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On reciprocal Behavior in Prisoner Dilemma game*

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Abstract. In this paper, we introduce the concept of *payoff distortion* in the standard prisoner's dilemma game when strategies are driven by psychological behaviors. This concept enables to take account each player's assessment of the other player's behavior and the asymmetry of information. We determine the conditions which allow that mutual cooperation constitutes the equilibrium. we particularly focus on the reciprocity in case of complete and incomplete information about the payoff distortion. We show that mutual cooperation is a Nash equilibrium with complete information and is a Bayesian equilibrium when each player believes that his opponent behaves with "large" reciprocity in incomplete information environment.

Keywords: Reciprocity, Behavior, Cooperation, prisoner's dilemma game.
JEL Classification : *C7, A13*

1 Introduction

The purpose of this paper is to determine in which cases cooperation can constitute the equilibrium of the prisoner's dilemma game when each player builds his choice on the other's behavior as well as on his own behavior. The concept of "behavior" is here totally linked with the psychological dimension of choices.

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In the non cooperative approach of the game theory, experimental exercises have revealed that players do not often reach the solution predicted by the theory (see for example Dawes and Thaler (1988)). In the particular framework of the prisoner's dilemma game, two explanations are proposed to argue that cooperation can emerge as the solution of the game. The repeated game theory (see Kreps and al. (1982)) is the first explanation. In the repeated games models, the assumption that each agent believes even weakly in the possibility that the other agents play cooperatively is crucial. The assumption of incomplete information combined with repeated shots imply incentives to cooperate. In such a framework, the players are supposed to be self-interested. The second explanation stipulates that all agents are not self-interested and that at least one part of them is motivated by the other agents' welfare in a framework of social interactions. Starting from this idea, two alternative approaches are suggested.

The first is based on the idea that at least some of the agents have 'social preferences', or more precisely, a utility function which depends on their own material utility but at the same time, on the payoffs obtained by the other players. This approach, as mentioned by Falk and Fischbacher (2002), only takes into account the players' objectives and not their intentions. A second approach called "reciprocity approach" based on intentions has been proposed. In this approach, one player's decision is influenced by the intentions of the other players. If this player estimates that the others will not play cooperatively, he will not make any effort either. This idea constitutes the background of the "fairness" equilibrium concept developed by Rabin (1993). This is the first contribution which presents the concept of reciprocity and analyzes the consequences of reciprocal behavior. A large number of articles have pointed out the existence of reciprocal behavior (see Fehr and Schmidt (2003) for a survey), altruist and selfish behaviors being sometimes considered as limit cases of reciprocal behavior. Then reciprocity appears as an individual psychological property which crucially alters the characteristics of the welfare.

The occurrence of cooperation in experimental games is always linked to the players' psychology and its influence on the payoff utilities: a player does not always appreciate a payoff similar to his opponent's. The choice of one player's strategy is totally determined by the payoff distortion of each player. In Mauroy (2002), the payoff distortion consists in a strictly increasing affine transformation of the scale of the payoffs, which characterizes the players' psychology and does not alter the game. Our article introduces an alternative way of modeling the payoff distortion. Note that the payoff distortion is not systematically common knowledge and depends on each player's psychology. The distinction between a complete information game and an incomplete information game is then essential. In the second type of game, the other players' payoff distortions are unknown or not completely known for at least one player.

In a standard prisoner's dilemma game, players base their choice exclusively on the payoff they obtain from these choices. In our article, the player determines his choice according to the following three elements: the states of the world, the actions, and the consequences on which he personally evaluates the probability of occurrence of each state of the world. In a context with uncertainty, it is straightforward to assume that an agent who interacts with others explains the other players' choices by his own beliefs. In a social interaction, an agent expresses his preferences on a set of alternatives by choosing an action which can be the outcome of elements which result from the belief about his own incentives as well as about those of the other players.

By the way, agents' beliefs are modified by the psychological behavior of each player. We suggest an alternative way of modeling reciprocity, keeping in mind that the idea of reciprocity is based on the players' intentions. Whereas the standard prisoner's dilemma game equilibria are based on an assessment of each player's utility, we introduce in this article the concept of "behavioral game" which differs from the "payoff games". Indeed, in a "payoff game", the agents, assumed to be rational, aim to maximize their material utility whereas in our model, each player's choice is assimilated to his "behavioral choice" and the probability to cooperate is linked to the adopted behavior. One agent's behavior is then assimilated to his degree of "reciprocity" towards the other agent. Moreover, our approach also differs from Rabin's in which both the material utility and the notion of fairness are combined to extract the equilibrium of the game. In our framework, we show that cooperation can emerge as a result of the game under some conditions which depend on the level of the reciprocal behavior of the players. Even if this result does not seem so surprising with complete information, we develop further our analysis and show the relationship between the degree of cooperation and the beliefs about the players' cooperation in the case of incomplete information.

Section 2 is devoted to the analysis of the payoff distortion according to the belief about the choices of the other player in the prisoner's dilemma game. The third section presents the reciprocity concept adopted in the paper and its consequences in both cases of complete and incomplete information.

2 Players' beliefs and behaviors

2.1 Behaviors

Traditionally, in the prisoner's dilemma game, a player has only two alternative choices: cooperate or defect. Axelrod (1984) has shown that the occurrence of cooperation is strictly linked with the existence of reciprocal

behaviors. One of his suggestions is that reciprocity can result either in cooperation or in defection.
Recent articles have shown the possibility to widen the standard game by allowing a variable degree of cooperation (Robert and Sherratt (1998); Wahl and Nowak (1999)).
In this section, we consider three main behaviors: **altruism**, **reciprocity** and **selfishness** for the following prisoner's dilemma game:

1/2	C^2	D^2
C^1	a, a	b, c
D^1	c, b	d, d

with $c > a > d > b$ and $2a > c + b$.

- **Altruism (Al)**

When altruist, the player chooses to cooperate whatever the choice of the other player. His probability of cooperation is:

$$p(C^i/C^j) = 1 \forall p(C^j) \Rightarrow p(D^i/C^j) = 0 \; \forall i, j$$

In a game with two altruist players, there exists only one Nash equilibrium (C, C) and the payoff is (a, a) for both players.

- **Reciprocity (R)**

According to the strict definition of reciprocity, a reciprocal player chooses a strategy strictly similar to that of the other player. In that sense, player i cooperates with a probability identical to the probability of cooperation of the other player. Then $p(C^i) = p(C^j) = p$ and $p(D^i) = p(D^j) = (1 - p)$. The distorted payoffs of two reciprocal players are given by the following matrix:

$$\begin{pmatrix} p.a, p.a & p.b, (1-p)c \\ (1-p)c, p.b & (1-p).d, (1-p).d \end{pmatrix}$$

The expected payoff of each player is:

$$r(p) = p[p.a+(1-p)c]+(1-p)[p.b+(1-p).d] = p^2.a+p(1-p)(c+b)+(1-p)^2.d \quad (1)$$

- **Selfishness (S)**

When selfishness is assumed, each player prefers to defect unconditionally.

$$p(C^i/C^j) = 0 \forall p(C^j) \Rightarrow p(D^i/C^j) = 1 \ \forall i, j$$

When both players are selfish, defection is the only possible profile of strategies and (D, D) is the unique Nash equilibrium of the game.

2.2 Behavioral analysis

In each case, we were able to determine, according to the behavior of the players, the matrix of the distorted payoffs. According to the information revealed to each player on the probability of cooperation of the other player, player i behaves with probability $p(C^i/C^j)$. This information can be, for example, based on previous interactions that the player has faced in the past. Player i can objectively derive this probability by estimating the behavior of his opponent during similar experiences. His action reflects his psychology towards this information.

When considering complete information, the matrix of the distorted payoffs is common knowledge for both players. In the opposite case, the information is quasi-complete and the choice is ambiguous.

In a first step, we still consider a framework with complete information in which each player knows the structure of the game. Taking account of the three behaviors just presented above and of the distorted payoffs, the interactions between both players can be described by the following matrix 3×3:

1/2	Al	R	S
Al	a, a	a, a	b, c
R	a, a	$r(p), r(p)$	d, d
S	c, b	d, d	d, d

Table 2

A selfish player obtains a payoff d if the other player is also selfish or reciprocal, a payoff c if the other player is altruist. Conversely, an altruist player receives a if the other player is altruist or reciprocal and b if the other player is selfish.

We are then able to rank the gains of the game according to the different behaviors that we consider.

Proposition 1 *The ranking of payoffs, for $0 < p < 1$, can be given as follows:*
if $2.d \leq (b+c)$ then, $a > r(p) > d$
if $2.d > (b+c)$ then, $a > r(p) > d$ if $p \in]p^*, 1[$ with $p^* = \frac{2.d-c-b}{a+d-c-b}$. Otherwise $a > d \geq r(p)$.

Proof. see Appendix A

According to this ranking, it appears that the result of a game between two non-selfish players and at least one altruist, is better than the gain obtained by two players who both adopt a reciprocal behavior.

These results mean that reciprocal behavior is beneficial for the players if the gains from common defection are quite low compared to the gains from defection from one player and cooperation from the other one. In the opposite case, the player's belief about the cooperation of the other player must be high enough.

2.3 Behaviors and social interactions

When an agent moves in a social framework, his payoff is the result of the interactions with all other individuals according to the distribution of the behaviors in the population. The purpose of this section is to show the necessary conditions which allow the reciprocal behavior to dominate the other ones.

Let q_1 denote the proportion of altruist agents; q_2, the proportion of individuals with a reciprocal behavior and $1 - q_1 - q_2$, the proportion of selfish agents. The "state of the population" is described by the vector $q = (q_1, q_2, 1 - q_1 - q_2)$.
Let $\mathbf{E}(\daleth, q)$ denote the expected value of the variable $\daleth$ when considering the state of the population q. In our case, the variable $\daleth$ characterizes the players' behavior. According to Table 1 we have:

$$\mathbf{E}(Al, q) = q_1(a-b) + q_2(a-b) + b$$
$$\mathbf{E}(R, q) = q_1(a-d) + q_2(r(p)-d) + d$$
$$\mathbf{E}(S, q) = q_1(c-d) + d$$

Proposition 2 *Assume that $p \neq 1$, there exists $\widetilde{p} < 1$ and $\widehat{p} < 1$ such that:*
i) Reciprocity dominates altruism if and only if $p > \widetilde{p}$.
ii) Reciprocity dominates selfishness if and only if $\frac{q_1}{q_2} < \frac{(a-d)}{(c-a)}$ and $p > \widehat{p}$.

Proof. see Appendix B

3 Reciprocity

In this section, our analysis focuses on the reciprocal behavior of an agent. In doing so, we adopt a larger definition of reciprocity than the definition given above and we alternatively study the case of complete information and incomplete information.

The way of modeling we adopt differs from both the standard and Rabin's approaches. In the standard game, agents are self-interested and their choices are motivated by their rationality towards the gain they obtain. Conversely, in the Rabin's approach, agents behave according to their rationality towards a combination of the material payoff and a "kindness gain" which let the material cost of sacrifice being smaller. As a result, each agent benefits from sacrificing a small piece of their own material well-being. This is the notion of equity. In what follows, we concentrate our approach on behavior. Agents are now motivated by their rationality towards their behavior.

Defining the behavior as the incentives which lead a player i to cooperate with a probability which is a function of the probability of the other player, it boils down to an application $\alpha_i : [0,1] \mapsto [0,1]$ with $\alpha_i(p(C^j)) = p(C^i)$. The reciprocal behaviour or reciprocity, is then defined by the following inequality:

$$p(C^i) = \alpha_i(p(C^j)) \geq p(C^j)$$

A player with a behavior α_i chooses a probability of cooperation $p(C^i)$ which is in line with his behavior. If his choice does not fit with his behavior, his utility falls. In this setting, $p(C^j)$ characterizes the intentions of the other player and $p(C^i)$ represents the intentions of the player i which can be assimilated to his behavior. We also distinguish two types of reciprocity, which leads to consider that each player can be of three types: strict reciprocity (SR) $(\alpha_i(p(C^j)) = p(C^j)\ \forall p(C^j))$, large reciprocity (LR) $(\alpha_i(p(C^j)) > p(C^j), \forall p(C^j))$ or non-reciprocity (NR) $\big(\alpha_i(p(C^j)) < p(C^j), \forall p(C^j)\big)$.

In the following analysis, we develop a game called the 'behavioral game' in which the equilibrium is only determined by the players' behavior.

3.1 Complete information

The case of complete information implies that each player i is completely informed about his own behavior α_i, about the behavior of his opponent α_j, about the set of strategies $[0,1]$ and about the payoff functions $u_i(s_i, s_j)$ and $u_j(s_j, s_i)$, with $s_i = \alpha_i(p(C^j)) = p(C^i)$, $s_j = \alpha_j(p(C^i)) = p(C^j)$, and where s_i and s_j stand for the strategies of players i and j.

3.1.1 Homogeneous behaviors: $\alpha_i = \alpha_j$

- Case 1: (Strict Reciprocity) $p\left(C^i\right) = \alpha_i(p(C^j)) = p(C^j)$

For each player i his best response $p(C^i)$ is equal to $p(C^j)$. In this case, no player benefits from deviating from the behavior of his opponent. We can show that (C, C) and (D, D) are Nash Equilibrium.

- Case 2: (Non-Reciprocity) $p\left(C^i\right) = \alpha_i(p(C^j)) < p(C^j)$

This type of behavior implies that the wish to cooperate of each player is lower than that expressed by the other player. Then if $p(C^j) \neq 0$, we have $\alpha_i(p(C^j)) = p(C^i) < p(C^j)$ and $\alpha_j(p(C^i)) = p(C^j) < p(C^i)$. We obtain $p(C^i) < p(C^j)$ and $p(C^j) < p(C^i)$. The unique equilibrium is then mutual defection.

- Case 3: (Large Reciprocity) $p\left(C^i\right) = \alpha_i(p(C^j)) > p(C^j)$

This situation is the opposite to the previous one. Each agent has a wish to cooperate higher than his opponent's $\alpha_i(p(C^j)) = p(C^i) > p(C^j)$ and $\alpha_j(p(C^i)) = p(C^j) > p(C^i)$. Consequently the unique equilibrium is mutual cooperation.

According to the study of these three cases, non surprisingly, mutual cooperation can be the result of the game if both players behave with reciprocity.

3.1.2 Heterogeneous behaviors: $\alpha_i \neq \alpha_j$

When heterogeneous behaviors are assumed, we have:

$$p(C^i) = \alpha_i(p(C^j)) \geq p(C^j) \text{ and } \alpha_j(p(C^i)) = p(C^j) < p(C^i)$$

The player j's payoff decreases if $p(C^j) \geq p(C^i)$, which implies that he does not benefit from attaching to the cooperation a probability at least equal to that of player i. The only point of equilibrium is then mutual defection.

This result shows that reciprocity from one of the players is not a sufficient condition to insure that cooperation constitutes the Nash equilibrium. Indeed, it is necessary that both players behave with reciprocity to let cooperation emerge as the Nash equilibrium.

3.1.3 Changing behaviour

These types of behaviors are quite difficult to study. We limit our analysis to the case of only one changing behavior (from large reciprocity to non reciprocity) whereas the other player cooperates with the same probability than his opponent (strict reciprocity).

Définition 1 *Changing behaviors are given by:*

$$\begin{aligned} p\left(C_i\right) &= \alpha_i(p(C^j)) = p(C^j) \\ \text{and } \alpha_j(p(C^i)) &< p(C^i) \text{ if } p(C^i) < p(C^i)^* \\ \text{and } \alpha_j(p(C^i)) &> p(C^i) \text{ if } p(C^i) \geqslant p(C^i)^* \end{aligned}$$

where $p(C^i)^* \in [0,1]$ is a threshold probability of cooperation.

According to this definition of changing behaviors, we can establish the conditions which allow the cooperative equilibrium to be the Nash equilibrium:

Proposition 3 *For changing behaviors defined by definition 1, mutual cooperation is a Nash equilibrium when $p(C^i) \geqslant p(C^i)^*$.*

Proof : To determine the equilibrium, we carry out a behavioral analysis. Since changing behaviors change along the interval [0,1], the level of reciprocity is crucial for the emergence of cooperation.

For probabilities $p(C^i) > p(C^i)^*$, we have $\alpha_j(p(C^i)) > p(C^i)$ and $\alpha_i(p(C^j)) = p(C^j)$. Then $(p(C^i), p(C^j)) = (1,1)$ is the only feasible Nash equilibrium. It is obvious that, for each player i, the best response to $p(C^j) = 1$ is $p(C^i) = \alpha_i(1) = 1$ then $p(C^i) = 1$, and by consequence mutual cooperation is a Nash equilibrium.

For probabilities $p(C^i) < p(C^i)^*$, the unique Nash equilibrium is $(p(C^i), p(C^j)) = (0,0)$. ■

This case points out the role played by one player's intentions on the behavior of the other and then on the emergence of cooperation. If the player with changing behavior feels that the intentions of the other player move sufficiently towards cooperation ($p(C^i) \geqslant p(C^i)^*$ or $p(C^i)$ is sufficiently close to 1) then he will opt for a reciprocal behavior and the result of the game is cooperation. Conversely, if the player estimates that the probability of cooperation of the other player is quite low ($p(C^i) < p(C^i)^*$ or far from 1) then, he will opt for a non reciprocal behavior and mutual cooperation does not emerge from this game.

3.2 Incomplete information

In the prisoner's dilemma game, a player i can doubt the reciprocity of his opponent, which is characterized by a degree of uncertainty on α_j. Nevertheless, for a given probability p, the player i knows $\alpha_i(p)$ since he knows his own type α_i. Conversely, when he takes p as a probability of cooperation, he does not know $\alpha_j(p)$. Then we face a game with incomplete information (Bayesian game). The Bayesian approach consists in attaching a distribution of probability to each uncertain element. When new information is given, an agent revises his beliefs according to the Bayes rule for conditional probabilities. In what follows, we consider a prisoner's dilemma game in which none of the players perfectly knows the behavior of the other. The player i's beliefs about the player j's behavior are defined as follows: i thinks that j is (SR) with probability p_1, (LR) with probability p_2 and (NR) with probability p_3. Similarly, player j has beliefs about the player i's behavior, with probabilities q_1, q_2 and q_3.

In this representation, the terminal nodes represent the payoffs of the game. For example, the couple (SR, SR) of behaviors leads to the probabilities of cooperation (p, p) (the players opt for the same probability of cooperation). In this case, the payoff is nothing but the expected payoff $(r(p), r(p))$ given in expression (1). Similarly, the couple (LR, SR) implies a profile of probabilities of cooperation $(1, 1)$. The players cooperate with a probability equal to 1 and both obtain a payoff a. Finally, the couple (NR, SR) gives $(0, 0)$ as probability to cooperate which implies that no player cooperates and they obtain a payoff (d, d).....and so on for the other possible cases of the game. We obtain the following result:

Proposition 4 *If each player believes that the other player can behave with large reciprocity, i.e. if $p_2 > 0$ and $q_2 > 0$, then the unique Bayesian Nash equilibrium is given by the couple of behaviors (LR, LR) and cooperation emerges.*

Proof : see Appendix C

With incomplete information, the belief about the opponent's behavior is the key point of the result. The beliefs about a potential large reciprocal behavior are sufficient to ensure that cooperation can emerge as Nash equilibrium. Reciprocity is then powerful in this type of models to reach cooperation.

4 Conclusion

Our analysis focuses on the incentives that lead the agent to cooperate. The action of each player depends on the behavior adopted by his opponent. If the players act in line with their rational behavior, their actions constitute an equilibrium.

The type of modeling presented in this paper is based on the players' intentions and points out the role of reciprocity in the emergence of cooperation. Reciprocity is strictly linked with the distribution of the behaviors in a population. The distortion of the payoffs proposed in this paper may be useful to explain the motives which lead the agents to act in one way or the other. Our approach of the distorted payoffs is based on the desire to catch the psychology of a player implicated in a prisoner's dilemma game. His beliefs about the other player's behavior is the best way to catch his psychology.

With complete or incomplete information, the mutual beliefs about reciprocity are crucial for the emergence of cooperation. The complexity of the individual behaviors leads to study the psychology of the agents and the sources of their motives. An agent can build his motives on the general interest, equity can replace individualism and confidence can dominate opportunism.

An extension to this approach of the distorted payoffs to the normal form games constitutes a path of future research that could be interesting to investigate.

5 Appendix

5.1 Appendix A: Proof of proposition 1

On one hand we have, since if $p \neq 1$:

$2.a > (b+c)$ and $a > d$, according to (1) we have,

$p^2.a + p(1-p)(c+b) + (1-p)^2.d < p^2.a + p(1-p)(2.a) + (1-p)^2.a = a$

thus $a > p^2.a + p(1-p)(c+b) + (1-p)^2.d = r(p)$ pour tout $p \in [0,1[$.

then $a > r(p)$ for each $p \in [0,1[$.
On the other hand,

$$\begin{aligned} r(p) - d &= p^2.a + p(1-p)(b+c) + (1-p)^2 d - d > 0 \\ &\Longleftrightarrow \\ p(c+b-a-d) &< c+b-2d \end{aligned}$$

If $2d < (b+c)$ it is always the case since $a > d$
If $2d > (b+c)$ then $a+d > c+b$ since $a > d$ and

$$p(c+b-a-d) < c+b-2d \Leftrightarrow p > \frac{2.d-c-b}{a+d-c-b}$$

Since we know $2.d > (b+c)$ and then $2a > b+c$, then $(a+d-c-b) > 0$ and $1 > \frac{2.d-c-b}{a+d-c-b} > 0$.

Then there exists $p^* = \frac{2.d-c-b}{a+d-c-b}$ such that if $p > p^*$ we get $r(p) > d$.■

5.2 Appendix B: Proof of proposition 2

• On one hand we have,

$$\mathbf{E}(R,q) > \mathbf{E}(Al,q) \Leftrightarrow \frac{1-q_1-q_2}{q_2} > \frac{a-r(p)}{d-b} \Leftrightarrow r(p) > a - \frac{1-q_1-q_2}{q_2}(d-b)$$

Using the expression of $r(p)$ we have

$$(p^2(a+d-c-b) + p(c+b-2d) + d) > a - \frac{1-q_1-q_2}{q_2}(d-b)$$

and we define

$$f(p) = p^2(a+d-c-b) + p(c+b-2d) + d - a + \frac{1-q_1-q_2}{q_2}(d-b)$$

Let us study the sign of $f(p)$

$$f'(p) = 2p(a+d-c-b) - (2d-c-b) > 0 \Leftrightarrow p > (<) \frac{2d-c-b}{2(a-b-c+d)} = \bar{p} \text{ if } (a-b-c+d) > (<) 0$$

If $(a-b-c+d) = 0$, $f(p)$ is a function easy to study.
If $(a-b-c+d) > 0$ then $\bar{p} < 1$ since $2a > (b+c)$, because

$$\frac{2d-c-b}{2(a-b-c+d)} < 1 \Leftrightarrow (-c-b) < (2a-2b-2c) \Leftrightarrow 2a > (b+c)$$

If $(a-b-c+d) < 0$, $\bar{p} > 1$ and $p < \bar{p}$.

Then we have 3 cases:

Case 1) when $2d > c+b$ then $(a-b-c+d) > 0$ since $a+d > 2d > c+b$, so $f'(p) > 0$ for $p > \bar{p} > 0$.

Case 2) when $2d < c+b$ and $(a-b-c+d) > 0$ then $f'(p) > 0$ $\forall p$ since $\bar{p} < 0$.

Case 3) when $2d < c+b$ and $(a-b-c+d) < 0$ then $f'(p) > 0$ $\forall p \in]0, \bar{p}[$.

For the boundary values of p we have:

$$f(1) = \frac{1 - q_1 - q_2}{q_2}(d - b) > 0$$

and

$$f(0) = \frac{(1 - q_1)(d - b) - q_2(a - b)}{q_2} > 0 \Leftrightarrow q_2 < \frac{(1 - q_1)(d - b)}{a - b}$$

For each case, since $f(1) > 0$, there exists $]\widetilde{p}, 1[$ on which $f(p)$ is increasing, continuous and positive. So, we can write $\mathbf{E}(R, q) > \mathbf{E}(Al, q)$ on $]\widetilde{p}, 1[$.

For case 3), if $f(0) > 0$, and since $f(p)$ is continuous and increasing on $]0, \bar{p}[$, then $f(p)$ is also positive on this interval. By the way, $\mathbf{E}(R, q) > \mathbf{E}(Al, q)$ for each p.

• On the other hand,

$$\mathbf{E}(R, q) > \mathbf{E}(S, q) \Leftrightarrow \frac{q_1}{q_2}(a - c) > d - r(p)$$

$$\Leftrightarrow r(p) > d + (c - a)\frac{q_1}{q_2}$$

This implies

$$(p^2(a + d - c - b) + p(c + b - 2d) + d) > d + (c - a)\frac{q_1}{q_2}$$

Or

$$g(p) = p^2(a + d - c - b) + p(c + b - 2d) + (a - c)\frac{q_1}{q_2}$$

$$g'(p) \quad = \quad 2p(a + d - c - b) + (c + b - 2d) > 0 \Leftrightarrow p > (<)\frac{2d - c - b}{2(a - b - c + d)} = \bar{p}$$

if $(a - b - c + d) \quad > \quad (<)0$

We find again the three previous cases

Case 1) when $2d > c + b$ then $(a - b - c + d) > 0$ since $a + d > 2d > c + b$, so $g'(p) > 0$ for $p > \bar{p} > 0$.

Case 2) when $2d < c + b$ and $(a - b - c + d) > 0$ then $g'(p) > 0$ $\forall p$ since $\bar{p} < 0$.

Case 3) when $2d < c + b$ and $(a - b - c + d) < 0$ then $g'(p) > 0$ $\forall p \in]0, \bar{p}[$ and $g(\bar{p}) > 0$

we have

$$g(1) = (a - d) + (a - c)\frac{q_1}{q_2} > 0 \Leftrightarrow \frac{q_1}{q_2} < \frac{a - d}{c - d}$$

$$g(0) = (a-c)\frac{q_1}{q_2} < 0$$

Then for each case, if $\frac{q_1}{q_2} < \frac{a-d}{c-d}$, then $g(1) > 0$ and there exists $1 > \widehat{p} > 0$ such that for each $p > \widehat{p}$ we have $g(p) > 0$, then $r(p) > d + (c-a)\frac{q_1}{q_2}$ and so $\mathbf{E}(R,q) > \mathbf{E}(S,q)$.

For case 3), $g(0) < 0$ and $g(\bar{p}) < 0$, and the sign of g is ambiguous for $p < \bar{p}$.■

5.3 Appendix C: Proof of proposition 5

The type of each player is given by t_k where $t_k \in \{SR, LR, NR\}$

The player k's expected payoff (distorted) is given by

$$S_k = \sum_{\{t_{-k} \in T_{-k})\}} u_k(x_k, s_{-k}(t_{-k}))P_k(t_{-k}/t_k)$$

In order to determine the equilibrium of the game, we compare the expected payoffs of the tree of the game for every possible type of behavior:

- For player i,

if $x_i = SR$ then $S_i = p_1 r(p) + p_2 a + p_3 d$,
if $x_i = LR$ then $S_i = p_1 a + p_2 a + p_3 d$
and if $x_i = NR$ then $S_i = p_1 d + p_2 d + p_3 d$.

- For player j,

if $x_j = SR$ then $S_j = q_1 r(p) + q_2 a + q_3 d$
if $x_j = LR$ then $S_j = q_1 a + q_2 a + q_3 d$
and if $x_j = NR$ then $S_j = q_1 d + q_2 d + q_3 d$.

Since $a > d$ and $a > r(p)$, LR represents the type of behavior which gives the highest level of expected payoff for each player. Then, if $p_2 > 0$ and $q_2 > 0$, the unique Bayesian Nash equilibrium is given by the couple of behaviors (LR, LR) and cooperation can emerge.■

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