the levels of quality. Specifically, the isoquants must have identical levels of joint-isoquality of the inputs. Previously, an Edgeworth-Duesenberry box was employed to illustrate the process of attaining identical levels of joint-isoquality. This process must now be repeated for isoquants.

In order for there to be an Edgeworth-Duesenberry box in the present study, it must be possible to construct an isoquant in quality-quality input space. Recall that isoquants are traditionally constructed in quantity-quantity input space. At this juncture, assume that firms can conceive of a tradeoff relationship between levels of quality: in the present case, between the level of quality of labor, v_l, and the level of quality of capital, v_k. Given limited resources, and a given state of technology, it does not seem unreasonable to assume that, for some constant quantity (and quality) of output, an increase in the quality of capital requires a decrease in the quality of labor, or *vice versa*.

Given the existence of isoquants in a quality-quality space, assume further that they are convex.[19] With this as background, the creation of an Edgeworth-Duesenberry box is possible and the locus of tangencies would create a corresponding contract curve. As previously, in order to assure identical joint-isoquality isoquants, the only acceptable point on the contract curve is in the middle of the box, where, e.g., $v_l^1 = v_l^2$ and $v_k^1 = v_k^2$, and where superscripts 1 and 2 indicate different firms.

As previously, it could be argued that creation of identical joint-isoquality isoquants is a Pareto optimal condition, and as such, the existence of identical joint-isoquality isoquants becomes a condition that precedes the traditional condition that MRTS (measured in quantities) are equal across firms. It should also be noted that these results hold without the issue identified earlier of subbudget decision making and the slope of the cost constraint.

The Soft Constraint and New Problems for Input-Cost Minimization

In consideration of two inputs, labor and capital, the traditional solution to input-cost minimization is given by, $(MP_l / p_l) = (MP_k / p_k)$, i.e., tangency of an isoquant with the iso-cost constraint gives $(p_l / p_k) = (MP_l / MP_k)$, which rearranges to the previous result. Under conditions of subbudgeting, however, where the slope of the constraint is $-(C_k / p_k)/(C_l / p_l)$, which rearranges to $-(C_k / p_k)(p_l / C_l)$, and ultimately becomes $[(C_k / C_l)(p_l / p_k)] = (MP_l / MP_k)$, input-cost minimization becomes $[(C_k p_l)/(C_l p_k)] = (MP_l / MP_k)$. Under this condition, the slope of the constraint can change without change in the ratio of prices, but rather due to change in the ratio of subbudgets (as explained earlier). All of this can occur under the traditional case, where it is implicitly assumed that identical joint-isoquality isoquants exist.

Now input-cost minimization becomes: $[MP_l/(C_k p_l)] = [MP_k/(C_l p_k)]$. Previously, an increase in p_l would lead to an increase in capital, *K*, and a decrease in labor, *L*. Now an increase in the subbudget for capital, C_k, will produce the same result. Furthermore, a decrease in C_l is equivalent to an increase in p_l. Clearly, in the HHD model, management decisions over subbudgets can have the same allocation effect (e.g., between labor and capital) as can a change in market-determined prices for labor and capital. Allocation issues are no longer determined exclusively by prices; they are also determined by management subbudget decisions, i.e., allocation decisions are no longer the product of impartial market forces. (As is well understood, management decisions are also influenced by relative levels of taxation, which are not argued to be determined by market forces.)

If the ratio of subbudgets remains constant, e.g., $(C_k / C_l) = \phi$, then the role of prices becomes more influential on allocation decisions, and in the extreme may approach the traditional model. Also important in this matter, however, are the growth rates of the MP_k and MP_l.

Change in the ratio of marginal productivities will change the shape and position of a (quantity-based) isoquant. At this juncture, the focus is on variable quality as applied to inputs, and it addresses the questions: How might change in input quality affect configuration of the isoquant? In a related manner, what is the relationship between Δv_l and ΔMP_l; and between Δv_k and ΔMP_k? And, as applied to labor and capital, what is the speed of learning as between labor and capital? Specifically, how fast can labor learn relative to the speed of learning by capital (technology)? Speed of learning most likely has a bearing on how quickly labor productivity can improve, and likewise, on how quickly the productivity of capital can improve, i.e., implicitly $(\Delta mp_l / \Delta v_l) > 0$ and likewise for capital. (These issues fall within Brems' concept of process improvement.[20]) In other words, change in the quality of labor and capital affect the shape and position of the (quantity-based) isoquant. Furthermore, any understanding of these learning relationships also depends on the relationship of labor to capital; specifically, is their relationship complementary or is it substitution? If the current trend is toward a substitution relationship between labor and capital, one should expect greater use of technology in the long-run (and more human unemployment).

The traditional long-term cost-minimization solution, $(MP_l / p_l) = (MP_k / p_k)$, given the increasingly rapid improvement of MP_k, accompanied by a pattern of decrease in p_k, essentially confronts human labor with the prospect that p_l must fall, and possibly in the extreme approach zero (i.e., unemployment again).

How extensively and intensively can the concepts of technological change and variable quality be applied to humans?[21] Will human workers be able to keep up with the rate of technological change and improved quality of technology? Are modern economies in the early stage of an extreme form of economic evolution (or punctuated equilibrium), particularly as manifest in labor markets? Are such markets approaching a "far-from-equilibrium" state, as introduced by Ilya Prigogine and others[22]? Do contemporary forms of economic growth arise from unstable (or nonequilibrium) states in markets, where the far-from-equilibrium states are created and driven by rapid improvement in technology? Does rapid improvement of quality play a role in the creation of far-from-equilibrium states? Given the accelerating pace of technological change, the conditions described above regarding labor markets may also apply to other markets, and the existence of far-from-equilibrium states may likewise arise in those markets.

In economics, far-from-equilibrium states may arise either under conditions of partial or general equilibria. As this paper has shown, however, attainment of general equilibrium appears extremely unlikely. The state of general disequilibrium seems far more likely, and may correspond to a multitude of far-from-general-equilibrium market states. As part of general disequilibrium, far-from-equilibrium states may arise within individual markets, where each market is analyzed under the assumptions of partial equilibrium. However, there may also exist interaction-effects between two or more far-from-partial-equilibrium market states, and it is conceivable that researchers are unaware of these effects.

Notes

1. On Kornai and Roland, see , Kornai 1959, 1980 and Kornai, Mátyás and Roland 2008; and Roland 2008.

2. See Houthakker, and Wadman 2000, Chapter 4. See also, Duesenberry, and Wadman 2000, Chapter 3. In many respects, Duncan Ironmonger was pursuing an approach similar to Duesenberry. For more information on Ironmonger, see Ironmonger, and Wadman 2000, Chapter 6, pp. 62-77. On the four-space diagram, see, Wadman 2009, Chapter 1.

3. On the open subspace in Hicksian space, see Wadman 2000, Chapter 4, pp.47-48 and Wadman 2009, Chapter 1, endnote 21.

4. For more information on joint-isoquality constraints and indifference maps, see Wadman 2000, Chapters 14 and 15, and Wadman 2009, Chapter 5.

5. Note that movement along a constraint requires a change in subbudgets. On the issue of a change in slope and movement along the constraint, note that this behavior of the constraint is related to the issue of a lump-sum tax. When consumers control subbudgets they control the slope of their budget constraint, and therefore also control the distribution of the burden of the a lump-sum tax. On lump-sum taxation, see Rosen and Tresch.

6. On General Equilibrium, see, Kuenne, Quirk and Saposnik, and Varian. See also, Henderson and Quandt, Chapters 5 and 7; and Nicholson, Chapters 18 and 19. As applied to welfare economics and public finance in general, see Tresch, and Rosen. For another approach to consumer theory and its impact on general equilibrium and welfare economics, see Steedman, and see Wadman 2002.

7. For a different view on the relationship of quality and price, see the "widgets in a box" argument in Fisher and Shell 1971, and see Wadman 2000, chapter 6, pp. 81-84. Note that the results presented in this paper are different from the widgets approach, i.e., in the widgets case, an increase in quality is seen as equivalent to a reduction in price, and therefore reflected in downward adjustments in price indices.
12. For further information on Houthakker prices, see Wadman 2000, Chapters 4 and 14.

8. Compare these results with Figure 14.6 in Wadman 2000, where a change in price is involved. See Wadman 2000, Chapter 14, Section 8 for more on the argument that adjustment of subbudgets may be a more accurate method for adjustment of price indices. It should be noted, however, that there may be practical, empirical and political problems associated with this form of price index adjustment.

9. Empirical support for this condition is at least implied in some of the early work by John Muellbauer. See Muellbauer, p. 980 and Wadman 2000, p. 87. Note that difference between income classes (or difference between income cohorts) now appear to matter in neo-classical economics. Traditionally, under identically sloped constraints, (p_i/p_j), income did not influence the rates of MRS, etc.

10. See Wadman 2000, Chapter 3, pp. 28-32.

11. See Wadman 2000, Chapter 14, Section 3.

12. See Hirshleifer, and Wadman 2000, Chapter 5.

13. For more on variable-quality demand, see Wadman 2000, pp. 238-239, especially Figure 14.34. See also Figure 9 in this paper.

14. On three isoquality demand curves, see Wadman 2000, p. 238.

15. Frederick Waugh came close to recognizing this problem. See Brems and Waugh. See also Wadman 2000, pp. 11-18.

16. On neighborhoods, windows and transaction regions, see Wadman 2000, chapters 11, 12 and 14, sections 3-4. For more information on equilibrium and the use of inequalities between consumers and producers, see Ulrich Witt's concepts of corridors and trajectories in Witt, 1985. In addition, note the following quote regarding John von Neumann and the use of inequalities, instead of equalities, in the context of economic equilibrium. This information was provided by Jacob Marschak:

> Yes, I remember those encounters in Berlin, 1926, most vividly. Of the participants, I cannot identify the Indian physicist. The others were: yourself, Szilard, Wigner, Neumann. We were sitting at an oval table and I recall how v Neumann was thinking aloud while running around the table. And I remember the issue. I was talking, in the "classical" Marshallian manner, of the demand and supply equations. Neumann got up and ran around the table, saying: "You can't mean this; you must mean inequalities, not equations. For a given amount of a good, the buyer will ofeer [sic] *at most* such-and-such price, the seller will ask *at least* such-and-such price!" This was (later?) pointed out by another mathematician, Abraham Wald, perhaps in the "Menger Semiar [sic]" in Vienna, and certainly in 1940 in USA.

This material is found in Philip Mirowski, *Machine Dreams*, p. 102. For more on the use of inequalities and their importance in transactions between consumers and producers, see the above chapters in Wadman 2000.

17. See portions of Chapter 5 in Wadman 2009 for a review of this topic; specifically, see pp. 92-94. See also Chapter 1 for a discussion of the reconstructed Hicksian consumption space.

18. Assume at this juncture that the level of quality of the output is constant and at the same level for both firms. Note that this is not joint-isoquality, because only one output is produced in the traditional input space diagram.

19. This is a rather complicated matter. Convexity of an isoquant in quality-quality input space is a new concept and needs further analysis. For present purposes, assume the isoquant is convex and otherwise behaves as a traditional isoquant.

20. On a related topic, see Wadman 2001. See also Brems, 1959 and Wadman 2000, p. 18.

21. See, again, Brems on the concept of process improvement as a manifestation of technological change. See also Wadman 2001.

22. See Nicolis and Prigogine, 1977, Chapters 1, 17, 18 and pp. 464-74; Prigogine, 1980; and Nicolis and Prigogine, 1989. For a more contemporary treatment, see Kauffman, especially pages 387-93. For an extensive background in economic evolution, see Witt, 2003. On punctuated equilibrium, see Dennett, pp. 282-312.

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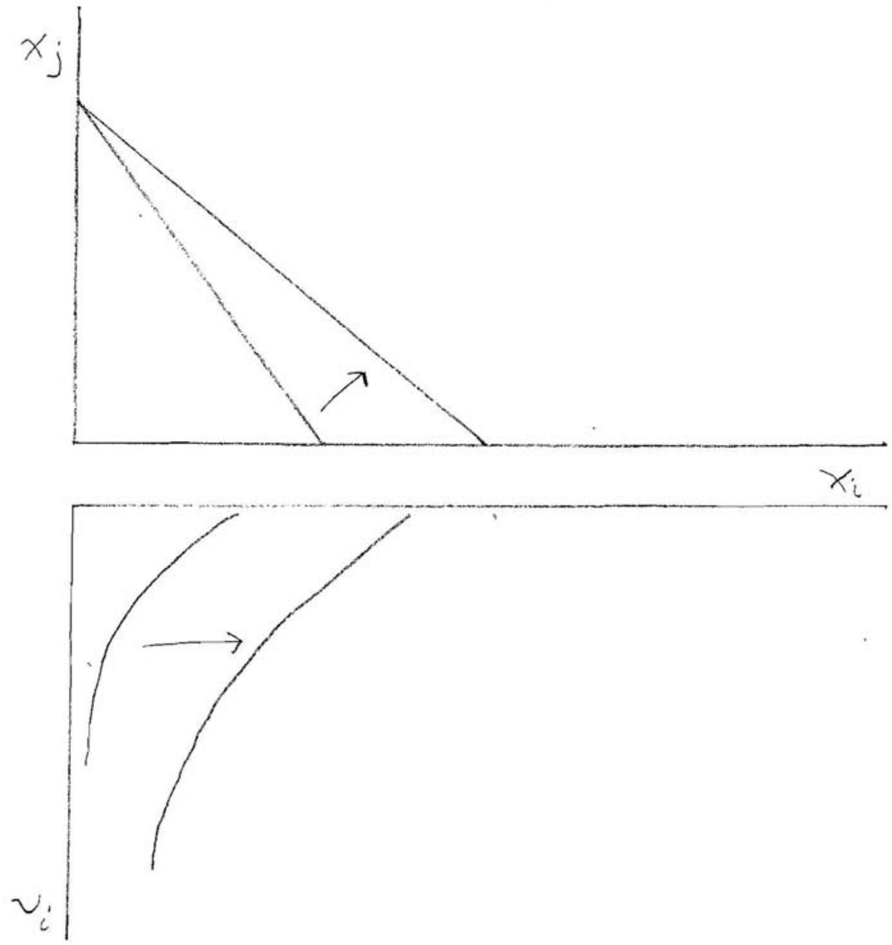
Figure 1



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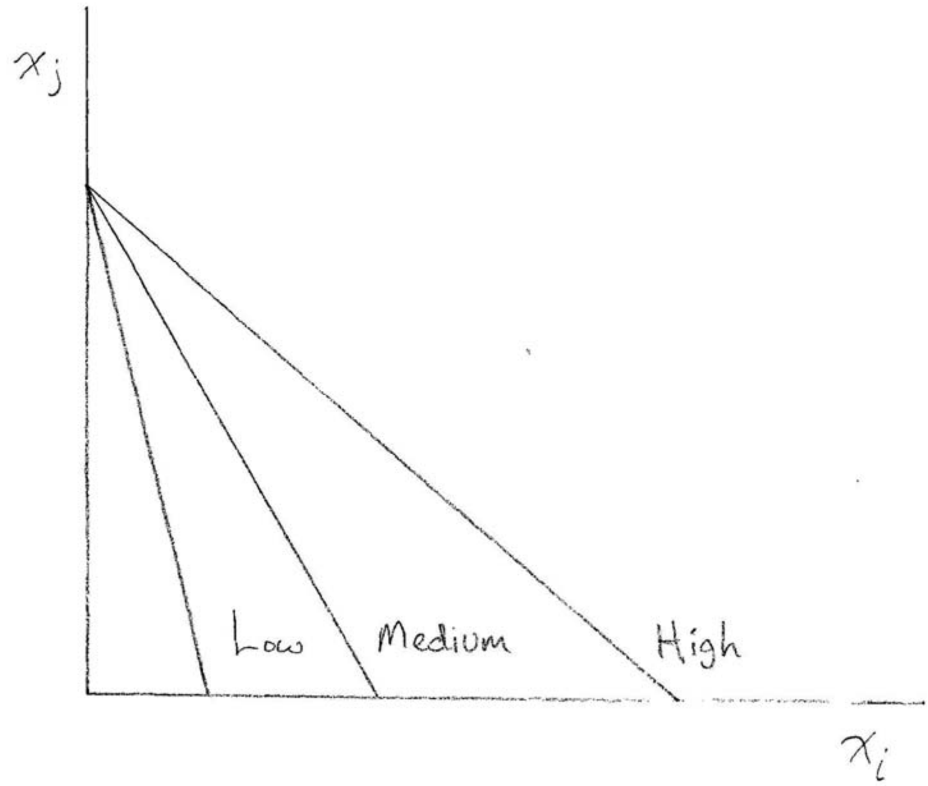

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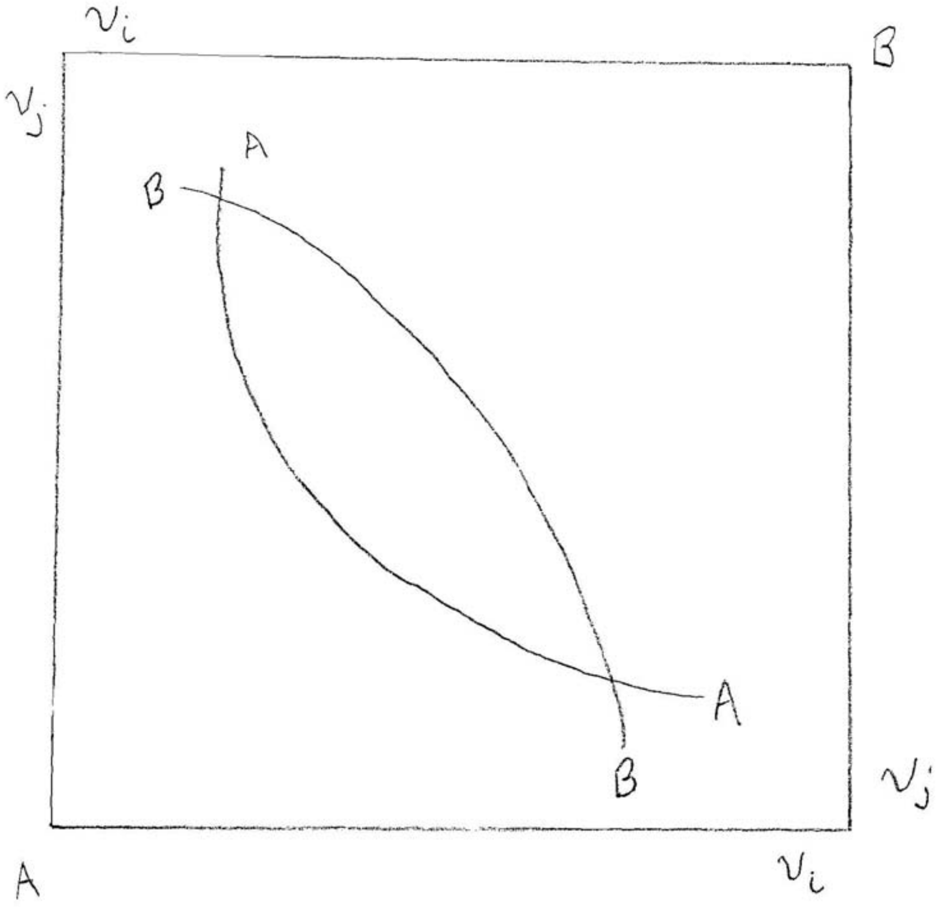

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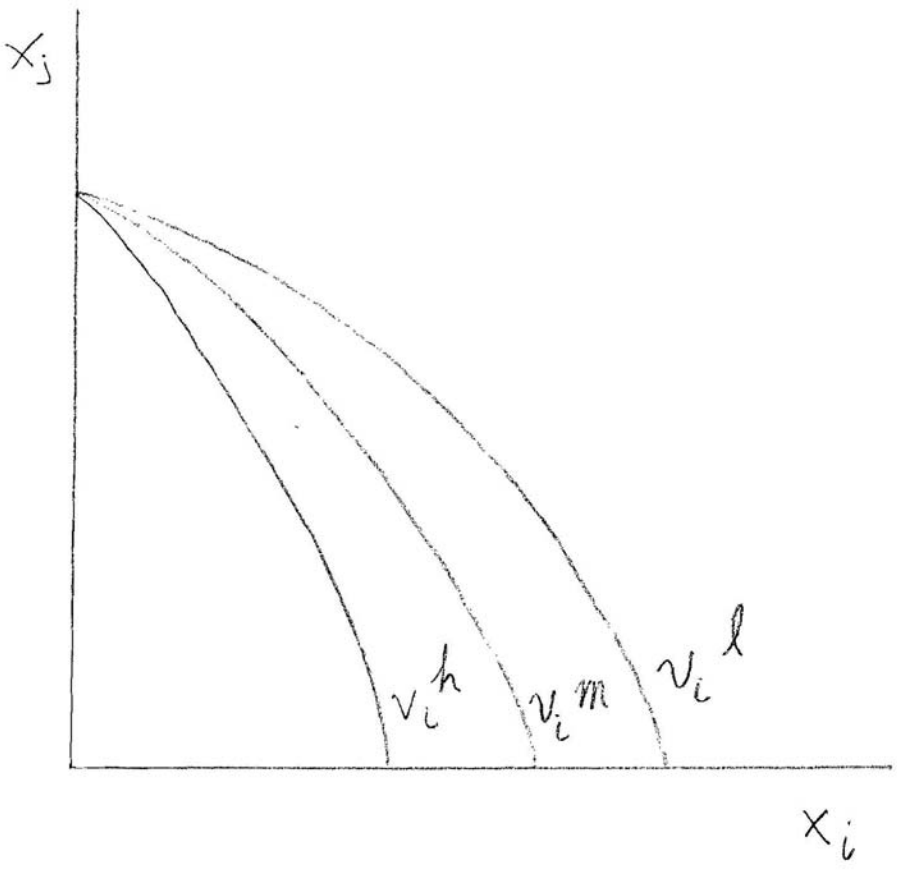

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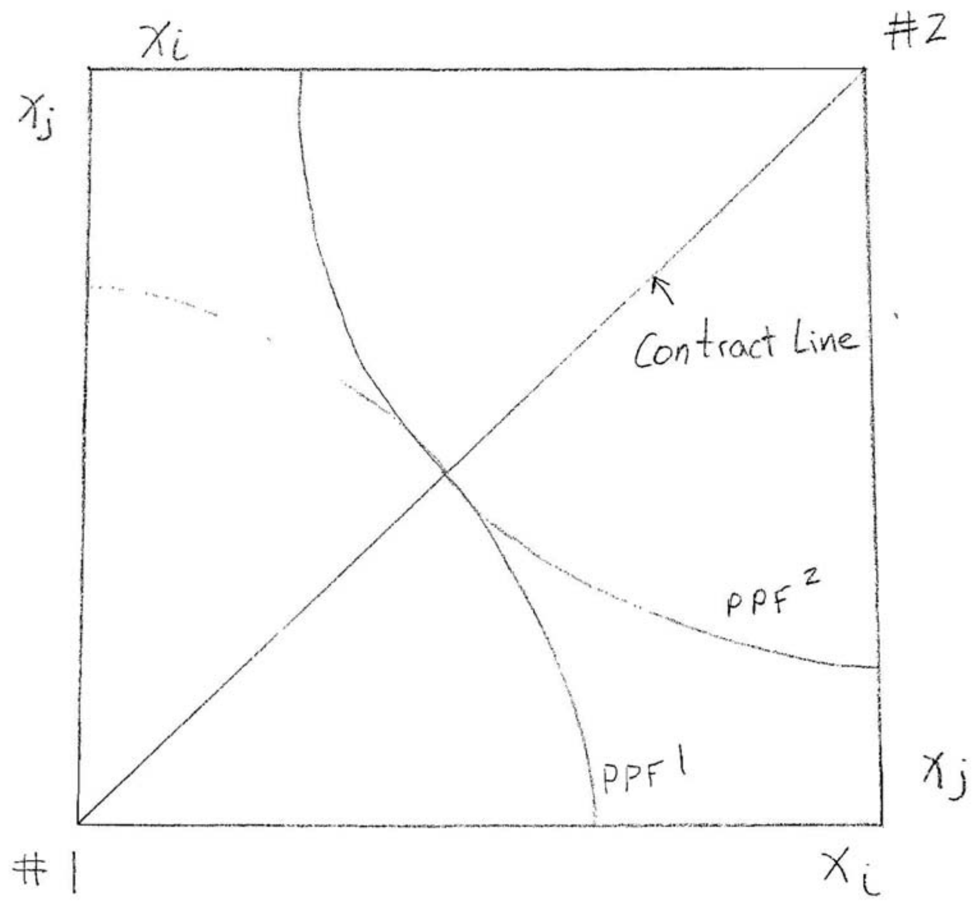

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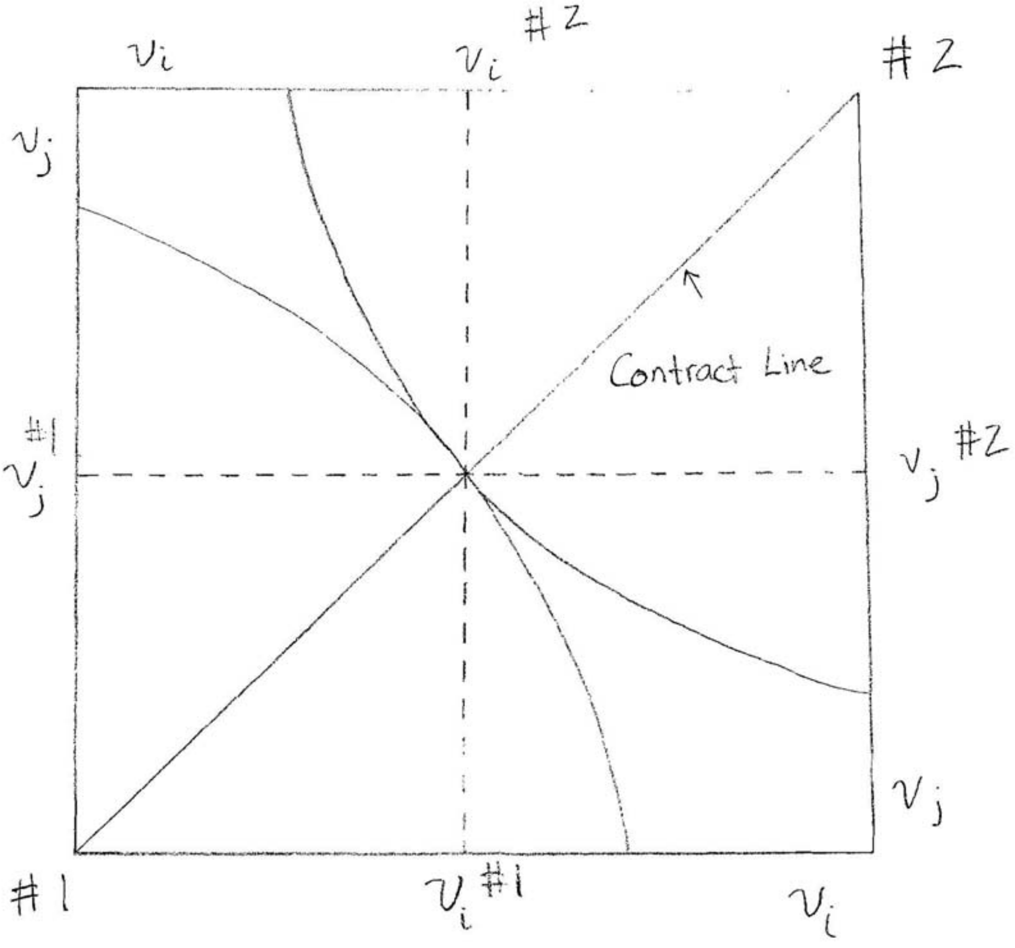

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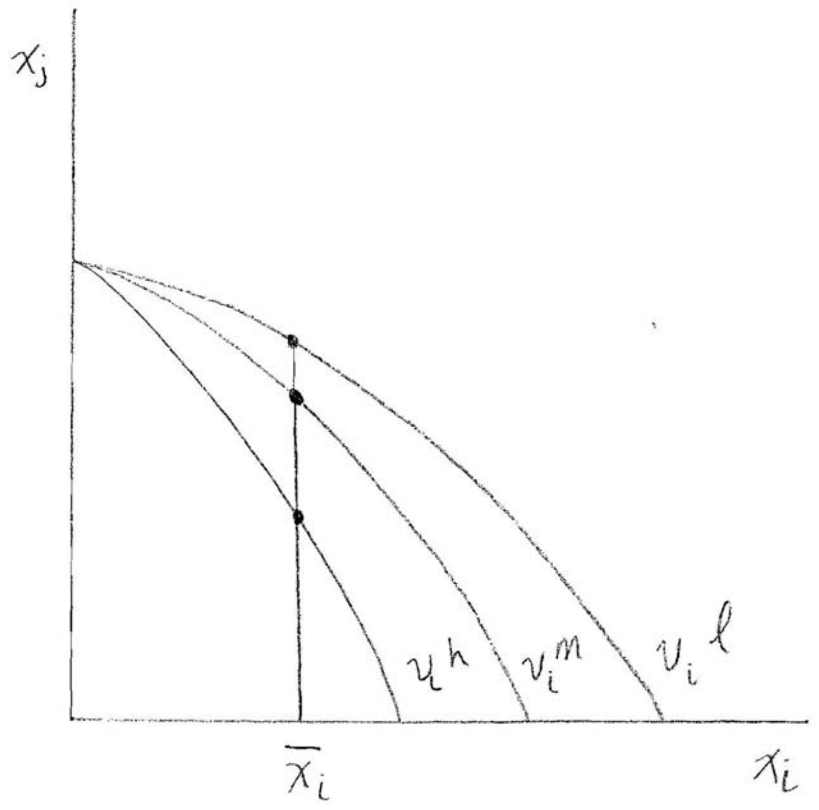

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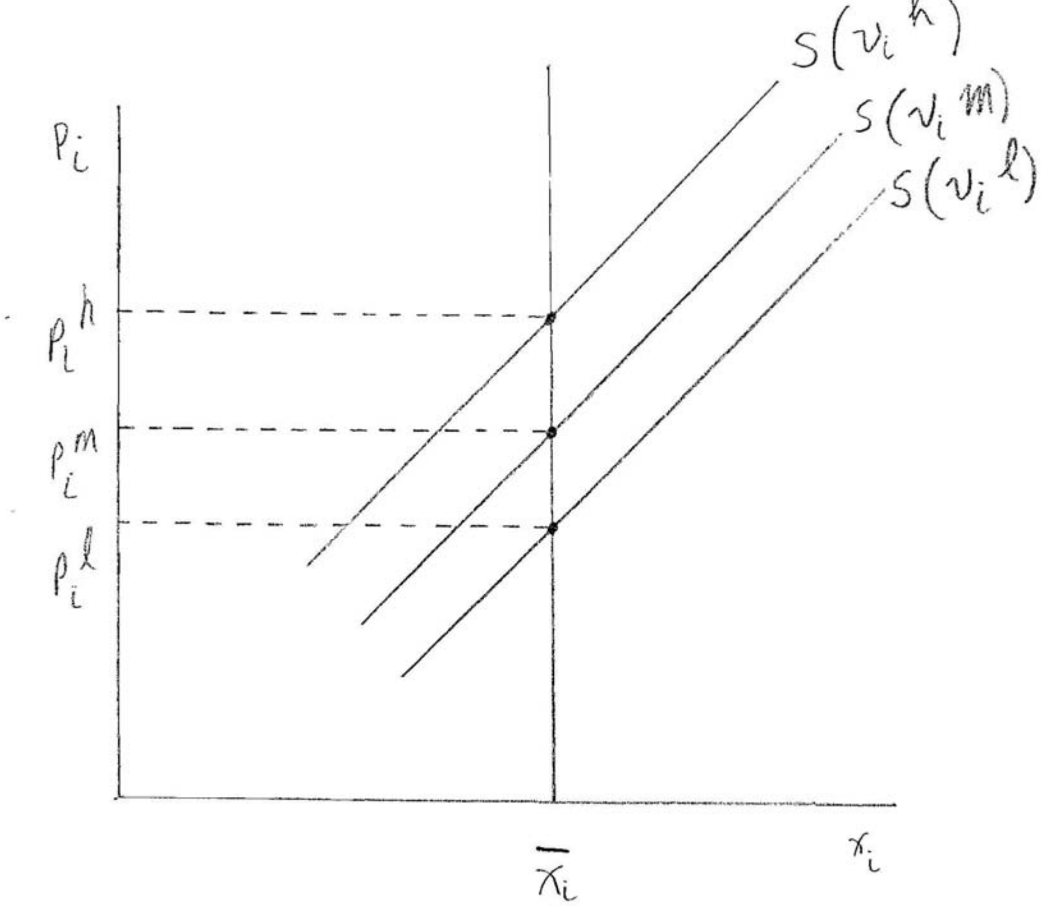

Figure 9