

# Quantity Competition under Asymmetric Information without Common Priors: An Indirect Evolutionary Approach

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Preliminary — Comments welcome.

## Abstract

The *common prior assumption* asserts that the beliefs of agents in different states of the world are their posteriors based on a *common prior* and possibly some private signal. Common priors are pervasive in most economic models of incomplete information, oligopoly models with asymmetrically informed firms being no exception. We dispose of the common prior assumption in a Cournot oligopoly with uncertain costs and allow firms to entertain arbitrary priors about the other firms' cost-types. Only Nature is aware of the true probability distribution of the costs and determines via the true distribution which priors will be evolutionarily stable. To check whether the evolutionarily stable priors satisfy the *commonness requirement* we present two alternative models. In the first model, firms believe that all other firms entertain the same beliefs about the distribution of marginal costs and Nature's priors are not the only evolutionarily stable priors. In a second model with the possibility of asymmetric priors Nature's priors are not evolutionarily stable.

KEY WORDS: (Indirect) evolution — Common prior assumption — Cournot competition

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# 1 Introduction

A major breakthrough leading to the predominance of the game theoretic approach in industrial organization has been the possibility to cope with the problem of incomplete information.<sup>1</sup> Generally incomplete information prevails if at least one player entertains only probabilistic beliefs regarding the rules of the game. As shown by Harsanyi (1967–1968), such rule uncertainty can be adequately captured by introducing a commonly known but purely fictitious chance move determining the rules of the game whose results are only partially revealed such that individual information deficits are preserved.

Harsanyi (1967–1968) has further argued that beliefs should be consistent in the sense that individual beliefs are just the marginals of the same probability distribution determining the rules of the game. This assumption, usually called the Common Prior Assumption (CPA) or ‘Harsanyi Doctrine’, has been defended on the methodological basis that it allows to ‘zero in on purely informational issues’ (Aumann, 1987), and on the more philosophical basis that ‘Under the CPA, differences in probabilities express differences in information *only*’ (Aumann, 1998: italics in original).<sup>2</sup>

Several authors have not been convinced by the latter argument. Morris (1995) and Gul (1998), for example, argued that there are no fundamental or rationality-based arguments to impose the CPA beyond the methodological argument that it allows to focus on information. More importantly, a well-known implication of the Harsanyi doctrine is that agents cannot ‘agree to disagree’ (Aumann, 1976): it cannot be common knowledge that one agent holds belief  $x$  about some event while another agent has belief  $y \neq x$ . However, mutual knowledge of disagreement is an everyday experience. It should also be noticed that, as shown by Maschler (1997), one does not need to assume common priors in order to develop Harsanyi’s theory for games with incomplete information.<sup>3</sup> In the inconsistent case the fictitious chance move would be described by a vector-valued probability distribution, one for each player (see Harrison and Kreps, 1978, Güth, 1985 and Harstad and Phelps, 1997 for early applications).

Here we will not engage in a debate whether differences in prior beliefs are essential to understanding economic phenomena or whether results consistent with heterogeneous prior beliefs render economic theorizing useless. By an indirect evolutionary framework to select among arbitrary prior beliefs, we will see whether the evolutionarily stable beliefs satisfy the consistency requirement or not. The indirect evolutionary approach usually assumes that under a process of random matching *phenotypes* interact repeatedly with each other in *basic games*. When playing the basic games phenotypes interact fully rationally with each other meaning that standard game

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<sup>1</sup>See, for example, the textbook by Tirole (1988).

<sup>2</sup>Common priors are pervasive in the literature on rational expectations, bargaining under incomplete information, auctions, signalling, etc. Actually, the vast majority of the asymmetric information literature in economics and game theory assumes the CPA.

<sup>3</sup>The case of distinct priors has also been considered by Harsanyi (1967–1968); he referred to it as the “inconsistent case”.

theory can be applied to predict the results of the interaction. Additionally, it is assumed that the basic games are embedded in an evolutionary process in which some of the parameters or rules of the game evolve. These changes which emerge as a function of the outcomes of rational play in the basic games are analyzed with tools borrowed from evolutionary game theory.<sup>4</sup>

We study a homogeneous market where three firms compete in quantities with uncertain constant marginal costs. Each firm knows its own cost but, due to arbitrary prior beliefs, two firms can entertain different beliefs concerning the cost type of the third firm. A firm's unit cost level can be either low, medium or high where it is commonly known that exactly one firm has low costs, i.e., cost types are dependent. Actually all three possible cases are equally likely what, however, is known only by Nature which determines the (reproductive) success of the interacting firms accordingly. Thus if a firm is a medium or high cost-type itself, it can entertain idiosyncratic and home grown beliefs who of its competitors is a low cost-type and who not. As standard in studies of indirect evolution, we first solve the markets defined by the possible constellations of idiosyncratic beliefs. The solutions of these markets then are used to define an evolutionary market game with the possible priors as mutants and the actually resulting expected profits (which can be assessed since Nature knows the true priors) as fitness measures.<sup>5</sup> The final task is then to explore which priors constellations are evolutionary stable.

To check whether the evolutionarily stable priors satisfy the consistency requirement we present two alternative models. In the first model, firms believe that all other firms entertain the same beliefs about the distribution of the marginal costs. This first model allows us to check whether Nature's priors are the only evolutionarily stable priors. In a second model, we allow for asymmetric priors and check whether Nature's priors are evolutionarily stable.

The paper is organized as follows. In Section 2 we first describe the market environment. A closed-form solution of the equilibrium for the two models is provided in Section 3. These results are then used in Section 4 to define and "solve" (derive the evolutionarily stable priors) the evolutionary games. Section 5 is devoted to some discussion about the generality of our results and concludes.

## 2 The market environment

We consider a linear homogeneous market in which three firms compete in quantities. Each firm  $i \in \{X, Y, Z\}$  incurs a constant cost  $c_i$  per unit of production which is known only to that firm

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<sup>4</sup>Other applications are, for instance, Güth and Yaari (1992)'s study on the evolution of reciprocal behavior, the evolution of altruism by Bester and Güth (1998), of cooperative commitments by Güth and Kliemt (2000), and the coevolution of trust and its dynamic interaction with legal institutions (Güth and Ockenfels, 2002). More closely related to our work, Güth and Huck (1997) provide an evolutionary justification for the theory of monopolistic competition.

<sup>5</sup>Using true profit expectations rather than true profits can be justified by assuming a large (infinite) population and random matching of sellers. Otherwise one would have to analyze a stochastic evolutionary process (see Güth and Kliemt, 2000, for an example).

and whose level can be either 0,  $\underline{c}$ , or  $\bar{c}$  with  $0 < \underline{c} < \bar{c} \leq 1/2$ .<sup>6</sup> Given firm  $i$ 's marginal cost, we refer to its sales amounts of the homogeneous product by  $q_i^{c_i}$  with  $q_i^{c_i} \geq 0 \forall i \in \{X, Y, Z\}$ . For a given profile of marginal costs  $(c_x, c_y, c_z)$ , the inverse demand function is defined by

$$p(q_x^{c_x}, q_y^{c_y}, q_z^{c_z}) = 1 - q_x^{c_x} - q_y^{c_y} - q_z^{c_z} \quad (1)$$

if  $q_x^{c_x} + q_y^{c_y} + q_z^{c_z} \leq 1$  what will be guaranteed by optimality. It is commonly known that  $(c_x, c_y, c_z) \in \mathcal{C} = \{(0, \bar{c}, \bar{c}), (0, \bar{c}, \underline{c}), (0, \underline{c}, \bar{c}), (0, \underline{c}, \underline{c}), (\bar{c}, 0, \bar{c}), (\bar{c}, 0, \underline{c}), (\underline{c}, 0, \bar{c}), (\underline{c}, 0, \underline{c}), (\bar{c}, \bar{c}, 0), (\bar{c}, \underline{c}, 0), (\underline{c}, \bar{c}, 0), (\underline{c}, \underline{c}, 0)\}$ . In words, the fact that only one of the three firms enjoys the null marginal cost whereas the two remaining firms can have medium or high marginal costs is assumed to be common knowledge.<sup>7</sup>

Additionally, we assume that each profile of marginal costs  $(c_x, c_y, c_z) \in \mathcal{C}$  is equally likely, this fact being only known by Nature.<sup>8</sup> Accordingly, the true expected profit functions which reflect the uniform distribution of the marginal costs vectors are symmetrically defined by

$$R_i(Q_i, Q_j, Q_k) = \frac{1}{3} \left[ q_i^0 \left( 1 - q_i^0 - \frac{1}{2} (Q_{-i}^{\underline{c}} + Q_{-i}^{\bar{c}}) \right) + q_i^{\underline{c}} (1 - \underline{c} - q_i^{\underline{c}}) + q_i^{\bar{c}} (1 - \bar{c} - q_i^{\bar{c}}) - (q_i^{\underline{c}} + q_i^{\bar{c}}) \left( \frac{1}{2} Q_{-i}^0 + \frac{1}{4} Q_{-i}^{\underline{c}} + \frac{1}{4} Q_{-i}^{\bar{c}} \right) \right], \quad (2)$$

for  $i, j, k \in \{X, Y, Z\}$  where  $i, j$ , and  $k$  all differ, with  $Q_i = (q_i^0, q_i^{\underline{c}}, q_i^{\bar{c}})$ ,  $Q_j = (q_j^0, q_j^{\underline{c}}, q_j^{\bar{c}})$ ,  $Q_k = (q_k^0, q_k^{\underline{c}}, q_k^{\bar{c}})$ ,  $Q_{-i}^0 = q_j^0 + q_k^0$ ,  $Q_{-i}^{\underline{c}} = q_j^{\underline{c}} + q_k^{\underline{c}}$ , and  $Q_{-i}^{\bar{c}} = q_j^{\bar{c}} + q_k^{\bar{c}}$ .

As the common prior assumption does not hold in our setting, firm  $i \in \{X, Y, Z\}$  can entertain any *type dependent* beliefs of the form that

- firm  $i$  with  $c_i > 0$  expects  $(c_j = 0, c_k = \bar{c})$  with probability  $r_i$ ,  $(c_j = \bar{c}, c_k = 0)$  with probability  $s_i$ ,  $(c_j = 0, c_k = \underline{c})$  with probability  $t_i$  and  $(c_j = \underline{c}, c_k = 0)$  with probability  $1 - r_i - s_i - t_i$  where  $0 \leq r_i, s_i, t_i \leq 1$  and  $r_i + s_i + t_i \leq 1$ ,
- firm  $i$  with  $c_i = 0$  expects  $c_j = c_k = \bar{c}$  with probability  $u_i$ ,  $(c_j = \underline{c}, c_k = \bar{c})$  with probability  $v_i$ ,  $(c_j = \bar{c}, c_k = \underline{c})$  with probability  $w_i$ , and  $c_j = c_k = \underline{c}$  with probability  $1 - u_i - v_i - w_i$  where  $0 \leq u_i, v_i, w_i \leq 1$  and  $u_i + v_i + w_i \leq 1$ ,

where  $i, j$  and  $k$  all differ and  $j, k \in \{X, Y, Z\}$ . Given its type dependent priors, firm  $i$ ,  $i \in$

<sup>6</sup>The industry is viable as soon as  $\bar{c} < 1$ . We restrict further the marginal costs so that, in equilibrium, quantities are positive.

<sup>7</sup>A firm with medium or high cost is just uncertain about whether one of its competitors has medium or high cost (the third one has null cost) whereas a firm with null cost only knows that both his competitors have no null cost.

<sup>8</sup>This latter assumption implies that firms have different beliefs prior to being endowed with their costs. Notice that even if the profile of marginal costs would be drawn from a commonly known distribution, firm's beliefs about the other firms' marginal costs would not be common knowledge as these costs are not independently distributed:  $\Pr(c_j = \underline{c}, c_k = \underline{c} \mid c_i = 0) = 1/4 > \Pr(c_j = \underline{c}, c_k = \underline{c} \mid c_i = \underline{c}) = 0$ . If firm  $i \in \{X, Y, Z\}$  has a null marginal cost then each  $c_{-i} = (c_j, c_k)$  where  $c_j, c_k \in \{\underline{c}, \bar{c}\}$  is equally likely. If firm  $i \in \{X, Y, Z\}$  has a strictly positive marginal cost then it knows that  $0 \in c_{-i}$  and each  $c_{-i}$  which contains 0 is equally likely.

$\{X, Y, Z\}$ , when endowed with marginal costs  $c_i$ ,  $c_i \in \{0, \underline{c}, \bar{c}\}$ , maximizes its conjectural profit  $\Pi_i^{c_i}$  which is given by<sup>9</sup>

$$\begin{aligned}\Pi_i^0(q_i^0, q_j^c, q_k^c, q_j^{\bar{c}}, q_k^{\bar{c}}) &= q_i^0 \left( 1 - q_i^0 - u_i Q_{-i}^{\bar{c}} - v_i (q_j^c + q_k^{\bar{c}}) - w_i (q_j^{\bar{c}} + q_k^c) - (1 - u_i - v_i - w_i) Q_{-i}^c \right), \\ \Pi_i^c(q_i^c, Q_j, Q_k) &= q_i^c \left( 1 - \underline{c} - q_i^c - r_i (q_j^0 + q_k^{\bar{c}}) - s_i (q_j^{\bar{c}} + q_k^0) - t_i (q_j^0 + q_k^c) - (1 - r_i - s_i - t_i) (q_j^c + q_k^0) \right), \\ \Pi_i^{\bar{c}}(q_i^{\bar{c}}, Q_j, Q_k) &= q_i^{\bar{c}} \left( 1 - \bar{c} - q_i^{\bar{c}} - r_i (q_j^0 + q_k^{\bar{c}}) - s_i (q_j^{\bar{c}} + q_k^0) - t_i (q_j^0 + q_k^c) - (1 - r_i - s_i - t_i) (q_j^c + q_k^0) \right),\end{aligned}$$

where  $i, j$  and  $k$  all differ and  $j, k \in \{X, Y, Z\}$ .

### 3 Market equilibrium analyses

In this section, we first compute a closed-form solution of the equilibrium quantities by restricting the analysis to symmetric priors. We then compute a closed-form solution of the equilibrium quantities by enlarging the analysis to asymmetric priors.<sup>10</sup>

#### 3.1 First model

In our first analysis we assume that firms believe that all firms entertain the same priors. Formally, firm  $i$ ,  $i \in \{X, Y, Z\}$ , believes  $(r_i, s_i, t_i, u_i, v_i, w_i) = (r_j, s_j, t_j, u_j, v_j, w_j) = (r_k, s_k, t_k, u_k, v_k, w_k) = (r, s, t, u, v, w)$  where  $i, j$  and  $k$  all differ and  $j, k \in \{X, Y, Z\}$ . The market equilibrium quantities are solutions of the following system of equations:  $\partial \Pi_i^0(q_i^{0*}, q_j^{c*}, q_k^{c*}, q_j^{\bar{c}*}, q_k^{\bar{c}*}) / \partial q_i^{0*} = 0$ ,  $\partial \Pi_i^c(q_i^{c*}, Q_j^*, Q_k^*) / \partial q_i^{c*} = 0$ ,  $\partial \Pi_i^{\bar{c}}(q_i^{\bar{c}*}, Q_j^*, Q_k^*) / \partial q_i^{\bar{c}*} = 0$ ,  $r_i = r_j = r_k = r$ ,  $s_i = s_j = s_k = s$ ,  $t_i = t_j = t_k = t$ ,  $u_i = u_j = u_k = u$ ,  $v_i = v_j = v_k = v$ , and  $w_i = w_j = w_k = w$ , for  $i \in \{X, Y, Z\}$  where  $i, j$  and  $k$  all differ and  $j, k \in \{X, Y, Z\}$ .<sup>11</sup> Hence, in an equilibrium where priors are symmetric, firm  $i$ ,  $i \in \{X, Y, Z\}$ , when endowed with its marginal cost chooses

$$\begin{aligned}q_i^{0*}(r, s, u, v, w) &= \frac{\Delta (3(2u + v + w) - 2(r + s)) + 4\underline{c} + 2}{8} \\ q_i^{c*}(r, s, u, v, w) &= \frac{\Delta (2(r + s) - (2u + v + w)) - 4\underline{c} + 2}{8} \\ q_i^{\bar{c}*}(r, s, u, v, w) &= \frac{\Delta (2(r + s) - (2u + v + w)) - 4\bar{c} + 2}{8},\end{aligned}\tag{3}$$

where  $\Delta = \bar{c} - \underline{c}$ . In principle Equations (3) allows for  $q_i^{0*}, q_i^{c*}, q_i^{\bar{c}*} < 0$ . It will, however, be shown in Section 4 that this never occurs for evolutionarily stable priors.

<sup>9</sup>We speak of conjectural profit as firm  $i$ 's beliefs guide its behavior.

<sup>10</sup>Notice that due to the linearity of our models, existence and uniqueness of the equilibrium is guaranteed (see, for example, Vives, 1999, chap. 8).

<sup>11</sup>Given that  $\partial \Pi_i^0 / \partial q_i^{0*} = 1 - 2q_i^{0*} + q_j^{c*}(u + w - 1) - q_j^{\bar{c}*}(u + w) + q_k^{c*}(u + v - 1) - q_k^{\bar{c}*}(u + v)$ ,  $\partial \Pi_i^c / \partial q_i^{c*} = 1 - \underline{c} - 2q_i^{c*} - q_j^{0*}(r + t) + q_j^{c*}(r + s + t - 1) - sq_j^{\bar{c}*} + q_k^{0*}(r + t - 1) - tq_k^{c*} - rq_k^{\bar{c}*}$ ,  $\partial \Pi_i^{\bar{c}} / \partial q_i^{\bar{c}*} = 1 - \bar{c} - 2q_i^{\bar{c}*} - q_j^{0*}(r + t) + q_j^{c*}(r + s + t - 1) - sq_j^{\bar{c}*} + q_k^{0*}(r + t - 1) - tq_k^{c*} - rq_k^{\bar{c}*}$ , second order conditions are fulfilled.

Optimal true expected profits of firm  $i \in \{X, Y, Z\}$  are given by

$$\begin{aligned} R_i^* (Q_i^*, Q_{-i}^*) &= R_i^* (m_i, m, m) \\ &= \frac{1}{192} \left[ (2 + 4\bar{c} + \Delta (2C - 2D + B - 3A)) (2 + 4\underline{c} + \Delta (3A - B)) \right. \\ &\quad \left. + (2 - 4\underline{c} + \Delta (A - B - 2C)) (4(1 - \underline{c} - \bar{c}) + 2\Delta (B - A)) + 8(\Delta)^2 \right], \end{aligned} \quad (4)$$

with  $m_i = (r_i, s_i, t_i, u_i, v_i, w_i)$ ,  $m = (r, s, t, u, v, w)$ , and  $Q_{-i}^* = (Q_{-i}^{0*}, Q_{-i}^{\underline{c}*}, Q_{-i}^{\bar{c}*})$  where  $A = 2u_i + v_i + w_i$ ,  $B = 2(r_i + s_i)$ ,  $C = 2u + v + w$ , and  $D = 2(r + s)$ . The optimal expected price is given by

$$\begin{aligned} p^* (q_i^{0*}, q_i^{\underline{c}*}, q_i^{\bar{c}*}) &= 1 - q_i^{0*} (r, s, u, v, w) - q_i^{\underline{c}*} (r, s, u, v, w) - q_i^{\bar{c}*} (r, s, u, v, w) \\ &= p^* (r, s, u, v, w) = \frac{1}{4} + \frac{4\bar{c} - \Delta (C + D)}{8}. \end{aligned} \quad (5)$$

If firms' prior beliefs would be equal to Nature's priors then the market solution would be given by

$$(q_i^{0N}, q_i^{\underline{c}N}, q_i^{\bar{c}N}) = \left( \frac{1 + \underline{c} + \bar{c}}{4}, \frac{1 - 2\underline{c}}{4}, \frac{1 - 2\bar{c}}{4} \right),$$

for  $i \in \{X, Y, Z\}$ . True expected profits of firm  $i \in \{X, Y, Z\}$  would be given by

$$R_i^N (Q_i^N) = \frac{1}{3} \left[ (q_i^{0N})^2 + (q_i^{\underline{c}N})^2 + (q_i^{\bar{c}N})^2 \right] = R_i^N (\underline{c}, \bar{c}) = \frac{5(\underline{c}^2 + \bar{c}^2) - 2(\underline{c} + \bar{c}) + 2\underline{c}\bar{c} + 3}{48},$$

and the expected price would be equal to

$$p^N (\underline{c}, \bar{c}) = \frac{1}{4} + \frac{\underline{c} + \bar{c}}{4}.$$

### 3.2 Second model

In our evolutionary analysis we are primarily interested in evolutionarily stable monomorphisms, i.e., in stable belief constellations where all firms entertain the same beliefs. Therefore, we can derive the market equilibrium quantities by assuming partially symmetric priors.<sup>12</sup>

The market equilibrium quantities in our second model are solutions of the following system of equations:  $\partial \Pi_i^0 (q_i^{0**}, q_j^{\underline{c}*}, q_k^{\bar{c}*}, q_j^{\underline{c}*}, q_k^{\bar{c}*}) / \partial q_i^{0**} = 0$ ,  $\partial \Pi_i^{\underline{c}} (q_i^{\underline{c}*}, Q_j^{**}, Q_k^{**}) / \partial q_i^{\underline{c}*} = 0$ ,  $\partial \Pi_i^{\bar{c}} (q_i^{\bar{c}*}, Q_j^{**}, Q_k^{**}) / \partial q_i^{\bar{c}*} = 0$ ,  $r_j = r_k = r$ ,  $s_j = s_k = s$ ,  $t_j = t_k = t$ ,  $u_j = u_k = u$ ,  $v_j = v_k = v$ , and  $w_j = w_k = w$ , for  $i \in \{X, Y, Z\}$  where  $i, j$  and  $k$  all differ and  $j, k \in \{X, Y, Z\}$ .<sup>13</sup> We refer to  $m_i = (r_i, s_i, t_i, u_i, v_i, w_i)$  as firm  $i$ 's beliefs type, and to  $m = (r, s, t, u, v, w)$  as the same belief type of sellers  $j$  and  $k$ . In evolutionary terminology we study how an  $m_i$ -mutant fares when invading

<sup>12</sup>The assertion that considering fully asymmetric priors would not modify the set of evolutionarily stable priors can be checked by looking at the formal definition of a *neutrally evolutionarily stable strategy* (Equation 9 on page 8).

<sup>13</sup>Again, one can easily check that the second order conditions are fulfilled.

an  $m$ -monomorphic population. In an equilibrium where priors are partially asymmetric, firm  $i$ ,  $i \in \{X, Y, Z\}$ , when endowed with its marginal cost chooses

$$\begin{aligned}
q_i^{0**}(m_i, m) &= \frac{1}{4} + \frac{\underline{c}}{2} + \frac{\Delta(B - 3D + 4A + 2C - A')}{8(2 + r + t)} \\
q_j^{0**}(m_i, m) &= q_k^{0**}(m_i, m) = \frac{1}{4} + \frac{\underline{c}}{2} + \frac{\Delta(-B - D + 6C - B')}{8(2 + r + t)} \\
q_i^{\underline{c}**}(m_i, m) &= \frac{1}{4} - \frac{\underline{c}}{2} + \frac{\Delta(3B - D - 2C + C')}{8(2 + r + t)} \\
q_j^{\underline{c}**}(m_i, m) &= q_k^{\underline{c}**}(m_i, m) = \frac{1}{4} - \frac{\underline{c}}{2} + \frac{\Delta(-B + 3D - 2C + D')}{8(2 + r + t)} \\
q_i^{\bar{c}**}(m_i, m) &= \frac{1}{4} - \frac{\bar{c}}{2} + \frac{\Delta(3B - D - 2C + C')}{8(2 + r + t)} \\
q_j^{\bar{c}**}(m_i, m) &= q_k^{\bar{c}**}(m_i, m) = \frac{1}{4} - \frac{\bar{c}}{2} + \frac{\Delta(-B + 3D - 2C + D')}{8(2 + r + t)}
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
A' &= (r + t)(2s_i - 10u_i + 4u - 5v_i + 2v - 5w_i + 2w + 2r_i), \\
B' &= (r + t)(2s_i - 2u_i - 4u - v_i - 2v - w_i - 2w + 2r_i), \\
C' &= (r + t)(2s_i + 2u_i - 4u + v_i - 2v + w_i - 2w + 2r_i), \\
D' &= (r + t)(2s_i - 6u_i + 4u - 3v_i + 2v - 3w_i + 2w + 2r_i).
\end{aligned}$$

Optimal true expected profits of firm  $i \in \{X, Y, Z\}$  are given by

$$\begin{aligned}
&R_i^{**}(Q_i^{**}, Q_{-i}^{**}) \\
&= R_i^{**}(m_i, m, m) \\
&= \frac{1}{192} [4(3 + 2\bar{c}^2 + 4(\bar{c} - 1)\underline{c} + 6\underline{c}^2) \\
&- \frac{1}{(2 + r + t)^2} (\Delta^2((-4A + A')^2 + 5B^2 - 12C^2 + 2C'^2 + 4CD - 11D^2 + 2B'(2C - C' + D) \\
&\quad + B(-6B' + 4C + 8C' + 10D) + 2D'(4A - A' + 4B + C' - 4D))] \\
&+ \frac{1}{2 + r + t} (4\Delta(\Delta(4A - A') + B'(1 - \bar{c} - \underline{c}) + C(-2 + 4\bar{c} + 8\underline{c}) \\
&\quad + B(3 + 2\bar{c} - 4\underline{c}) + C'\Delta + D(-5 - 2\bar{c}) + D'(-2 + \bar{c} - \underline{c})))]
\end{aligned} \tag{7}$$

The equilibrium price expectation (based on actual probabilities of cost types) is given by

$$\begin{aligned}
p^{**}(Q_i^{**}, Q_{-i}^{**}) &= 1 - \frac{1}{3} \left( q_i^{0**}(m_i, m) + q_i^{c**}(m_i, m) + q_i^{\bar{c}**}(m_i, m) \right) \\
&\quad - \frac{2}{3} \left( q_j^{0**}(m_i, m) + q_j^{c**}(m_i, m) + q_j^{\bar{c}**}(m_i, m) \right) \\
&= p^{**}(m_i, m) \\
&= \frac{1}{4} + \frac{\bar{c}(24 + 2r + 12t) + 10r\underline{c} - \Delta(2r_i + 2E + F)}{24(2 + r + t)},
\end{aligned} \tag{8}$$

where  $E = s_i + 4u_i + 2v_i + 2w_i + 5s + 2u + v + w$  and  $F = 3(r+t)(2r_i + 2s_i - 2u_i + 4u - v_i + 2v - w_i + 2w)$ .

## 4 The evolutionary games

In this section the basic idea is that prior beliefs which are more profitable than others will grow or spread over time (or will be imitated). According to our two models we have to distinguish two evolutionary games. The mutant or strategy space for both evolutionary games is  $\mathcal{M} = \{(r, s, t, u, v, w) \mid r + s + t \leq 1, u + v + w \leq 1, 0 \leq r, s, t, u, v, w \leq 1\}$ .

Together with the true expected profit functions—Equation (4) for the first model and Equation (7) for the second model—this defines an evolutionary game  $G^k$  with  $k \in \{*, **\}$  indicating the model type. The function  $R_i^k(\cdot)$  measures the true market success, i.e.,—in evolutionary terms—the reproductive success of firm  $i \in \{X, Y, Z\}$ . Since the markets are symmetric due to  $R_X^k(Q_X, Q_Y, Q_Z) = R_Y^k(Q_Y, Q_X, Q_Z)$ ,  $R_X^k(Q_X, Q_Y, Q_Z) = R_Z^k(Q_Z, Q_Y, Q_X)$  and  $R_Y^k(Q_X, Q_Y, Q_Z) = R_Z^k(Q_X, Q_Z, Q_Y)$ , the evolutionary games are symmetric and can be described as  $G^k = (\mathcal{M}; R^k(m_X, m_Y, m_Z))$ .

We first show that Nature's priors are not the only neutrally evolutionarily stable strategies (Maynard Smith, 1982) when firms' priors are restricted to be symmetric. We then determine the set of neutrally evolutionarily stable priors of our second evolutionary game, and find out that Nature's priors are not part of this set, i.e., Nature's priors are not neutrally evolutionarily stable if firms can entertain asymmetric priors.

A neutrally evolutionarily stable strategy for the evolutionary game  $G^k$  is a strategy  $\tilde{m} = (\tilde{r}, \tilde{s}, \tilde{t}, \tilde{u}, \tilde{v}, \tilde{w}) \in \mathcal{M}$  such that, for all  $m = (r, s, t, u, v, w) \neq \tilde{m}$ :

$$R^k(\tilde{m}, \tilde{m}, \tilde{m}) \geq R^k(m, \tilde{m}, \tilde{m}), \tag{9}$$

$$R^k(\tilde{m}, \tilde{m}, \tilde{m}) = R^k(m, \tilde{m}, \tilde{m}) \Rightarrow R^k(\tilde{m}, m, \tilde{m}) \geq R^k(m, m, \tilde{m}),$$

$$R^k(\tilde{m}, \tilde{m}, \tilde{m}) = R^k(m, \tilde{m}, \tilde{m}) \text{ and } R^k(\tilde{m}, m, \tilde{m}) = R^k(m, m, \tilde{m}) \Rightarrow R^k(\tilde{m}, m, m) \geq R^k(m, m, m).$$



## 4.1 First evolutionary game

From Equation (9), we see that in addition to being a symmetric equilibrium of the evolutionary game, a neutrally evolutionarily stable strategy  $\tilde{m}^*$  must achieve a greater success than an alternative best reply  $m$  when interacting with a mixture composed of itself and the alternative best reply, and when interacting with two alternative best replies which achieve an identical success against  $(m, \tilde{m}^*)$ . The reproductive success of a firm is measured by the function  $R^*(\cdot)$  which has been deduced by assuming symmetric priors implying that a neutrally evolutionarily stable strategy is simply a symmetric equilibrium of  $G^*$ .

We denote by  $\mathcal{M}^* \subset \mathcal{M}$  the set of symmetric Nash equilibria of the evolutionary game  $G^*$ , each element of this set being a solution of the following system of equations:  $\frac{\partial R_i^*(m_i^*, m^*, m^*)}{\partial r_i^*} = 0$ ,  $\frac{\partial R_i^*(m_i^*, m^*, m^*)}{\partial s_i^*} = 0$ ,  $\frac{\partial R_i^*(m_i^*, m^*, m^*)}{\partial t_i^*} = 0$ ,  $\frac{\partial R_i^*(m_i^*, m^*, m^*)}{\partial u_i^*} = 0$ ,  $\frac{\partial R_i^*(m_i^*, m^*, m^*)}{\partial v_i^*} = 0$ ,  $\frac{\partial R_i^*(m_i^*, m^*, m^*)}{\partial w_i^*} = 0$ , and  $m_i^* = m^*$ .<sup>14</sup> We obtain that any profile of priors  $m^* = (r^*, s^*, t^*, u^*, v^*, w^*) \in \mathcal{M}^*$  is characterized by  $2(r^* + s^*) = 1$  and  $2u^* + v^* + w^* = 1$ . According to our findings, even though Nature's prior beliefs are neutrally evolutionarily stable, firms' priors which do not put equal weight on each possible costs' profile can be neutrally evolutionarily stable. For example,  $m = (r, s, t, u, v, w) = (\frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0)$  is neutrally evolutionarily stable, implying that each firm  $i \in \{X, Y, Z\}$  when endowed with a strictly positive cost puts equal weight on  $(c_j = 0, c_k = \bar{c})$  and  $(c_j = \underline{c}, c_k = 0)$ , and when endowed with a null cost puts equal weight on  $(c_j = \bar{c}, c_k = \bar{c})$  and  $(c_j = \underline{c}, c_k = \underline{c})$ . In such a case, the following costs' profiles are excluded *a priori*:  $(0, \bar{c}, \underline{c})$ ,  $(0, \underline{c}, \bar{c})$ ,  $(\bar{c}, 0, \underline{c})$ ,  $(\underline{c}, 0, \bar{c})$ ,  $(\bar{c}, \bar{c}, 0)$ , and  $(\underline{c}, \bar{c}, 0)$  where the first argument of the costs' profile is  $c_i$ , the second argument is  $c_j$  and the third argument is  $c_k$ .

## 4.2 Second evolutionary game

The set  $\mathcal{M}^{**} \subset \mathcal{M}$  of symmetric Nash equilibria of the evolutionary game  $G^{**}$ , which is defined analogously, is given by

$$\begin{aligned} u^{**} &= \frac{1}{2}(1 - v^{**} - w^{**}) + \frac{(r^{**} + t^{**})(2 + \underline{c} + \bar{c})}{5\Delta} \\ s^{**} &= \frac{6 - 7\underline{c} + 3\bar{c} - r^{**}(4 - 8\underline{c} + 12\bar{c}) - t^{**}(4 + 2\underline{c} + 2\bar{c})}{10\Delta} \end{aligned} \quad (10)$$

Any  $m^{**} \in \mathcal{M}^{**}$  is a neutrally evolutionarily stable strategy of  $G^{**}$  since in the  $m^{**}$ -monomorphic population no mutant  $m \in \mathcal{M}$  with  $m \neq m^{**}$  earns a higher success than  $m^{**}$ .

The rational expectation hypothesis which is frequently employed (nearly universally in game theory) would claim that at least in the long run conjectural beliefs converge to the true ones. That this is not so obvious is revealed by

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<sup>14</sup>Second order conditions are fulfilled as  $\partial^2 R_i^*/\partial r_i^{*2} < 0$ ,  $\partial^2 R_i^*/\partial s_i^{*2} < 0$ ,  $\partial^2 R_i^*/\partial t_i^{*2} = 0$ ,  $\partial^2 R_i^*/\partial u_i^{*2} < 0$ ,  $\partial^2 R_i^*/\partial v_i^{*2} < 0$ , and  $\partial^2 R_i^*/\partial w_i^{*2} < 0$ .

**Theorem** *There exists no admissible cost parameters  $(\underline{c}, \bar{c})$  yielding a  $\tilde{m} \in \mathcal{M}^{**}$  with  $\tilde{r} = \tilde{s} = \tilde{t} = \tilde{u} = \tilde{v} = \tilde{w} = \frac{1}{4}$ .*

The proof is obvious.

In view of the analysis done by Güth and Peleg (2001) this result had to be expected. It is essentially due to our (rather strong) assumption that idiosyncratic beliefs of the three sellers are commonly known. Thus changing one's own beliefs does not only influence own behavior but the behavior of all the three sellers as revealed by the market equilibrium derived in Section 3. Clearly such beliefs have the property of commitment devices what questions their adjustment to the true beliefs. This illustrates that the rational expectation hypothesis must exclude any possibility to signal own or detect others' beliefs,<sup>15</sup> an equally strong assumption.

## 5 Discussion

Why should Nature guarantee a prior-symmetry of the three firms? Since there seems to be no convincing answer to this question the most bothering restriction of our evolutionary analysis is the symmetry of the true probabilities of the twelve possible cost constellations. This does not question at all the equilibrium analysis done in Section 3 nor causes any problem when trying to define the reproductive success measures where one simply substitutes the same probability (1/12) by the asymmetric ones.

Thus the essential problem is only the usual a priori-symmetry in evolutionary game theory. In the context at hand this means to maintain the one population-evolutionary setup in spite of the obvious true asymmetry of the three firms. This can be achieved by assuming that each population member is characterized by idiosyncratic belief parameters  $m_X, m_Y, m_Z$  being assigned to firm  $X, Y$ , and  $Z$ . Thus the individual beliefs  $m$  of one specific firm are more complex in the sense that they specify three sets of parameters  $r_i, s_i, t_i, u_i, v_i$ , and  $w_i$ , i.e., altogether eighteen parameters instead of six. This shows that generalizing our analysis such that Nature chooses cost parameters in non-symmetric ways causes many computational but no additional conceptual problems. An interesting question would be, for instance, when such asymmetry still allows for monomorphic neutrally evolutionarily stable strategies.

Another possibility is to maintain Nature's symmetry but to impose stronger notions of evolutionary stability, i.e., to refine the concept of neutral evolutionary stability in order to derive more informative predictions from evolutionary stability. Similar to the refinement approach (Selten, 1975) in orthodox game theory one can, for instance, rely on rare trembles where these however may not reflect just a philosophical idea but rather capture rare "mistakes" or "mutations" which actually occur. In the case at hand the obvious idea would be that a monomorphic population does not exclude nearby beliefs. More formally, one could require that for all small  $\epsilon > 0$  there exists a  $\epsilon$ -neighborhood  $N(\tilde{m})$  of the monomorphic population  $\tilde{m}$  such that  $\tilde{m}$  is a best reply of

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<sup>15</sup>This follows from the general result of Güth and Peleg (2001).

firm  $X$  against (one or) both other firms entertaining a belief  $m \in N(\tilde{m})$  (and the other the  $\tilde{m}$  belief).

This already indicates that there exists quite some arbitrariness in refining the neutrally evolutionary stability concept for structurally richer markets as in our study (for a refinement approach in a simpler set up see Güth and Huck, 1997). In the equilibrium analysis for one model in Section 3 it is simpler to assume that in the  $\tilde{m}$ -monomorphic population, invaded by a rare mutant belief  $m$ , both other firms  $Y$  and  $Z$  rely on the same neighboring belief  $\tilde{m} \in N(\tilde{m})$ .

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