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Inconsistent Incomplete Information:
A Betting Experiment

Werner Güth* Loreto Llorente Erviti† Anthony Ziegelmeyer‡

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Abstract

We study two person-betting games with inconsistent commonly know beliefs, using an experimental approach. In our experimental games, participants bet against one another, each bettor choosing one of two possible outcomes, and payoff odds are know at the time bets are placed. Bettors' beliefs are always commonly known. Participants play a series of betting games, in some of which the occurrence probabilities of the two outcomes differ between bettors (inconsistent beliefs) while in others the same occurrence probabilities prevail for both bettors (consistent beliefs). In the betting games with consistent commonly know beliefs, we observe that participants refrain from betting. In the betting games with inconsistent commonly know beliefs, we observe significant betting rates and the larger the discrepancy between the two bettors' subjective expectations the larger the volume of bets. Our experimental results contrast with the existing evidence on zero-sum betting games according to which participants' irrational inclination to bet is difficult to eliminate.

Keywords: Betting; Common prior; Harsanyi consistency; Experimental Economics.

JEL Classification: C72; C92; D84.

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1 Introduction

A fundamental—and controversial—assumption in game theory and economic theory is the *common prior assumption* (CPA) claiming that rational agents in models with asymmetric information have a common prior. Conceptually, CPA may be interpreted as saying that all agents are born equal in terms of their beliefs about the world so that different beliefs are solely due to different private signals. Harsanyi's (1967–1968) revolutionary work on games with incomplete information became the first work in the economics literature to address the issue head on. Harsanyi showed that a game with incomplete information can be reduced to a standard game of imperfect information with fictitious move by nature, if and only if individuals rely on a common prior over payoffs in some state space (i.e., have "consistent" beliefs). Aumann (1976) showed that if individuals' posterior beliefs are derived from a common prior, and there is common knowledge of those posteriors, then the posteriors must be the same.

Aumann's work stimulated work on *no trade* results which establish that, in the absence of *ex ante* gains from trade, asymmetric information cannot generate trade. In particular, Sebenius and Geanakoplos (1983)— extending Aumann's argument— showed that (under the common prior assumption) it cannot be common knowledge that risk neutral individuals are prepared to bet against each other, that is, that one individual's posterior beliefs exceed another's. Milgrom and Stokey (1982) showed an analogous result in a more general setting of risk averse traders. Since no-trade results can be shown to underlie many important results in microeconomic theory, it had by now become clear that the common prior assumption was critical.

In recent years, CPA and its consequences have been increasingly scrutinized. Morris (1995) argues that "rationality" arguments in favor of CPA are rather weak, and he gives examples of work where explicit modelling of heterogeneous prior beliefs seems to help our understanding. A more radical criticism of CPA has been made by Gul (1998) who questions its very meaningfulness in situations of incomplete information. Beside skepticism concerning the CPA in situations of incomplete information, several authors have considered inconsistent incomplete information and suggested different ways to represent this inconsistency (e.g., Maschler (1997) suggests representing inconsistent incomplete information in extensive form by one game tree for each player). Watanabe, Nonoyama, and Mori (1994) formulate a model of a parimutuel system which considers a horse race with two horses and where the subjective winning probabilities of the bettors are assumed to be inconsistent. Other early studies of strategic interaction with inconsistent beliefs are Güth (1985) and Harstad and Phlips (1997).

Here we do not engage in a discussion for and against such conclusions from commonly known rationality. Rather we want to explore experimentally a situation where the commonly known beliefs are inconsistent. This means that player i knows that another player j entertains different probabilistic expectations concerning chance moves but that this does not induce him to update his own beliefs. Formally this means that the fictitious chance move is governed not by just one probability assignment but rather by a vector of such probability assignments, generally one for each player, and that these subjective expectations are commonly known.

More precisely, this paper investigates the impact of inconsistent beliefs on bettors' behavior. The rules are those in the *Pelota's betting system*, two-team games on whose bettors bet one against each other. Given the market's odds, each bettor chooses one of the two possible events. The amount at stake is the minimum of both bettors' willingness to bet and the payoff after uncertainty is resolved depends on the odds. Data and analyses of the experiment reported here are the first evidence that has been produced regarding betting with inconsistent beliefs.

Indeed, all previous experimental studies on betting induce consistent beliefs. Piron and Smith (1995) design an experiment to test the hypothesis that the so-called *favorite-longshot bias* (favorites win more often than the betting odds indicate) occurs because horserace wagering is a consumption activity (Asch-Quandt Betting Hypothesis). Their experimental results do not confirm this hypothesis. Hurley and McDonough (1995) also examine the *favorite-longshot bias* in parimutuel betting concluding that this bias is not explained by costly information and transaction costs. Sonsino, Erev, and Gilat (2000) study the potential descriptive implications of the Sebenius and Geanakoplos (1983) no-betting conjecture. In contradiction with the no-betting conjecture, they find high persistent betting rates that cannot be explained simply because subjects love betting. Ziegelmeyer, Bédoussac Broihanne, and Koessler (2004) investigate sequential parimutuel betting in the laboratory with symmetrically informed players and offer a theoretical explanation of the subjects' behavior by relying on probability weighting functions.

In Section 2 we describe the experimental betting market and we characterize the equilibrium bets in case of inconsistent beliefs. After having introduced the details of our experimental design in Section 3, we discuss the results in Section 4. Section 5 concludes.

2 Theory

In this section we first explain concisely how "Basque *Pelota*" betting markets operate (see Llorente and Aizpurua (2008) for more details). Next, we formally describe a simplified version of the Basque *Pelota* betting market which corresponds to the betting game implemented in the laboratory. Finally, we characterize the equilibria of our betting game assuming bettors are (subjective) expected wealth maximizers.

2.1 Basque *Pelota* Betting Markets

Basque *Pelota* is a court sport played with a ball against walls. Basque *Pelota* matches are of different varieties. Here we briefly describe the variety called "*remonte*" which is played on a two walled long court (at least 54 meters), one frontal wall and a lateral wall at the left of the frontal one. The back side of the court has typically a wall in closed courts but is simply delimited by a line on the floor in common rural open-air courts.

In *remonte* matches, there are two opposing teams and each team usually has two players. The "*reds*" and the "*blues*" alternate hurling and catching a ball (the *pelota*) with an enlarged basket against a wall. When one team misses, the other team scores a point. The team that reaches a pre-set number of points wins the match.

Professional matches are open to betting on the results. Bets may be placed on either team to win the match before every point is played at fixed locked-in payoff odds, until the outcome of the match. For a bet to take place, two participants must be involved where one participant places money on the *reds* while the other participant places money on the *blues*. Participants place bets through a middleman who gets a 16% commission of the money won in each bet. A participant can place many bets but only those matched by another participant take place. Payoffs on bets made during the match are settled at the end of the match based on the odds at each betting point.

Let us illustrate how the payoffs of a single bet between two participants are determined. Assume that the *reds* are clearly leading and are most likely to win the match. According to the *Pelota* betting system, the payoff odds for the reds equal 100 whereas the payoff odds for the blues are strictly lower than 100, for example 50.[1] In this example, the participant who bets on the *reds* looses 100 euros if the *blues* win the match. If the *reds* win the match then the participant wins 50 euros minus the 16% commission, i.e. 42 euros. The participant who bets on the *blues* looses 50 euros if the *reds* win the match. If the *blues* win the match then the participant wins 100 euros minus 16%, i.e. 84 euros.

2.2 The Betting Game

Here we model participants' betting behavior at a certain point in the *remonte* match which means that we take the payoff odds as given. We denote by O_R the *reds* odds, i.e. the amount of money a participant (henceforth player) risks if he places a single bet on the *reds* (assuming that the bet is matched by another player). Similarly, O_B is the amount of money a player risks if he places a single bet on the *blues*. Without loss of generality, we adopt the following normalization: $O_R = 1$ and $O_B \in]0,1]$. In other words, we assume that if there is a favorite team in the considered betting period then this is the red team.

Each of the two players, player X and player Y, has an initial wealth denoted by $E \in \mathbb{N}_+$. Player X believes the *reds* (respectively *blues*) will win the match with probability p_X (respectively $1 - p_X$) where $0 < p_X < 1$. Player Y believes the *blues* (respectively *reds*) will win the match with probability p_Y (respectively $1 - p_Y$) where $0 < p_Y < 1$. In the inconsistent case, the two players assign different probabilities to each team to win, $p_X \neq 1 - p_Y$, so that we have vector valued probabilities capturing the players' beliefs. In the consistent case, the two players have identical subjective prior beliefs, i.e. $p_X = 1 - p_Y$.

Given the odds, the two players simultaneously place bets on each team. Player X's strategy, denoted by s_X, is given by (x_R, x_B) where $E \geq x_R \geq 0$ denotes X's number of (unmatched) bets placed on *reds* and $E/O_B \geq x_B \geq 0$ denotes X's number of (unmatched) bets placed on *blues*. Similarly, player Y's strategy, denoted by s_Y, is given by (y_R, y_B) where $E \geq y_R \geq 0$ denotes Y's number of (unmatched) bets placed on *reds* and $E/O_B \geq y_B \geq 0$ denotes Y's number of (unmatched) bets placed on *blues*. If $x_R = 0$ and $y_R = 0$, or $x_B = 0$ and $y_B = 0$, then no bet takes place. Denote player X's strategy set by $S_X = \{(x_R, x_B) \,|\, x_R \in \{0, ..., E\}$ and $x_B \in \{0, ..., E/O_B\}$, player Y's strategy set by $S_Y = \{(y_R, y_B) \,|\, y_R \in \{0, ..., E\}$ and $y_B \in \{0, ..., E/O_B\}$, and the set of strategy profiles by $S = S_X \times S_Y$.

Abstracting from commissions and given players' decisions, i.e. given a strategy profile $s = (s_X, s_Y)$, the subjective (final) expected wealth of player X is given by

$$W_X(s_X, s_Y) = E + [p_X\, O_B - (1 - p_X)] \min\{x_R, y_B\} + [(1 - p_X) - O_B\, p_X] \min\{x_B, y_R\}$$

or

$$W_X(s_X, s_Y) = E + (p_X(O_B + 1) - 1)(\min\{x_R, y_B\} - \min\{x_B, y_R\}). \tag{1}$$

Analogously, player Y's subjective expected wealth is given by

$$W_Y(s_X, s_Y) = E + (p_Y(O_B + 1) - O_B)(\min\{x_R, y_B\} - \min\{x_B, y_R\}). \tag{2}$$

We assume that players aim at maximizing their subjective expected wealth and that players' beliefs, preferences, and rationality are common knowledge.

[1] Odds of the favorite team always equal 100 whereas odds of the longshot team are between 2 and 90.

2.3 Equilibria of the Betting Game

We denote by $G(O_B) = \langle \{X, Y\}, (S_X, S_Y), (W_X, W_Y) \rangle$ the betting game. The strategy profile $s^* = (s^*_X, s^*_Y) \in S$ is a (Nash) equilibrium of $G(O_B)$ if and only if $\forall s_X \in S_X$, $W_X(s^*_X, s^*_Y) \geq W_X(s_X, s^*_Y)$, and $\forall s_Y \in S_Y$, $W_Y(s^*_X, s^*_Y) \geq W_Y(s^*_X, s_Y)$. Proposition 1 characterizes the equilibria of the betting game.

Proposition 1 *Let $S^* \subset S$ be the set of equilibria, $0 \leq x^*_R, y^*_R \leq E$ and $0 \leq x^*_B, y^*_B \leq E/O_B$.*

1. *If $p_X > 1/(O_B+1)$ and $p_Y > O_B/(O_B+1)$ then $S^* = ((x^*_R, x^*_B), (0, y^*_B)) \cup ((x^*_R, 0), (y^*_R, y^*_B))$ where $x^*_R < E$ implies $y^*_B = x^*_R$ and $x^*_R = E$ implies $y^*_B \geq x^*_R$. In all the bets which take place, player X places money on the* reds *while player Y places money on the* blues.
2. *If $p_X > 1/(O_B+1)$ and $p_Y < O_B/(O_B+1)$ then $S^* = ((x^*_R, 0), (y^*_R, 0))$. No bet takes place.*
3. *If $p_X < 1/(O_B+1)$ and $p_Y > O_B/(O_B+1)$ then $S^* = ((0, x^*_B), (0, y^*_B))$. No bet takes place.*
4. *If $p_X < 1/(O_B+1)$ and $p_Y < O_B/(O_B+1)$ then $S^* = ((x^*_R, x^*_B), (y^*_R, 0)) \cup ((0, x^*_B), (y^*_R, y^*_B))$ where $y^*_R < E$ implies $x^*_B = y^*_R$ and $y^*_R = E$ implies $x^*_B \geq y^*_R$. In all the bets which take place, player X places money on the* blues *while player Y places money on the* reds.

In the inconsistent case, bets take place whenever the winning probability assigned by one player to the *reds* added to the winning probability assigned by the other player to the *blues* exceeds 1 and the two players disagree sufficiently. In the consistent case, we obtain the traditional no-betting result since either $p_X > \frac{1}{O_B+1}$ and $p_Y < \frac{O_B}{O_B+1}$ or $p_X < \frac{1}{O_B+1}$ and $p_Y > \frac{O_B}{O_B+1}$.

The Possibility of Mistakes

In the equilibria of the betting game described above, some unappealing strategies are used only because each player is absolutely sure of what the other player does. Here, we assume that players are more cautious and we characterize the equilibria of the betting game that are robust to the possibility that, with some very small probability, players make mistakes.

In the betting game, these trembling-hand perfect (Nash) equilibria are those equilibria not involving play of a weakly dominated strategy (Mas-Colell, Whinston, and Green, 1995, p. 259). It is easy to show that: (i) If $p_X > 1/(O_B+1)$ (respectively $p_X < 1/(O_B+1)$) then the only not weakly dominated strategy for player X is $s_X = (E, 0)$ (respectively $s_X = (0, x_B)$ where $x_B \geq E$); (ii) If $p_y < O_B/(O_B+1)$ (respectively $p_y > O_B/(O_B+1)$) then the only not weakly dominated strategy for player Y is $s_Y = (E, 0)$ (respectively $s_Y = (0, y_B)$ where $y_B \geq E$). Proposition 2 characterizes the trembling-hand perfect equilibria of the betting game.

Proposition 2 *Let $S^{**} \subset S^*$ be the set of trembling-hand perfect equilibria, $0 \leq x^*_R, y^*_R \leq E$ and $0 \leq x^*_B, y^*_B \leq E/O_B$.*

1. *If $p_X > 1/(O_B+1)$ and $p_Y > O_B/(O_B+1)$ then $S^{**} = ((E, 0), (0, y^*_B))$ where $y^*_B \geq E$. In each equilibrium, E bets take place.*
2. *If $p_X > 1/(O_B+1)$ and $p_Y < O_B/(O_B+1)$ then $S^{**} = ((E, 0), (E, 0))$. In equilibrium, no bet takes place since both players place E bets on the* reds.
3. *If $p_X < 1/(O_B+1)$ and $p_Y > O_B/(O_B+1)$ then $S^{**} = ((0, E), (0, E))$. In equilibrium, no bet takes place since both players place E bets on the* blues.
4. *If $p_X < 1/(O_B+1)$ and $p_Y < O_B/(O_B+1)$ then $S^{**} = ((0, x^*_B), (E, 0))$ where $x^*_B \geq E$. In each equilibrium, E bets take place.*

3 Experimental Design

We implemented a large set of parameterized versions of the betting game in the laboratory in order to study the impact of three variables on the participants' behavior. First, the degree of inconsistency in the players' beliefs varies among the experimental betting games. In particular, in some games players have inconsistent beliefs while in others there is a common prior. Second, we consider different levels for the payoff odds of the *blues* (O_B). Finally, in experimental betting games with inconsistent beliefs, we distinguish between games where both players get the same equilibrium expected wealth and games with asymmetric equilibrium expected wealths. Overall, our design consists of 21 experimental versions of the betting game which cover rather completely its relevant parameter space.

3.1 The 21 Experimental Betting Games

Our design builds on 8 experimental betting games with identical odds for both teams i.e. $O_B = 1$. Among those games where players have inconsistent beliefs (inconsistent cases), we distinguish between the situations where both players get the same equilibrium expected wealth (symmetric situations) and situations with asymmetric equilibrium expected wealths (asymmetric situations). Note that when $O_B = 1$ an asymmetric situation is simply characterized by $p_X \neq p_Y$. Table 1 summarizes the features of these 8 games.[2]

Inconsistent cases	Asymmetric situations	Game 1	$p_X = .75$	$p_Y = .50$	$W_X^* = 3/2\,E$	$W_Y^* = E$
		Game 2	$p_X = .50$	$p_Y = .65$	$W_X^* = E$	$W_Y^* = 4/3\,E$
		Game 3	$p_X = .65$	$p_Y = .75$	$W_X^* = 4/3\,E$	$W_Y^* = 3/2\,E$
	Symmetric situations	Game 4	$p_X = .75$	$p_Y = .75$	$W_X^* = 3/2\,E$	$W_Y^* = 3/2\,E$
		Game 5	$p_X = .65$	$p_Y = .65$	$W_X^* = 4/3\,E$	$W_Y^* = 4/3\,E$
Consistent cases		Game 6	$p_X = .50$	$p_Y = .50$	$W_X^* = E$	$W_Y^* = E$
		Game 7	$p_X = .35$	$p_Y = .65$	$W_X^* = E$	$W_Y^* = E$
		Game 8	$p_X = .75$	$p_Y = .25$	$W_X^* = E$	$W_Y^* = E$

Table 1: The 8 experimental betting games with equal odds ($O_B = 1$).

Except in game 7 (respectively game 8), player X (respectively player Y) has an incentive to bet on the *reds* (respectively the *blues*). The incentive is weak in games 2 and 6 (respectively games 1 and 6). The last two columns of Table 1 show the players' equilibrium wealths where W_X^* (respectively W_Y^*) denotes player X's (respectively player Y's) subjective expected wealth in the trembling-hand perfect equilibria of the corresponding game. We now detail the trembling-hand perfect equilibrium outcomes of the experimental betting games with equal odds.

In game 1, we assume that E bets take place. Indeed, we assume that player Y places between E and E/O_B bets on the *blues*, though his expected wealth is invariant with the number of bets which take place. Anticipating this, player X places E bets on the *reds*. Similarly, in game 2, we assume that E bets take place since player X places E bets on the *reds*, though his expected wealth is invariant with the number of bets which take place, and player Y anticipates this and places at least E bets on the *blues*.

In games 3, 4, and 5, E bets take place since player X places E bets on the *reds* while player Y places between E and E/O_B bets on the *blues*.

[2] The largest winning probability equals 3/4 because otherwise in the additional games where $O_B < 1$ players' beliefs would be almost degenerate.

In game 6, we assume that no bet takes place since none of the players has a strict incentive to place bets on either of the two teams.

Finally, in game 7 (respectively game 8), no bet takes place since both players have an incentive to place bets on the *blues* (respectively *reds*).

In addition to the 8 experimental betting games with equal odds, we implemented betting games with $O_B = .67$ and others with $O_B = .54$.

The 8 experimental betting games with $O_B = .67$, referred to as game 9 to game 16, are parameterized so that the trembling-hand perfect equilibrium outcome of experimental betting game k is identical to the trembling-hand perfect equilibrium outcome of experimental betting game $k+8$ where $k \in \{1, \ldots, 8\}$. Identical predictions are achieved with different odds by modifying players' beliefs. For example, the couple of probabilities $(p_X = .50, p_Y = .65)$ in game 2 becomes $(p_X = .60, p_Y = .60)$ in game 10.

Similarly, we modified players' beliefs to achieve identical predictions in betting games with $O_B = .54$ as in the original games with equal odds. However, to prevent beliefs from becoming degenerate, a smaller set of betting games with $O_B = .54$ has to be considered. We obtain 5 experimental betting games with $O_B = .54$, referred to as game 17 to game 21, such that the trembling-hand perfect equilibrium outcome of game 17 (respectively 18, 19, 20, and 21) is identical to the trembling-hand perfect equilibrium outcome of game 2 (respectively 3, 5, 6, and 7). For example, the couple of probabilities $(p_X = .50, p_Y = .65)$ in game 2 becomes $(p_X = .65, p_Y = .55)$ in game 17.

Tables 2 and 3 summarize the features of these additional betting games.

Inconsistent cases	Asymmetric situations	Game 9	$p_X = .90$	$p_Y = .40$	$W_X^* = 3/2\,E$	$W_Y^* = E$
		Game 10	$p_X = .60$	$p_Y = .60$	$W_X^* = E$	$W_Y^* = 4/3\,E$
		Game 11	$p_X = .80$	$p_Y = .70$	$W_X^* = 4/3\,E$	$W_Y^* = 3/2\,E$
	Symmetric situations	Game 12	$p_X = .90$	$p_Y = .70$	$W_X^* = 3/2\,E$	$W_Y^* = 3/2\,E$
		Game 13	$p_X = .80$	$p_Y = .60$	$W_X^* = 4/3\,E$	$W_Y^* = 4/3\,E$
Consistent cases		Game 14	$p_X = .60$	$p_Y = .40$	$W_X^* = E$	$W_Y^* = E$
		Game 15	$p_X = .40$	$p_Y = .60$	$W_X^* = E$	$W_Y^* = E$
		Game 16	$p_X = .90$	$p_Y = .10$	$W_X^* = E$	$W_Y^* = E$

Table 2: The 8 experimental betting games with $O_B = .67$.

Inconsistent cases	Asymmetric situations	Game 17	$p_X = .65$	$p_Y = .55$	$W_X^* = E$	$W_Y^* = 4/3\,E$
		Game 18	$p_X = .85$	$p_Y = .70$	$W_X^* = 4/3\,E$	$W_Y^* = 3/2\,E$
	Symmetric situation	Game 19	$p_X = .85$	$p_Y = .55$	$W_X^* = 4/3\,E$	$W_Y^* = 4/3\,E$
Consistent cases		Game 20	$p_X = .65$	$p_Y = .35$	$W_X^* = E$	$W_Y^* = E$
		Game 21	$p_X = .45$	$p_Y = .55$	$W_X^* = E$	$W_Y^* = E$

Table 3: The 5 experimental betting games with $O_B = .54$.

3.2 Practical Procedures

The two sessions of the computerized experiment were conducted at the Experimental Laboratory of the Max Planck Institute of Economics in Jena (Germany). All 48 subjects were undergraduate students from various disciplines at the University of Jena, none of whom had previously played a betting game in the laboratory. At the beginning of each session, the 24 subjects were divided into three matching groups comprising 4 X players and 4 Y players. Subjects then played twice in a row the 21 parameterized versions of the betting game described above in two different random orders.[3] In each betting game, X players were randomly matched with Y players of their matching group. The actual size of the matching groups was not revealed to the subjects in order to discourage repeated interaction effects.

Subjects played the 42 betting games on a computer terminal, which was physically isolated from other terminals. Communication, other than through the decisions made, was not allowed. Subjects were instructed about the rules of the game and the use of the computer program through written instructions, which were read aloud by an assistant (who was chosen at random from the group of subjects at the beginning of the session). A short questionnaire and three dry runs followed.[4]

We used the term "market" to refer to a particular betting game. In each market, we implemented the betting game in the following way. At the beginning of the market, two random choices were made between color A and color B according to the objective probability distribution of each player (in case both players had the same probability distributions, only one random choice was made). For example, assume color A corresponds to the *reds*, the probability that color A is randomly selected for player X in Market 1 (game 3) is given by 65% (respectively 25% for player Y). Subjects were not aware of the chosen color until the end of the market. Notice that we used different colors in each market to avoid probability matching over the supergame by the subjects.[5] Subjects were then asked to allocate up to 100 Experimental Currency Units ($E = 100$ ECUs) on color A and B. At the end of each market, all subjects observed on their screens the final return of each color, the winning color and their earnings.

At the end of each session, subjects received 0.03 euros per ECU of three randomly selected markets. We chose this incentive structure (Random Lottery Incentive System) to encourage subjects to think carefully about each bet while controlling for income effects.

4 Results

We structure the presentation of our results by first analyzing individual betting behavior and then discussing the corresponding volume of bets in the different games.

[3]The first random sequence of betting games is given by game 3, 16, 7, 13, 20, 8, 2, 12, 5, 19, 9, 10, 21, 4, 15, 17, 1, 11, 18, 6, and game 14. The second random sequence of betting games is given by game 8, 4, 21, 10, 16, 9, 6, 12, 7, 17, 20, 11, 1, 2, 15, 18, 3, 14, 5, 13, and game 19.

[4]In each session, subjects read the instructions on their own, listened to the assistant reading the instructions aloud, and answered a questionnaire. The questionnaire mainly checked subjects' understanding of the calculation of earnings. Subjects who made mistakes in answering the questionnaire were paid the minimum compensation of 3 euros and were replaced by subjects who were randomly selected among those who made no mistakes on the questionnaire. Thus, in each session, all subjects who were retained for participation in the remainder of the experiment had made no mistakes on the questionnaire. The three betting games used for the dry runs were game 5 ($O_B = 1$), game 20 ($O_B = .54$), and game 11 ($O_B = .67$).

[5]The gambler's fallacy means to believe that the probability of an event decreases when the event has occurred recently even though the probability of the event is objectively known to be independent across trials. By using different colors in each market we try to reduce the influence of the gambler's fallacy on betting behavior.

4.1 Individual Betting Behavior

Expect in games 7, 15 and 21, a participant who acts as player X and values positively her own expected wealth should place more bets on the *reds* than on the *blues*. On the other hand, expect in games 8 and 16, a participant who acts as player Y and values positively her own expected wealth should place more bets on the *blues* than on the *reds*. Therefore, our analysis of individual betting behavior is based on the participants' net bets i.e. the difference $x_R - x_B$ for participants who act as player X and the difference $y_B - y_R$ for participants who act as player Y. From now on, we denote by mg_X and mg_Y player X and player Y's marginal potential gain from a net bet i.e. $mg_X = p_X(O_B + 1) - 1$ and $mg_Y = p_Y(O_B + 1) - O_B$.

Table 4 summarizes the participants' net bets in the 21 last markets of the experimental sessions.[6] There are several general patterns apparent. First, as expected, the larger the potential increase in expected wealth from net bets the larger the average net bets. Player X's average net bets increase from 8.19 in games where mg_X approximately equals $(-1/3)$ to 54.69 in games where $mg_X = 1/2$. Similarly, player Y's average net bet increases from 4.63 in games where $mg_Y = (-1/2)$ to 53.08 in games where mg_Y approximately equals 1/2. Second, for a given marginal potential gain from a net bet, participants have the tendency to place larger net bets the lower O_B. For example, player X's average net bets increase from 38.96 in game 3 where mg_X approximately equals 1/3 and $O_B = 1$ to 69.79 in game 18 where mg_X approximately equals 1/3 and $O_B = 0.54$. Note that in games where the marginal potential gain from a net bet is identical, the same amounts of net bets lead to the same levels of expected wealths whatever O_B. On the other hand, the variance of the players' distribution of wealth decreases with O_B. The second observation therefore suggests that participants value negatively the variance in wealth induced from net bets i.e. participants exhibit risk aversion. Third, for a given increase in expected wealth from net bets, the lower the equilibrium wealth the lower the average net bets. When $mg_X = 1/2$, player X's average net bets decrease from 60.96 in games where $W_X^* = 3/2\,E$ to 42.17 in games where $W_X^* = E$. When mg_Y approximately equals 1/3, player Y's average net bets decrease from 39.63 in games where $W_Y^* = 4/3\,E$ to 25.71 in games where $W_Y^* = E$. The third observation implies that larger net bets take place in inconsistent games than in consistent ones which suggests that participants take into account the other player's incentives when deciding about their betting strategy.

The three observations are summarized in our first result and are confirmed by a regression analysis which we include in the result's support.

Result 1: *The marginal potential gain from a net bet has a strong and positive impact on the level of observed net bets. For a given marginal potential gain from a net bet, payoff odds of the blues (O_B) has a negative impact on the level of observed net bets. For a given marginal potential gain from a net bet and given payoff odds of the blues, larger net bets are observed in games where players have inconsistent beliefs than in games with consistent beliefs.*

Support: Table 5 reports the results of simple OLS regressions with robust standard errors clustered on matching groups as the independent units of observation.

Model 1 includes five explanatory variables plus a constant. The variable "Marginal potential gain" which corresponds to the marginal potential gain from a net bet is highly significantly positive and its estimated coefficient indicates that the marginal potential gain has a strong impact on the level of observed net bets. The dummy variable "Inconsistent beliefs" takes value one in games where

[6] Qualitatively similar results are obtained by considering all the 42 markets.

		Average net bets (1^{st} quartile; median; 3^{rd} quartile)					
		Player X			Player Y		
Game	O_B	W_X^*	mg_X	$x_R - x_B$	W_Y^*	mg_Y	$y_B - y_R$
1	1	3/2 E	0.50	**53.58** (27.50; 55.00; 79.50)	E	0.00	**9.79** (0.00; 0.00;8.75)
9	0.67	3/2 E	0.50	**60.83** (26.25; 75.00; 100.00)	E	0.00	**8.13** (0.00; 0.00;10.00)
2	1	E	0.00	**5.58** (0.00; 0.00; 0.00)	4/3 E	0.30	**24.33** (3.75; 20.00;32.50)
10	0.67	E	0.00	**23.75** (0.00; 22.50; 36.25)	4/3 E	0.33	**42.54** (8.75; 35.00;52.50)
17	0.54	E	0.00	**25.63** (0.00; 20.00; 40.00)	4/3 E	0.31	**28.50** (0.00; 20.00;50.00)
3	1	4/3 E	0.30	**38.96** (13.75; 37.50; 56.25)	3/2 E	0.50	**41.25** (20.00; 40.00;51.25)
11	0.67	4/3 E	0.34	**56.25** (27.50; 60.00; 92.50)	3/2 E	0.50	**57.38** (30.00; 49.50;82.50)
18	0.54	4/3 E	0.31	**69.79** (57.50; 87.50; 100.00)	3/2 E	0.54	**76.13** (38.75; 74.00;100.00)
4	1	3/2 E	0.50	**60.96** (36.25; 71.50; 100.00)	3/2 E	0.50	**42.88** (17.50; 37.00;62.50)
12	0.67	3/2 E	0.50	**68.46** (40.00; 77.50; 100.00)	3/2 E	0.50	**52.96** (25.00; 49.50;75.00)
5	1	4/3 E	0.30	**27.92** (0.00; 20.00; 45.00)	4/3 E	0.30	**35.83** (8.75; 30.00;50.00)
13	0.67	4/3 E	0.34	**58.75** (20.00; 75.00; 100.00)	4/3 E	0.33	**52.63** (21.25; 32.50;81.25)
19	0.54	4/3 E	0.31	**54.75** (18.75; 50.00; 100.00)	4/3 E	0.31	**53.96** (0.00; 45.00;92.50)
6	1	E	0.00	**7.08** (0.00; 0.00; 0.00)	E	0.00	**9.17** (0.00; 0.00;0.00)
14	0.67	E	0.00	**8.25** (0.00; 0.00; 4.75)	E	0.00	**10.54** (0.00; 0.00;5.00)
20	0.54	E	0.00	**25.63** (0.00; 17.50; 32.50)	E	0.00	**19.17** (0.00; 0.00;22.50)
7	1	E	-0.30	**13.96** (0.00; 0.00; 5.00)	E	0.30	**22.29** (7.50; 20.00;22.50)
15	0.67	E	-0.33	**5.21** (0.00; 0.00; 0.00)	E	0.33	**30.88** (0.00; 30.00;50.00)
21	0.54	E	-0.31	**5.42** (0.00; 0.00; 0.00)	E	0.31	**23.96** (0.00; 12.50;40.00)
8	1	E	0.50	**37.88** (0.00; 27.50; 74.25)	E	-0.50	**0.08** (0.00; 0.00;0.00)
16	0.67	E	0.50	**46.46** (0.00; 40.00; 90.00)	E	-0.50	**9.17** (0.00; 0.00;0.00)

Table 4: Individual net bets.

players have inconsistent beliefs and zero otherwise. Clearly, larger net bets are observed in games where players have inconsistent beliefs than in games with consistent beliefs. The dummy variable "Repetition" takes value one in the last 21 markets of an experimental session and zero otherwise. Individual betting behavior is similar in the first and second half of an experimental session. The estimated coefficient of the payoff odds of the *blues* (O_B) indicates that the level of observed nets is strongly negatively affected by O_B. The dummy variable "Asymmetric situation" takes value one in games where players have asymmetric equilibrium expected wealths and zero otherwise. Individual betting behavior is similar in symmetric and asymmetric situations.

Model 2 is the same as Model 1 except that we drop the insignificant variables "Repetition" and "Asymmetric situation". Estimated coefficients of the significant variables are almost unchanged.

	Dependent variable: Individual net bets	
	Model 1	Model 2
Marginal potential gain	49.22***	49.82***
	(4.75)	(4.52)
Inconsistent beliefs	10.18***	8.68***
	(2.60)	(2.33)
Repetition	2.12	
	(1.51)	
O_B	-25.45***	-25.33***
	(1.98)	(2.01)
Asymmetric situation	-2.11	
	(1.76)	
Constant	35.60***	36.57***
	(2.69)	(2.67)
Observations	2016	2016
R-squared	0.21	0.21

Notes: OLS regressions with robust standard errors (clustered on matching groups) in parentheses. *** significant at 1%.

Table 5: Individual betting behavior.

4.2 Volume of Bets

We now turn to the analysis of the aggregate betting behavior by focusing on the volume of bets in the different games. Given the individual bets, the volume of bets equals $\min\{x_R, y_B\} - \min\{x_B, y_R\}$. According to our first result, larger individual net bets are observed in games where players have inconsistent beliefs than in games with consistent beliefs. Therefore, we expect the degree of divergence in players' beliefs to have a positive impact on the volume of bets (in a game where players have consistent beliefs, the degree of divergence in players' beliefs is zero). We define the degree of divergence in players' beliefs by $(W_X^* + W_Y^* - 2\,E)/E$ which allows us to investigate the additional impact of O_B. Our second result summarizes the determinants of the volume of bets.

Result 2: *The more players disagree about the winning outcome the larger the observed volume of bets. For a given degree of divergence in players' beliefs, payoff odds of the blues (O_B) has a negative impact on the observed volume of bets.*

Support: Table 6 reports the results of simple OLS regressions with robust standard errors clustered on matching groups as the independent units of observation.

Model 1 includes four explanatory variables plus a constant. The variable "Divergence in beliefs" which corresponds to the degree of divergence in players' beliefs is highly significantly positive and its estimated coefficient indicates that the degree of divergence has a strong impact on the observed volume of bets. The dummy variable "Repetition" takes value one in the last 21 markets of an experimental session and zero otherwise. A slightly lower volume of bets is observed in the second half than in the first half of an experimental session. The estimated coefficient of the payoff odds of the *blues* (O_B) indicates that the observed volume of bets is strongly negatively affected by O_B. The dummy variable "Asymmetric situation" takes value one in games where players have asymmetric equilibrium expected wealths and zero otherwise. Observed volume of bets is not significantly influenced by the asymmetry in equilibrium wealths.

Model 2 is the same as Model 1 except that we drop the insignificant variable "Asymmetric situation". Estimated coefficients of the significant variables are almost unchanged.

	Dependent variable: Volume of bets	
	Model 1	Model 2
Divergence in beliefs	22.39***	22.31***
	(2.96)	(3.11)
Repetition	-3.00***	-3.00***
	(0.81)	(0.81)
O_B	-17.28***	-17.27***
	(2.02)	(2.04)
Asymmetric situation	-0.17	
	(0.99)	
Constant	34.78***	34.73***
	(4.80)	(4.81)
Observations	1008	1008
R-squared	0.08	0.08

Notes: OLS regressions with robust standard errors (clustered on matching groups) in parentheses. *** significant at 1%.

Table 6: Aggregate betting behavior.

5 Conclusion

We study two person-betting games with inconsistent commonly know beliefs, using an experimental approach. In our experimental games, participants bet against one another, each bettor choosing one of two possible outcomes, and payoff odds are know at the time bets are placed. Bettors' beliefs are always commonly known. Participants play a series of betting games, in some of which the occurrence probabilities of the two outcomes differ between bettors (inconsistent beliefs) while in others the same occurrence probabilities prevail for both bettors (consistent beliefs).

We observe a higher volume of bets in games where bettors have inconsistent beliefs than in games with consistent beliefs. In fact, the larger the discrepancy between the two bettors' subjective expectations the larger the volume of bets. Moreover, our experimental results suggest that bettors exhibit risk aversion and they contrast with the existing evidence on zero-sum betting games according to which participants' irrational inclination to bet is difficult to eliminate.

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