Asymmetric Information without Common Priors: An Indirect Evolutionary Analysis of Quantity Competition

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Abstract

The *common prior assumption* justifies private beliefs as posterior probabilities when updating a *common prior* based on individual information. Common priors are pervasive in most economic models of incomplete information and oligopoly models with asymmetrically informed firms. We dispose of the common prior assumption for a homogeneous oligopoly market with uncertain costs and firms entertaining arbitrary priors about other firms' cost-type to analyze which priors will be evolutionarily stable when truly expected profit measures (reproductive) success. When firms believe that all other firms entertain the same beliefs Nature's priors are not the only evolutionarily stable priors. In a second model allowing for asymmetric priors Nature's priors are not even evolutionarily stable.

KEYWORDS: (Indirect) evolution; Common prior assumption; Cournot competition

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1 Introduction

Incomplete information prevails if at least one player entertains only probabilistic beliefs regarding the rules of the game. Harsanyi (1967–1968) has captured such rule uncertainty by introducing a commonly known but purely fictitious chance move whose results are only partially revealed such that individual information deficits are preserved. Harsanyi (1967–1968) has further argued that beliefs should be consistent in the sense that individual beliefs are just the marginals of the same probability distribution determining the rules of the game.

This Common Prior Assumption (CPA) has been defended on the methodological basis that it allows to 'zero in on purely informational issues' (Aumann, 1987), and on the more philosophical basis that 'Under the CPA, differences in probabilities express differences in information *only*' (Aumann, 1998: italics in original).[1] Morris (1995) and Gul (1998), on the contrary, argue that there are no fundamental or rationality-based arguments to impose the CPA and that agents cannot 'agree to disagree' (Aumann, 1976): it cannot be common knowledge that one agent holds belief x about some event while another agent has belief $y \neq x$. Since mutual knowledge of disagreement is an everyday experience, one should not (need to) assume common priors when analyzing games with incomplete information.[2]

Here we rely on an indirect evolutionary framework to check whether the evolutionarily stable beliefs satisfy the consistency requirement or not. The indirect evolutionary approach assumes that phenotypes interact fully rationally and that in an evolutionary process some of the parameters or rules of the game evolve. Analytically, the indirect evolutionary approach can be seen as a formal methodology to endogenize usual premises of the neo-classical approach like beliefs.[3] Unlike the purely strategic approach where such premises are determined by earlier choices (see, e.g., Fershtman and Judd, 1987), indirect evolution does not require an overall game whose rules are commonly

[1] Common priors are pervasive in the literature on rational expectations, bargaining under incomplete information, auctions, signalling, etc. Actually, the vast majority of the asymmetric information literature in economics and game theory assumes the CPA. A novel positive foundation (for a compact state space requiring the elicitation of individual beliefs regarding the fundamentals of the interaction) has been recently provided by Heifetz (2006).

[2] The case of distinct priors has also been considered by Harsanyi (1967–1968); he referred to it as the "inconsistent case". In the inconsistent case the fictitious chance move has a commonly known vector-valued probability distribution, one for each player (see Harrison and Kreps, 1978 for an early application).

[3] There are by now several studies based on the indirect evolutionary approach initiated by Güth and Yaari (1992) which demonstrate that evolution may yield preferences that deviate from the underlying fitness function. An early example is given by Bester and Güth (1998) who demonstrate that altruism can be evolutionary stable. More recently, Heifetz and Segev (2004) show that a toughness bias in bargaining might be favored by evolution, and Huck, Kirchsteiger, and Oechssler (2005) establish that for a general class of evolutionary processes strictly positive endowment effects will survive in the long run. Fewer studies have been concerned by the evolution of beliefs. Gehrig, Güth, and Levinsky (2004) find that, in the case of demand uncertainty, evolutionary stable expectations converge to rational expectations *only* in large markets, and Güth and Huck (1997) provide an evolutionary justification for the theory of monopolistic competition.

known. The two approaches are, of course, similar in the sense that such premises serve as commitment devices when others can notice how they change individually. In this sense our study is one of belief commitment by (indirect) evolution rather than by strategic considerations.[4]

Rather than tackling the problem in full generality, we confine ourselves to analyzing one sufficiently complex market model namely a homogeneous market where three firms compete in quantities with uncertain constant marginal costs. Each firm knows its own costs but, due to arbitrary prior beliefs, two firms can entertain different beliefs concerning the cost type (low, medium or high) of the third firm knowing that exactly one firm has low costs. If a firm is a medium or high cost-type itself, it can entertain idiosyncratic and home grown beliefs who of its competitors is a low cost-type and who not. The solutions based on these beliefs are then used to define an evolutionary market game with the possible priors as strategies and the actually resulting expected profits as fitness measures.[5] We finally explore which priors constellations are evolutionarily stable.

To check whether the evolutionarily stable priors satisfy the consistency requirement we present two alternative models. In the first model, firms believe that all other firms entertain the same beliefs about the distribution of the marginal costs what allows to check whether Nature's priors are the only evolutionarily stable priors. By a second model with asymmetric priors we check whether Nature's priors are evolutionarily stable. In Section 2 we describe the market environment. A closed-form solution of the equilibrium for the two models is provided in Section 3. Section 4 defines and "solves" (derives the evolutionarily stable priors) the evolutionary games. Section 5 discusses the robustness of our results.

2 The market environment

We consider a linear homogeneous market in which three firms compete in quantities.[6] Each firm $i \in \{X, Y, Z\}$ incurs a constant cost c_i per unit of production which is known only to that firm and whose level can be either 0, $\underline{c}$, or $\bar{c}$ with $0 < \underline{c} < \bar{c} \leq 1/2$.[7] Given firm i's marginal cost, we refer to its sales amounts of the homogeneous product by $q_i^{c_i}$ with $q_i^{c_i} \geq 0 \; \forall \, i \in \{X, Y, Z\}$. For a given profile

[4]Choosing beliefs strategically seems rather strange as strategic specification of preferences (see Frank, 1987).

[5]Using true profit expectations rather than true profits can be justified by assuming a large (infinite) population and random matching of sellers. Otherwise one would have to analyze a stochastic evolutionary process (see Güth and Kliemt, 2000, for an example).

[6]Since we rely on the usual but questionable tradition that incomplete information only concerns other players' types, inconsistency of beliefs requires at least three firms since the two firms can entertain different beliefs regarding the third firm's type.

[7]The industry is viable as soon as $\bar{c} < 1$. We restrict further the marginal costs so that, in equilibrium, quantities are positive.

of marginal costs (c_x, c_y, c_z), the inverse demand function is defined by

$$p(q_x^{c_x}, q_y^{c_y}, q_z^{c_z}) = 1 - q_x^{c_x} - q_y^{c_y} - q_z^{c_z} \tag{1}$$

if $q_x^{c_x} + q_y^{c_y} + q_z^{c_z} \leq 1$ what will be guaranteed by optimality. It is commonly known that $(c_x, c_y, c_z) \in \mathcal{C} = \{(0, \bar{c}, \bar{c}),\ (0, \bar{c}, \underline{c}),\ (0, \underline{c}, \bar{c}),\ (0, \underline{c}, \underline{c}),\ (\bar{c}, 0, \bar{c}),\ (\bar{c}, 0, \underline{c}),\ (\underline{c}, 0, \bar{c}),\ (\underline{c}, 0, \underline{c}),\ (\bar{c}, \bar{c}, 0),\ (\bar{c}, \underline{c}, 0),\ (\underline{c}, \bar{c}, 0),\ (\underline{c}, \underline{c}, 0)\}$. In words, the fact that only one of the three firms enjoys the null marginal cost whereas the two remaining firms can have medium or high marginal costs is assumed to be common knowledge.[8] Allowing for only finitely many cost constellations allows us to formulate beliefs as finite probability vectors (as well as finitely dimensional mutant space for the evolutionary analysis) and thus to avoid all measure theoretic terminology.

Additionally, we assume that each profile of marginal costs $(c_x, c_y, c_z) \in \mathcal{C}$ is equally likely, this fact being only known by Nature.[9] Accordingly, the true expected profit functions which reflect the uniform distribution of the marginal costs vectors are symmetrically defined by

$$\begin{aligned} R_i\left(Q_i, Q_j, Q_k\right) \;=\; & \frac{1}{3}\Bigg[q_i^0\left(1 - q_i^0 - \frac{1}{2}\left(Q_{-i}^{\underline{c}} + Q_{-i}^{\bar{c}}\right)\right) + q_i^{\underline{c}}\left(1 - \underline{c} - q_i^{\underline{c}}\right) \\ & + q_i^{\bar{c}}\left(1 - \bar{c} - q_i^{\bar{c}}\right) - \left(q_i^{\underline{c}} + q_i^{\bar{c}}\right)\left(\frac{1}{2}Q_{-i}^0 + \frac{1}{4}Q_{-i}^{\underline{c}} + \frac{1}{4}Q_{-i}^{\bar{c}}\right)\Bigg], \end{aligned} \tag{2}$$

for $i, j, k \in \{X, Y, Z\}$ where i,j, and k all differ, with $Q_i = (q_i^0, q_i^{\underline{c}}, q_i^{\bar{c}})$, $Q_j = (q_j^0, q_j^{\underline{c}}, q_j^{\bar{c}})$, $Q_k = (q_k^0, q_k^{\underline{c}}, q_k^{\bar{c}})$, $Q_{-i}^0 = q_j^0 + q_k^0$, $Q_{-i}^{\underline{c}} = q_j^{\underline{c}} + q_k^{\underline{c}}$, and $Q_{-i}^{\bar{c}} = q_j^{\bar{c}} + q_k^{\bar{c}}$.

As the common prior assumption does not hold in our setting, firm $i \in \{X, Y, Z\}$ can entertain any *type dependent* beliefs of the form that

- firm i with $c_i > 0$ expects $(c_j = 0,\ c_k = \bar{c})$ with probability r_i, $(c_j = \bar{c},\ c_k = 0)$ with probability s_i, $(c_j = 0,\ c_k = \underline{c})$ with probability t_i and $(c_j = \underline{c},\ c_k = 0)$ with probability $1 - r_i - s_i - t_i$ where $0 \leq r_i, s_i, t_i \leq 1$ and $r_i + s_i + t_i \leq 1$,

- firm i with $c_i = 0$ expects $c_j = c_k = \bar{c}$ with probability u_i, $(c_j = \underline{c},\ c_k = \bar{c})$ with probability v_i, $(c_j = \bar{c},\ c_k = \underline{c})$ with probability w_i, and $c_j = c_k = \underline{c}$ with probability $1 - u_i - v_i - w_i$ where

[8] A firm with medium or high cost is just uncertain about whether one of its competitors has medium or high cost (the third one has null cost) whereas a firm with null cost only knows that both his competitors have no null cost.

[9] This latter assumption implies that firms have different beliefs prior to being endowed with their costs. Notice that even if the profile of marginal costs would be drawn from a commonly known distribution, firm's beliefs about the other firms' marginal costs would not be common knowledge as these costs are not independently distributed: $\Pr\left(c_j = \underline{c}, c_k = \underline{c} \mid c_i = 0\right) = 1/4 > \Pr\left(c_j = \underline{c}, c_k = \underline{c} \mid c_i = \underline{c}\right) = 0$. If firm $i \in \{X, Y, Z\}$ has a null marginal cost then each $c_{-i} = (c_j, c_k)$ where $c_j, c_k \in \{\underline{c}, \bar{c}\}$ is equally likely. If firm $i \in \{X, Y, Z\}$ has a strictly positive marginal cost then it knows that $0 \in c_{-i}$ and each c_{-i} which contains 0 is equally likely.

$0 \leq u_i, v_i, w_i \leq 1$ and $u_i + v_i + w_i \leq 1$,

where i, j and k all differ and $j, k \in \{X, Y, Z\}$. Given its type dependent priors, firm i, $i \in \{X, Y, Z\}$, when endowed with marginal costs c_i, $c_i \in \{0, \underline{c}, \bar{c}\}$, maximizes its conjectural profit $\Pi_i^{c_i}$ which is given by[10]

$$\Pi_i^0 \left(q_i^0, q_j^{\underline{c}}, q_k^{\underline{c}}, q_j^{\bar{c}}, q_k^{\bar{c}}\right) = q_i^0 \left(1 - q_i^0 - u_i Q_{-i}^{\bar{c}} - v_i \left(q_j^{\underline{c}} + q_k^{\bar{c}}\right) - w_i \left(q_j^{\bar{c}} + q_k^{\underline{c}}\right) - (1 - u_i - v_i - w_i) Q_{-i}^{\underline{c}}\right),$$
$$\Pi_i^{\underline{c}} \left(q_i^{\underline{c}}, Q_j, Q_k\right) = q_i^{\underline{c}} \left(1 - \underline{c} - q_i^{\underline{c}} - r_i \left(q_j^0 + q_k^{\bar{c}}\right) - s_i \left(q_j^{\bar{c}} + q_k^0\right) - t_i \left(q_j^0 + q_k^{\underline{c}}\right) - (1 - r_i - s_i - t_i) \left(q_j^{\underline{c}} + q_k^0\right)\right),$$
$$\Pi_i^{\bar{c}} \left(q_i^{\bar{c}}, Q_j, Q_k\right) = q_i^{\bar{c}} \left(1 - \bar{c} - q_i^{\bar{c}} - r_i \left(q_j^0 + q_k^{\bar{c}}\right) - s_i \left(q_j^{\bar{c}} + q_k^0\right) - t_i \left(q_j^0 + q_k^{\underline{c}}\right) - (1 - r_i - s_i - t_i) \left(q_j^{\underline{c}} + q_k^0\right)\right),$$

where i, j and k all differ and $j, k \in \{X, Y, Z\}$.

3 Market equilibrium analyses

In this section, we first compute a closed-form solution of the equilibrium quantities by restricting the analysis to symmetric priors. We then compute a closed-form solution of the equilibrium quantities by enlarging the analysis to asymmetric priors where, for the sake of simplicity, we will always rely on interior solutions, both when analyzing markets and when deriving evolutionarily stable beliefs.[11]

3.1 First model: Conjectural symmetry by false consensus

In our first analysis firms are assumed to believe that other firms share their prior expectations. Formally, firm i, $i \in \{X, Y, Z\}$, believes $(r_i, s_i, t_i, u_i, v_i, w_i) = (r_j, s_j, t_j, u_j, v_j, w_j) = (r_k, s_k, t_k, u_k, v_k, w_k)$ $= (r, s, t, u, v, w)$ where i, j and k all differ and $j, k \in \{X, Y, Z\}$. In psychology one refers to such belief formations as false consensus effect (see Engelmann and Strobel, 2000 for a critical review and evidence). When not being aware what others (should) believe the idea is to rely on the only sample information available, namely one's own belief formation. This, of course, implies that everybody expects symmetric beliefs but that different beliefs are expected by different firms.

The market equilibrium quantities are solutions of the following system of equations: $\partial \Pi_i^0 \left(q_i^{0^*}, q_j^{\underline{c}^*}, q_k^{\underline{c}^*}, q_j^{\bar{c}^*}, q_k^{\bar{c}^*}\right) / \partial q_i^{0^*} = 0$, $\partial \Pi_i^{\underline{c}} \left(q_i^{\underline{c}^*}, Q_j^*, Q_k^*\right) / \partial q_i^{\underline{c}^*} = 0$, $\partial \Pi_i^{\bar{c}} \left(q_i^{\bar{c}^*}, Q_j^*, Q_k^*\right) / \partial q_i^{\bar{c}^*} = 0$, $r_i = r_j = r_k = r$, $s_i = s_j = s_k = s$, $t_i = t_j = t_k = t$, $u_i = u_j = u_k = u$, $v_i = v_j = v_k = v$, and $w_i = w_j = w_k = w$, for $i \in \{X, Y, Z\}$ where i, j and k all differ and $j, k \in \{X, Y, Z\}$.[12]

[10] We speak of conjectural profit as firm i's beliefs guide its behavior.

[11] Notice that due to the linearity of our models, existence and uniqueness of the equilibrium is guaranteed (see, for example, Vives, 1999, chap. 8).

[12] Given that $\partial \Pi_i^0 / \partial q_i^{0^*} = 1 - 2q_i^{0^*} + q_j^{\underline{c}^*}(u + w - 1) - q_j^{\bar{c}^*}(u + w) + q_k^{\underline{c}^*}(u + v - 1) - q_k^{\bar{c}^*}(u + v)$, $\partial \Pi_i^{\underline{c}} / \partial q_i^{\underline{c}^*} =$

Hence, in an equilibrium where priors are symmetric, firm i, $i \in \{X, Y, Z\}$, when endowed with its marginal cost chooses

$$\begin{aligned} q_i^{0^*}(r,s,u,v,w) &= \frac{\Delta\left(3\left(2u+v+w\right)-2\left(r+s\right)\right)+4\underline{c}+2}{8} \\ q_i^{\underline{c}^*}(r,s,u,v,w) &= \frac{\Delta\left(2\left(r+s\right)-\left(2u+v+w\right)\right)-4\underline{c}+2}{8} \\ q_i^{\bar{c}^*}(r,s,u,v,w) &= \frac{\Delta\left(2\left(r+s\right)-\left(2u+v+w\right)\right)-4\bar{c}+2}{8}, \end{aligned} \tag{3}$$

where $\Delta = \bar{c} - \underline{c}$. In principle Equations (3) allows for $q_i^{0^*}, q_i^{\underline{c}^*}, q_i^{\bar{c}^*} < 0$. It will, however, be shown in Section 4 that this never occurs for evolutionarily stable priors.

Optimal true expected profits of firm $i \in \{X, Y, Z\}$ are given by

$$\begin{aligned} R_i^*\left(Q_i^*, Q_{-i}^*\right) &= R_i^*\left(m_i, m, m\right) \\ &= \frac{1}{192}\Big[\left(2+4\bar{c}+\Delta\left(2C-2D+B-3A\right)\right)\left(2+4\underline{c}+\Delta\left(3A-B\right)\right) \\ &+ \left(2-4\underline{c}+\Delta\left(A-B-2C\right)\right)\left(4(1-\underline{c}-\bar{c})+2\Delta\left(B-A\right)\right)+8\left(\Delta\right)^2\Big], \end{aligned} \tag{4}$$

with $m_i = (r_i, s_i, t_i, u_i, v_i, w_i)$, $m = (r, s, t, u, v, w)$, and $Q_{-i}^* = \left(Q_{-i}^{0^*}, Q_{-i}^{\underline{c}^*}, Q_{-i}^{\bar{c}^*}\right)$ where $A = 2u_i + v_i + w_i$, $B = 2\left(r_i + s_i\right)$, $C = 2u + v + w$, and $D = 2\left(r + s\right)$. The optimal expected price is given by

$$\begin{aligned} p^*\left(q_i^{0^*}, q_i^{\underline{c}^*}, q_i^{\bar{c}^*}\right) &= 1 - q_i^{0^*}(r,s,u,v,w) - q_i^{\underline{c}^*}(r,s,u,v,w) - q_i^{\bar{c}^*}(r,s,u,v,w) \\ &= p^*\left(r,s,u,v,w\right) = \frac{1}{4} + \frac{4\bar{c} - \Delta\left(C+D\right)}{8}. \end{aligned} \tag{5}$$

If firms' prior beliefs would be equal to Nature's priors then the market solution would be given by

$$\left(q_i^{0^N}, q_i^{\underline{c}^N}, q_i^{\bar{c}^N}\right) = \left(\frac{1+\underline{c}+\bar{c}}{4}, \frac{1-2\underline{c}}{4}, \frac{1-2\bar{c}}{4}\right),$$

for $i \in \{X, Y, Z\}$. True expected profits of firm $i \in \{X, Y, Z\}$ would be given by

$$R_i^N\left(Q_i^N\right) = \frac{1}{3}\left[\left(q_i^{0^N}\right)^2 + \left(q_i^{\underline{c}^N}\right)^2 + \left(q_i^{\bar{c}^N}\right)^2\right] = R_i^N\left(\underline{c}, \bar{c}\right) = \frac{5(\underline{c}^2+\bar{c}^2) - 2(\underline{c}+\bar{c}) + 2\underline{c}\bar{c} + 3}{48},$$

$1 - \underline{c} - 2q_i^{\underline{c}^*} - q_j^{0^*}(r+t) + q_j^{\underline{c}^*}(r+s+t-1) - sq_j^{\bar{c}^*} + q_k^{0^*}(r+t-1) - tq_k^{\underline{c}^*} - rq_k^{\bar{c}^*}$, $\partial\Pi_i^{\bar{c}}/\partial q_i^{\bar{c}^*} = 1 - \bar{c} - 2q_i^{\bar{c}^*} - q_j^{0^*}(r+t) + q_j^{\underline{c}^*}(r+s+t-1) - sq_j^{\bar{c}^*} + q_k^{0^*}(r+t-1) - tq_k^{\underline{c}^*} - rq_k^{\bar{c}^*}$, second order conditions are fulfilled.

and the expected price would be equal to

$$p^N\left(\underline{c},\bar{c}\right)=\frac{1}{4}+\frac{\underline{c}+\bar{c}}{4}.$$

3.2 Second model: The case of asymmetric priors

In our evolutionary analysis we are primarily interested in evolutionarily stable monomorphisms, i.e., in stable belief constellations where all firms entertain the same beliefs. Therefore, we can derive the market equilibrium quantities by assuming partially symmetric priors.[13] Of course, deriving an equilibrium presupposes that whatever beliefs the different firms entertain this is commonly known, i.e., the sellers agree to disagree (in case of inconsistency). This does not deny that one revises one's beliefs when becoming aware of others' beliefs but only that such revisions lead finally to consistency. Becoming aware of others' beliefs could be due to communication among the firms or signaling, e.g. by organizational measures.

The market equilibrium quantities in our second model are solutions of the following system of equations:
$\partial\Pi_i^0\left(q_i^{0^{**}},q_j^{\underline{c}^{**}},q_k^{\underline{c}^{**}},q_j^{\bar{c}^{**}},q_k^{\bar{c}^{**}}\right)/\partial q_i^{0^{**}}=0$, $\partial\Pi_i^{\underline{c}}\left(q_i^{\underline{c}^{**}},Q_j^{**},Q_k^{**}\right)/\partial q_i^{\underline{c}^{**}}=0$, $\partial\Pi_i^{\bar{c}}\left(q_i^{\bar{c}^{**}},Q_j^{**},Q_k^{**}\right)/\partial q_i^{\bar{c}^{**}}$ $=0$, $r_j=r_k=r$, $s_j=s_k=s$, $t_j=t_k=t$, $u_j=u_k=u$, $v_j=v_k=v$, and $w_j=w_k=w$, for $i\in\{X,Y,Z\}$ where i,j and k all differ and $j,k\in\{X,Y,Z\}$.[14]

We refer to $m_i=(r_i,s_i,t_i,u_i,v_i,w_i)$ as firm i's beliefs type, and to $m=(r,s,t,u,v,w)$ as the same belief type of sellers j and k. In evolutionary terminology we study how an m_i-mutant fares when invading an m-monomorphic population. In an equilibrium where priors are partially asymmetric,

[13] The assertion that considering fully asymmetric priors would not modify the set of evolutionarily stable priors can be checked by looking at the formal definition of a *neutrally evolutionarily stable strategy* (Equation 9 on page 9).

[14] Again, one can easily check that the second order conditions are fulfilled.

firm i, $i \in \{X, Y, Z\}$, when endowed with its marginal cost chooses

$$
\begin{aligned}
q_i^{0^{**}}(m_i, m) &= \frac{1}{4} + \frac{\underline{c}}{2} + \frac{\Delta(B - 3D + 4A + 2C - A')}{8(2 + r + t)} \\
q_j^{0^{**}}(m_i, m) &= q_k^{0^{**}}(m_i, m) = \frac{1}{4} + \frac{\underline{c}}{2} + \frac{\Delta(-B - D + 6C - B')}{8(2 + r + t)} \\
q_i^{\underline{c}^{**}}(m_i, m) &= \frac{1}{4} - \frac{\underline{c}}{2} + \frac{\Delta(3B - D - 2C + C')}{8(2 + r + t)} \\
q_j^{\underline{c}^{**}}(m_i, m) &= q_k^{\underline{c}^{**}}(m_i, m) = \frac{1}{4} - \frac{\underline{c}}{2} + \frac{\Delta(-B + 3D - 2C + D')}{8(2 + r + t)} \\
q_i^{\bar{c}^{**}}(m_i, m) &= \frac{1}{4} - \frac{\bar{c}}{2} + \frac{\Delta(3B - D - 2C + C')}{8(2 + r + t)} \\
q_j^{\bar{c}^{**}}(m_i, m) &= q_k^{\bar{c}^{**}}(m_i, m) = \frac{1}{4} - \frac{\bar{c}}{2} + \frac{\Delta(-B + 3D - 2C + D')}{8(2 + r + t)}
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
A' &= (r + t)(2s_i - 10u_i + 4u - 5v_i + 2v - 5w_i + 2w + 2r_i), \\
B' &= (r + t)(2s_i - 2u_i - 4u - v_i - 2v - w_i - 2w + 2r_i), \\
C' &= (r + t)(2s_i + 2u_i - 4u + v_i - 2v + w_i - 2w + 2r_i), \\
D' &= (r + t)(2s_i - 6u_i + 4u - 3v_i + 2v - 3w_i + 2w + 2r_i).
\end{aligned}
$$

Optimal true expected profits of firm $i \in \{X, Y, Z\}$ are given by

$$
\begin{aligned}
& R_i^{**}\left(Q_i^{**}, Q_{-i}^{**}\right) \\
= \; & R_i^{**}(m_i, m, m) \\
= \; & \frac{1}{192}\left[4\left(3 + 2\bar{c}^2 + 4(\bar{c} - 1)\underline{c} + 6\underline{c}^2\right)\right. \\
- \; & \frac{1}{(2 + r + t)^2}\left(\Delta^2\left((-4A + A')^2 + 5B^2 - 12C^2 + 2C'^2 + 4CD - 11D^2 + 2B'(2C - C' + D)\right.\right. \\
& \qquad \left.\left. + B(-6B' + 4C + 8C' + 10D) + 2D'(4A - A' + 4B + C' - 4D)\right)\right) \\
+ \; & \frac{1}{2 + r + t}\left(4\Delta\left(\Delta(4A - A') + B'(1 - \bar{c} - \underline{c}) + C(-2 + 4\bar{c} + 8\underline{c})\right.\right. \\
& \qquad \left.\left.\left. + B(3 + 2\bar{c} - 4\underline{c}) + C'\Delta + D(-5 - 2\bar{c}) + D'(-2 + \bar{c} - \underline{c})\right)\right)\right].
\end{aligned}
\tag{7}
$$

The equilibrium price expectation (based on actual probabilities of cost types) is given by

$$\begin{aligned} p^{**}\left(Q_i^{**}, Q_{-i}^{**}\right) &= 1-\frac{1}{3}\left(q_i^{0^{**}}(m_i,m)+q_i^{\underline{c}^{**}}(m_i,m)+q_i^{\bar{c}^{**}}(m_i,m)\right) \\ &\quad -\frac{2}{3}\left(q_j^{0^{**}}(m_i,m)+q_j^{\underline{c}^{**}}(m_i,m)+q_j^{\bar{c}^{**}}(m_i,m)\right) \\ &= p^{**}(m_i,m) \\ &= \frac{1}{4}+\frac{\bar{c}\,(24+2r+12t)+10r\underline{c}-\Delta\,(2r_i+2E+F)}{24(2+r+t)}, \end{aligned} \tag{8}$$

where $E = s_i+4u_i+2v_i+2w_i+5s+2u+v+w$ and $F = 3(r+t)(2r_i+2s_i-2u_i+4u-v_i+2v-w_i+2w)$.

4 The evolutionary games

In this section the basic idea is that prior beliefs which are more profitable than others will grow or spread over time (or will be imitated). According to our two models we have to distinguish two evolutionary games. The mutant or strategy space for both evolutionary games is $\mathcal{M} = \{(r,s,t,u,v,w) \mid r+s+t \leq 1, u+v+w \leq 1, 0 \leq r,s,t,u,v,w \leq 1\}$.

Together with the true expected profit functions— Equation (4) for the first model and Equation (7) for the second model—this defines an evolutionary game G^k with $k \in \{*, **\}$ indicating the model type. The function $R_i^k(.)$ measures the true market success, i.e.,—in evolutionary terms—the reproductive success of firm $i \in \{X,Y,Z\}$. Since the markets are symmetric due to $R_X^k(Q_X,Q_Y,Q_Z) = R_Y^k(Q_Y,Q_X,Q_Z)$, $R_X^k(Q_X,Q_Y,Q_Z) = R_Z^k(Q_Z,Q_Y,Q_X)$ and $R_Y^k(Q_X,Q_Y,Q_Z) = R_Z^k(Q_X,Q_Z,Q_Y)$, the evolutionary games are symmetric and can be described as $G^k = \left(\mathcal{M}; R^k(m_X,m_Y,m_Z)\right)$.

We first show that Nature's priors are not the only neutrally evolutionarily stable strategies (Maynard Smith, 1982) when firms' priors are restricted to be symmetric. We then determine the set of neutrally evolutionarily stable priors of our second evolutionary game, and find out that Nature's priors are not part of this set, i.e., Nature's priors are not neutrally evolutionarily stable if firms can entertain asymmetric priors.

A neutrally evolutionarily stable strategy for the evolutionary game G^k is a strategy $\tilde{m} = \left(\tilde{r},\tilde{s},\tilde{t},\tilde{u},\tilde{v},\tilde{w}\right) \in \mathcal{M}$ such that, for all $m = (r,s,t,u,v,w) \neq \tilde{m}$:

$$R^k(\tilde{m},\tilde{m},\tilde{m}) \geq R^k(m,\tilde{m},\tilde{m}), \tag{9}$$

$$R^k(\tilde{m},\tilde{m},\tilde{m}) = R^k(m,\tilde{m},\tilde{m}) \Rightarrow R^k(\tilde{m},m,\tilde{m}) \geq R^k(m,m,\tilde{m}),$$

$$R^k(\tilde{m},\tilde{m},\tilde{m}) = R^k(m,\tilde{m},\tilde{m}) \text{ and } R^k(\tilde{m},m,\tilde{m}) = R^k(m,m,\tilde{m}) \Rightarrow R^k(\tilde{m},m,m) \geq R^k(m,m,m).$$

We thus extend the usual stability concept of evolutionary games by considering three players-encounters rather than bilateral ones. Otherwise we rely on the standard assumptions of evolutionary game theory (Weibull, 1995).

4.1 The evolutionary game under symmetric priors

From Equation (9), we see that in addition to being a symmetric equilibrium of the evolutionary game, a neutrally evolutionarily stable strategy $\tilde{m}^*$ must achieve a greater success than an alternative best reply m when interacting with a mixture composed of itself and the alternative best reply, and when interacting with two alternative best replies which achieve an identical success against $(m, \tilde{m}^*)$. The reproductive success of a firm is measured by the function $R^*(.)$ which has been deduced by assuming symmetric priors implying that a neutrally evolutionarily stable strategy is simply a symmetric equilibrium of G^*.

We denote by $\mathcal{M}^* \subset \mathcal{M}$ the set of symmetric Nash equilibria of the evolutionary game G^*, each element of this set being a solution of the following system of equations: $\frac{\partial R_i^*(m_i^*,m^*,m^*)}{\partial r_i^*} = 0$, $\frac{\partial R_i^*(m_i^*,m^*,m^*)}{\partial s_i^*} = 0$, $\frac{\partial R_i^*(m_i^*,m^*,m^*)}{\partial t_i^*} = 0$, $\frac{\partial R_i^*(m_i^*,m^*,m^*)}{\partial u_i^*} = 0$, $\frac{\partial R_i^*(m_i^*,m^*,m^*)}{\partial v_i^*} = 0$, $\frac{\partial R_i^*(m_i^*,m^*,m^*)}{\partial w_i^*} = 0$, and $m_i^* = m^*$.[15] We obtain that any profile of priors $m^* = (r^*, s^*, t^*, u^*, v^*, w^*) \in \mathcal{M}^*$ is characterized by $2(r^* + s^*) = 1$ and $2u^* + v^* + w^* = 1$. According to our findings, even though Nature's prior beliefs are neutrally evolutionarily stable, firms' priors which do not put equal weight on each possible costs' profile can be neutrally evolutionarily stable. For example, $m = (r, s, t, u, v, w) = \left(\frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0\right)$ is neutrally evolutionarily stable, implying that each firm $i \in \{X, Y, Z\}$ when endowed with a strictly positive cost puts equal weight on $(c_j = 0, c_k = \bar{c})$ and $(c_j = \underline{c}, c_k = 0)$, and when endowed with a null cost puts equal weight on $(c_j = \bar{c}, c_k = \bar{c})$ and $(c_j = \underline{c}, c_k = \underline{c})$. In such a case, the following costs' profiles are excluded *a priori*: $(0, \bar{c}, \underline{c})$, $(0, \underline{c}, \bar{c})$, $(\bar{c}, 0, \underline{c})$, $(\underline{c}, 0, \underline{c})$, $(\bar{c}, \bar{c}, 0)$, and $(\underline{c}, \bar{c}, 0)$ where the first argument of the costs' profile is c_i, the second argument is c_j and the third argument is c_k.

[15] Second order conditions are fulfilled as $\partial^2 R_i^* / \partial r_i^{*2} < 0$, $\partial^2 R_i^* / \partial s_i^{*2} < 0$, $\partial^2 R_i^* / \partial t_i^{*2} = 0$, $\partial^2 R_i^* / \partial u_i^{*2} < 0$, $\partial^2 R_i^* / \partial v_i^{*2} < 0$, and $\partial^2 R_i^* / \partial w_i^{*2} < 0$.

4.2 The evolutionary game under asymmetric priors

The set $\mathcal{M}^{**} \subset \mathcal{M}$ of symmetric Nash equilibria of the evolutionary game G^{**}, which is defined analogously, is given by

$$\begin{aligned} u^{**} &= \frac{1}{2}(1 - v^{**} - w^{**}) + \frac{(r^{**} + t^{**})(2 + \underline{c} + \bar{c})}{5\Delta} \\ s^{**} &= \frac{6 - 7\underline{c} + 3\bar{c} - r^{**}(4 - 8\underline{c} + 12\bar{c}) - t^{**}(4 + 2\underline{c} + 2\bar{c})}{10\Delta}. \end{aligned} \tag{10}$$

Any $m^{**} \in \mathcal{M}^{**}$ is a neutrally evolutionarily stable strategy of G^{**} since in the m^{**}-monomorphic population no mutant $m \in \mathcal{M}$ with $m \neq m^{**}$ earns a higher success than m^{**}.

The rational expectation hypothesis which is frequently employed (nearly universally in game theory) would claim that at least in the long run conjectural beliefs converge to the true ones. Such a claim is invalidated by our theoretical analysis as stated in the following proposition.

Proposition 1 *There exists no admissible cost parameters* $(\underline{c}, \bar{c})$ *yielding a* $\tilde{m} \in \mathcal{M}^{**}$ *with* $\tilde{r} = \tilde{s} = \tilde{t} = \tilde{u} = \tilde{v} = \tilde{w} = \frac{1}{4}$.

The proof derives trivially from the characterization of the set $\mathcal{M}^{**}$.

5 Discussion

We conclude by discussing the robustness of our results which are derived under the assumption that the idiosyncratic beliefs of the three firms are commonly known. Thus changing one's own beliefs does not only influence own behavior but the behavior of all the three firms as revealed by the market equilibrium derived in Section 3. Clearly such beliefs have the property of commitment devices. The literature on preference evolution (or commitment, see, e.g., Fershtman and Judd, 1987) has shown that in strategic interactions, a wide array of distortions may be evolutionarily stable *under* the assumption that players' preferences are perfectly observable.[16]

One bothering restriction of our evolutionary analysis is the symmetry of the true probabilities of the twelve possible cost constellations,[17] resembling the usual a priori-symmetry in evolutionary

[16] Recently, Heifetz, Shannon, and Spiegel (2004) have shown that the evolutionary emergence of dispositions is generic and that dispositions may remain evolutionarily viable even when the players' preferences are only *imperfectly* observed. Together with Theorem 11 in Güth and Peleg (2001), this illustrates that the rational expectation hypothesis denies any possibility to signal own or detect others' beliefs.

[17] This does not question at all the equilibrium analysis done in Section 3 nor causes any problem when trying to define the reproductive success measures where one simply substitutes the same probability (1/12) by the asymmetric ones.

game theory. Maintaining the one population-evolutionary setup in spite of the obvious true asymmetry of the three firms can be achieved by assuming that each population member is characterized by idiosyncratic belief parameters m_X, m_Y, m_Z being assigned to firm X, Y, and Z. Thus the individual beliefs m of one specific firm are more complex in the sense that they specify three sets of parameters r_i, s_i, t_i, u_i, v_i, and w_i, i.e., altogether eighteen parameters instead of six. This shows that generalizing our analysis such that Nature chooses cost parameters in non-symmetric ways causes computational but no additional conceptual problems.

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