Evolution in Imperfect Commitment Bargaining
– Strategic versus Ignorant Types –

September 26, 2003

Abstract:

To commit credibly in bargaining is crucial: In the ultimatum game with its one–sided early commitment power the "proposer" gets (nearly) the whole pie while the "responder" is left with (almost) nothing. When both parties commit simultaneously the (a)symmetric Nash(1950)–bargaining solution, however, predicts (more or less) equal shares. Relying on a continuous connection of these two games by a one parameter–family of games (Fischer et al. 2003), we distinguish two behavioral dispositions, namely (1) neglecting early commitment and (2) reacting to it strategically. Based on their payoff implications we derive the evolutionarily stable behavioural disposition. This sheds new light on the hypothesis that in experiments participants neglect subtle strategic aspects like the sequence of moves.

Keywords: bargaining, imperfect commitment, ultimatum game, Nash demand game, evolutionary game theory.

JEL-classification: C72; C78

1 Introduction

Bargaining may extend over many rounds. If one round requires little time as usually is the case in experimental studies of bargaining, costs of delaying an agreement are negligible. All what then matters strategically is who commits first in the final round. The two prominent paradigms for capturing what happens in the last round are the Nash(1950, 1953)–demand game and the ultimatum game (Güth 1976; Güth et al. 1982). Whereas the Nash-demand game relies on simultanous commitments of both parties, in ultimatum bargaining one party (the proposer) can preempt the other (the responder) by a take it or leave it-offer.

According to game theory the fixed pie p is distributed rather unevenly in the ultimatum game with the proposer getting nearly the whole pie whereas in the Nash-demand game all individually rational distributions (i.e. each party receiving a positive amount) are strict equilibria. In the latter case one of the strict equilibria is selected by the Nash(1950, 1953)–bargaining solution, namely the 50:50–split of the pie.[1]

The extreme sensitivity of the game theoretic solution to the time structure of commitments has no direct analogue in experimentally observed behaviour. In ultimatum experiments proposers usually leave a considerable share of the pie to responders, on average close to 40% (see Güth 1995 and Roth 1995 for surveys). The reason is that low offers, e.g. below 1/3 of the pie, are frequently rejected. More specifically, the modal and median outcome is the 50:50–split of the pie which is also the modal and median outcome of the simultaneous demand game (see Kalisch et al. 1954, Nydegger and Own 1974 and Roth and Malouf 1979 for early experiments inspired by the demand game and the Nash–bargaining solution). In view of such results it has been argued that boundedly rational participants do not always pay attention to strategic aspects like sequential decision making but often generate their choices by imagining a more abstract situation like the demand game (see Pull 1999 and Selten 2000).

For the purpose of our evolutionary analysis we do not concentrate on the two extreme decision processes, namely the simultaneous demand game and the sequential ultimatum game. We rather consider a one parameter–family of games, theoretically and experimentally studied in Fischer et al. (2003) which connects the two games continuously. The basic idea is that

although one party (X) always commits first to its demand x, this decision will only be revealed to the second mover (Y) with probability w where $0 \leq w \leq 1$. If x is revealed to Y, player Y can either accept the residual $y(x) = p - x$ (yielding x for X and $y(x)$ for Y) or not (implying conflict with 0-payoff for both). If x is not revealed, then Y has to choose its own demand y. Although X and Y decide one after another in the latter case, they state their demands independently as in the case of the demand game.[2]

In section 2 we define the class of games G(w) and their risk-dominant solution (Harsanyi and Selten 1988). Here, we also report the main experimental finding that participants tend to neglect subtle strategic aspects in the sense of a varying parameter w.

To justify such an insensitivity evolutionarily we distinguish in section 3 between two behavioral dispositions, namely satisficing I–types who do not react at all to w ("Invariant-types") and R–types who strategically react to parameter w ("Reacting-types").[3] With the help of the solution payoffs for all possible encounters, we define an evolutionary game whose (evolutionarily stable) equilibria and dynamics are derived in section 4. The basic conclusions from this analysis are discussed in section 6.

2 G(w)–games and Experiments

Two parties X and Y can share a pie (monetary reward) p that – without loss of generality – is set to $p = 1$. The decision process is as follows:

- First, X chooses his demand x with $0 \leq x \leq 1$.

- Then, a chance move

    - either reveals x to Y with probability $w \in [0, 1]$

    - or not (with probability $1 - w$).

- After

    - learning about x player Y can accept $y(x) = 1 - x$ (so that X earns x and Y earns $y(x)$) or not, implying 0–payoffs for both,

– otherwise player Y chooses his demand y with $0 \leq y \leq 1$. If $x + y \leq 1$, X earns x and Y earns y, if not both get 0–payoffs.

Whereas G(1) can be solved by repeated elimination of dominated strategies[4], every outcome (x, y) with $x, y > 0$ and $x + y = 1$ is a strict equilibrium outcome in games G(w) with $w < 1$. One can first truncate G(w)–games by replacing their proper subgames occurring with probability w by their subgame perfect equilibrium payoffs. By doing so, so called N–truncations are formed, in which the second mover with probability w automatically accepts any offer $y(x) = 1 - x$ ($x \geq 0$) and with probability $(1 - w)$ states her own demand y independently. By applying risk dominance (Fischer et al. 2003) one can solve these N–truncations uniquely. More specifically, $(x^\star(w), y^\star(w))$ with

$$x^\star(w) = \frac{1}{2 - w} \text{ and } y^\star(w) = \frac{1 - w}{2 - w}$$

risk dominates all other strict equilibria.[5] The combination of the risk dominant solution of the N–truncations with the acceptance of revealed demands x defines the solution for the G(w)–games. Therefore we have a unique risk dominant solution for all G(w)–games where the first mover demands $x^\star(w)$ and the uninformed second mover demands $y^\star(w)$ whereas the informed responder always accepts. The risk dominant solution continuously connects the Nash–bargaining solution $x^\star(0) = 1/2 = y^\star(0)$ for $w = 0$ with the ultimatum behaviour $x^\star(1) = 1, y^\star = 0$ for $w = 1$.

Contrary to this prediction, Fischer et al. (2003) in their experimental analysis of G(w)–games only observe mild reactions to changes in w. Subjects are confronted with four different levels of w ($w \in \{.1, .3, .7, .9\}$) repeatedly.[6] Though on average observed reactions are qualitatively as predicted and statistically significant, their size is far below the predicted levels or even negligible. Furthermore, many subjects show identical median demands for all w's, and learning strengthens w–invariance with equal splits mostly being modal and median demands for all w's.

3 Evolutionary Setup

In what follows we allow for asymmetric solutions even in the case of the demand game. Hence, instead of restricting ourselves to the limit case $x^\star(w = 0) = y^\star(w = 0) = 1/2$ and specifying w–invariance by postulating $x(w) = y(w) = 1/2$ for all $w \in [0, 1]$, we allow for all common w–invariant aspiration levels or "acceptance thresholds" a of second mover Y with $0 \le a \le 1/2$. For each aspiration level a we will explore whether this $I(a)$–type of w–insensitivity or the R–type playing game theoretically or a bimorphism of both survives evolutionary selection.[7]To do so we first derive for all aspiration levels $a \in [0, 1/2]$ and all possible encounters of types the solution payoffs. We then define the evolutionary game with behavioural dispositions as strategies.

While R–types are assumed to play according to the risk dominant solution (section 2) in both roles, $I(a)$–types are insensitive to w in the following way:

- In the role of X an $I(a)$–type with $0 \le a \le 1/2$ demands $x = 1 - a$.

- In the role of Y an $I(a)$–type only accepts those offers $y(x) = 1 - x$ with $y(x) \ge a$ if x is learned and demands $y = a$ if not.

Regardless of its own type an individual can encounter an $I(a)$–type or an R–type. Hence, the mutant or strategy set of the evolutionary game is $M = \{I(a), R\}$. What remains is to define the success function $S(m, \tilde{m})$ for all pairs $m, \tilde{m} \in M$. We define $S(m, \tilde{m})$ as the expected payoff of an m–type when confronting an $\tilde{m}$–type. This neglects the stochastic nature of individual payoffs which can be justified by assuming (as usual in evolutionary game theory) an infinite population with random matching. More specifically, we assume a one-population interpretation such that each member of the infinite population plays the $G(w)$–game twice, once in the role of X and once in the role of Y with two randomly selected partners. Instead of performing the analysis for one arbitrary probability $w \in [0, 1]$ only, we rely on the whole continuum in the sense that all probabilities $w \in [0, 1]$ are equally likely where the random choice of w is independently performed for each randomly matched pair.[8]

Clearly, an $I(a)$–type earns 1 against other $I(a)$–types, namely $1 - a$ as X and a as Y. Similarly, an R–type earns 1 when confronting his own type twice, i.e. $x^\star(w)$ as X and $y^\star(w) =$

$1 - x^\star(w)$ as Y. Hence, it only remains to define $S(I(a), R)$ and $S(R, I(a))$ in order to derive the success matrix $\boldsymbol{S}$. For these cases we must first find the boundary value of w up to which there is agreement (disagreement). If type R plays as X against $I(a)$, they find an agreement for

$$\frac{1-w}{2-w} \geq a \;\; \text{or} \;\; w \leq \frac{1-2a}{1-a} = \beta \,. \tag{1}$$

If type R plays as Y against $I(a)$ and learns about $x = a$, he always accepts the offer, yielding a for him and $1 - a$ for $I(a)$. If, however, Y of type R does not learn about x, an agreement with an $I(a)$–type only results for $w \geq \beta$. In detail the success functions of mixed encounters are:

$$\begin{aligned} S(R, I(a)) &= \int_0^\beta \frac{1}{(2-w)} \, dw + \int_0^1 wa \, dw + \int_\beta^1 \frac{(1-w)^2}{(2-w)} \, dw \\ &= \frac{a^3 + a^2 - a}{2(1-a)^2} + \ln(2) =: \rho \end{aligned} \tag{2}$$

and

$$\begin{aligned} S(I(a), R) &= \int_0^\beta w(1-a) \, dw + \int_\beta^1 (1-a) \, dw + \int_0^\beta w\frac{1-w}{2-w} \, dw + \int_0^\beta (1-w)a \, dw \\ &= \frac{2 - 4a + a^2}{1-a} - 2\ln(2 - 2a) =: \tau \end{aligned} \tag{3}$$

The success matrix $\boldsymbol{S}$ is therefore given by

$$\boldsymbol{S} = \begin{pmatrix} 1 & \rho \\ \tau & 1 \end{pmatrix} \tag{4}$$

In figure 1 the success levels $\rho = S(R, I(a))$ and $\tau = S(I(a), R)$ are plotted as functions of a showing that ρ is monotonically decreasing in a whereas τ is inversely hump-shaped.

[Insert Figure 1 about here.]

4 Evolutionary Stability of Types

The payoff matrix $\boldsymbol{S}$ has three symmetric equilibria: two strict ones in pure and one in mixed strategies. The two equilibria in pure strategies are the monomorphisms $\sigma_R = (R,\ R)$ and $\sigma_I = (I(a),\ I(a))$. The equilibrium in mixed strategies $s_m = (p^*,\ 1 - p^*)$, with $p^* \in [0, 1]$ as the probability for the R–type satisfies (see also figure 2):

$$p^* = \frac{1 - \rho}{2 - \tau - \rho} \tag{5}$$

An evolutionarily stable strategy s^* has to satisfy

$$s^* \boldsymbol{S} s^{*\prime} \geq s \boldsymbol{S} s^{*\prime} \qquad \text{for all } s = (q, 1 - q) \text{ with } q \in [0, 1] \tag{6}$$

$$\text{and } \ s^* \boldsymbol{S} s' \ > s \boldsymbol{S} s' \qquad \text{for all } s \neq s^* \text{ with } s^* \boldsymbol{S} s^{*\prime} = s \boldsymbol{S} s^{*\prime} \tag{7}$$

Due to their strictness, the two monomorphisms with the equilibrium strategies $s_R^\star = (1, 0)$, resp. $s_I^\star = (0, 1)$ obviously satisfy these conditions and are therefore evolutionarily stable. The intuition behind this result is that a mutant invading a monomorphic population will always earn less due to $\rho,\ \tau < 1$ and therefore eventually disappear.

[Insert Figure 2 about here.]

By definition the equilibrium in mixed strategies s_m satisfies $s_m \boldsymbol{S} s'_m = s \boldsymbol{S} s'_m$ for all s, i.e. all mixed strategies s are best replies to s_m. However, it does not satisfy condition (7): When confronting the alternative strategy s_R, strategy s_m earns $s_m \boldsymbol{S} s'_R = 1 - (\tau - 1)^2/(2 - \tau - \rho)$, which due to $0 < (\tau,\ \rho) < 1$ is smaller than 1. The fitness of s_R in the s_R monomorphic population however is $(1, 0)\boldsymbol{S}(1, 0)' = 1$. Consequently s_m is not evolutionarily stable.

To study the population dynamics leading to an R, resp. $(1, 0)$– or to an $I(a)$, resp. $(0, 1)$–monomorphism, we reformulate parts of our setup. A mixed strategy vector of the form $s = (p,\ 1 - p)$ now represents the distribution of strategies in the population. This means that the population is composed of a fraction p of R–types and a fraction $1 - p$ of $I(a)$–types. We are now able to introduce the replicator dynamics in continuous time for our evolutionary

setup:

$$\frac{dp}{dt} = p_t \frac{(1,0)\boldsymbol{S}s_t' - s_t\boldsymbol{S}s_t'}{s_t\boldsymbol{S}s_t'} \quad \text{and} \quad \frac{d(1-p)}{dt} = (1-p_t)\frac{(0,1)\boldsymbol{S}s_t' - s_t\boldsymbol{S}s_t'}{s_t\boldsymbol{S}s_t'} \tag{8}$$

The fraction of subjects playing strategy R increases (decreases) if their expected payoff is higher (lower) than the average payoff of the entire population. In this dynamic setting our game now has three rest points, which are equivalent to the three equilibria $(1,0)$, $(0,1)$ and s_m. The first two are also asymptotically stable, which follows from their strictness (implying generic stability sets). The equilibrium in mixed strategies s_m satisfies $s_m\boldsymbol{S}s_m' = s\boldsymbol{S}s_m'$ for all s including $s = (1,0)$, and therefore is a rest point, too. However, as

$$\frac{dp}{dt} \begin{cases} > 0 & \text{for } p > p^* \\[2mm] = 0 & \text{for } p = p^* \\[2mm] < 0 & \text{for } p < p^* \end{cases}$$

it is not asymptotically stable. The one–dimensional dynamics following from this are plotted in figure 3 for $a = 1/2$.

[Insert Figure 3 about here.]

5 Rational Satisficing

Although satisficing is not in line with payoff maximization as such, it could result from fast learning. More specifically, one could assume that for any in evolutionary time t given population composition $s_t = (p_t,\, 1 - p_t)$ the aspiration level a quickly (or compared to evolutionary time, "instantaneously") adjusts in the sense that $I(a)$–types learn to use better levels of a given the population composition s_t.

Individual learning in general might lead to encounters of an $I(a)$–type and an $I(\tilde{a})$–type with $a > \tilde{a}$ and, of course, of both types with R–types. It can be shown that in an $I(a)$, $I(\tilde{a})$–encounter, the $I(a)$ type earns more[9] but that $I(\tilde{a})$ earns more than $I(a)$ when both confront an R–type and R-types are suffiently frequent.[10]

Thus whether learning will increase or decrease the a–level depends crucially on the relative frequency of $I(a)$– or R–types. In an $I(a)$–monomorphic population the generally prevailing aspiration level a will increase until $a = 1/2$. Conversely, when an R–monomorphism has evolved the more successful mutant would have to rely on $a = 0$ (see the $S(I(a), R)$ curve in figure 1). This, however, does not suffice to render the invasion by $I(0)$–types as successful, i.e. the R–monomorphism will not be endangered even by the best adjusted $I(a)$–mutant.

6 Discussion

In our evolutionary analysis of how different behavioural dispositions, namely "satisficing" and "playing strategically", evolve we show that the two monomorphisms are stable. A bi-morphism could only prevail temporarily. Furthermore, assuming that there is quick learning by all $I(a)$–type players, an aspiration level of $a = 1/2$ would be the observed outcome in an $I(a)$–monomorphism where one should keep in mind that we have imposed $a \in [0, 1/2]$. In our view, this suggests a new and innovative justification of experimentally observed equal splitting as reported in the introduction. Since in an R–monomorphic population a mutant would converge to $a = 0$ and since neither the R– nor the $I(0)$–behavior are observed in bargaining experiments, observed bargaining behaviour can be explained by an universal $I(1/2)$–disposition when playing such games. Although the R-monomorphism is also stable, it is not confirmed experimentally in G(w)–games. One reason could be that our "game of life" comprises other games discouraging the R–disposition. When behaving strategically in private relationships, one may for instance be punished by ostracism. bur since our setup did not capture such possibilities, this is just speculation.

Notes

[1]Later generalizations allow for incomplete information (see Harsanyi and Selten 1972).

[2]Game theoretically the timing of decisions does not matter as long as it is not possible for one party to condition its choice on the other's decision like in the ultimatum game, where the responder can reject certain demands and accept others. One speaks of the positional order protocol when it is commonly known that players decide sequentially but do not learn the earlier choices. Hence, $w = 1$ corresponds to the ultimatum game and

$w = 0$ to the demand game and the two extremal games are continuously connected by the class of games $G(w)$, defined for all $w \in [0, 1]$.

[3]Note that with strategically we mean behaviour according to the game theoretic prediction assuming common knowledge of rationality.

[4]For Y all strategies rejecting $y(x) = 1 - x$ with $x < 1$ are dominated. Anticipating this for X all strategies x with $x < 1$ are dominated so that (in the continuous case) $x^\star = 1$ and acceptance of all $y(x) = 1 - x$ is the solution.

[5]Risk dominance (see Harsanyi and Selten 1988) is usually intransitive, so that here the risk dominant solution is very intuitive.

[6]In addition Fischer et al. (2003) report on a $G(w)$–experiment where they employed the strategy method, i.e. they confronted each subject with all four w's in each round. In this experiment, they observed even smaller (and insignificant) reactions to changes in w.

[7]What we explore first are therefore encounters of individuals with identical thresholds a in the interval $0 \leq a \leq 1/2$. Later we will allow for different a–levels among the $I(a)$–types.

[8]We therefore avoid or at least weaken the common drawback of most evolutionary studies which analyze the evolution of fundamental behavioural aspects for a highly specific game only. Getting closer to the "game of life" (see also Güth and Napel 2002) requires that players face a variety of games.

[9]For $1/2 \geq a > \tilde{a} \geq 0$ an X–player of type $I(a)$ earns $1 - a$ for sure whereas the Y–player of type $I(\tilde{a})$ only earns $wa + (1-w)\tilde{a}$. If the two types reverse their roles both earn nothing. Thus due to $1 - a \geq 1/2 > wa + (1-w)\tilde{a}$ the $I(a)$–type is more successful than his more modest counterpart $I(\tilde{a})$.

[10]Given a population composition of $s_t = (p_t, 1 - p_t)$ with a p_t close to 1, the profit maximizing level a must maximize $\pi = p_t \tau + 1 - p_t$. As the function $\tau(a)$ has a global inflection point at $a = 0$, this constitutes the maximum point for $a \in [0, 0.5]$ (see figure 1). Thus the lower level fares better against R when p_t is sufficiently large.

References

Fischer, S., W. Güth, W. Müller and A. Stiehler 2003. 'From Ultimatum to Nash Bargaining: Theory and Experimental Evidence' *Discussion Papers on Strategic Interaction* 7–2003.

Güth, W. 1976. 'Towards a More General Study of v. Stackelberg-Situations' *Journal of Institutional & Theoretical Economics* 132(4): 592–608.

Güth, W. 1995. 'On Ultimatum Bargaining Experiments-A Personal Review' *Journal of Economic Behavior & Organization* 27(3): 329–44.

Güth, W. and S. Napel 2002. 'Inequality Aversion in a Variety of Games – An Indirect Evolutionary Analysis' *Discussion Papers on Strategic Interaction* 23–2002.

Güth, W., R. Schmittberger and B. Schwarze 1982. 'An Experimental Analysis of Ultimatum Bargaining' *Journal of Economic Behavior & Organization* 3(4): 367–88.

Harsanyi, J. C. and R. Selten 1972. 'A Generalized Nash Solution for Two–Person Bargaining Games with Incomplete Information' *Management Science* 18(5): 80–106.

Harsanyi, J. C. and R. Selten 1988. *A general theory of equilibrium selection in games* Cambridge, Mass. and London: MIT Press.

Kalisch, G. K., J. Milnor, J. F. Nash and E. Nering 1954. 'Some Experimental n-Person Games' in: Thrall, R., C. Coombs and R. Davis (eds.) 'Decision Processes,' Wiley.

Nash, J. 1950. 'The Bargaining Problem' *Econometrica* 18(2): 155–162.

Nash, J. 1953. 'Two-Person Cooperative Games' *Econometrica* 21(1): 128–140.

Nydegger, R. V. and G. Own 1974. 'Two-Person Bargaining: An Experimental Test of the Nash Axioms' *International Journal of Game Theory,* 3: 239 – 249.

Pull, K. 1999. 'What is the fair wage? – a model of as–if–co–operation' *Quint – Essenzen* 58. Discussion Paper.

Roth, A. and M. Malouf 1979. 'Game-Theoretic Models and the Role of Information in Bargaining' *Psychological Review* 86: 574–594.

Roth, A. E. 1995. 'Bargaining Experiments' in: Kagel, J. H. and A. E. Roth (eds.) 'The handbook of experimental economics,' Princeton University Press 253–348.

Selten, R. 2000. 'Thünen–Vorlesung: Eingeschränkte Rationalität und Ökonomische Motivation' *Zeitschrift für Wirtschafts– und Sozialwissenschaften* 9(special issue): 129–157.
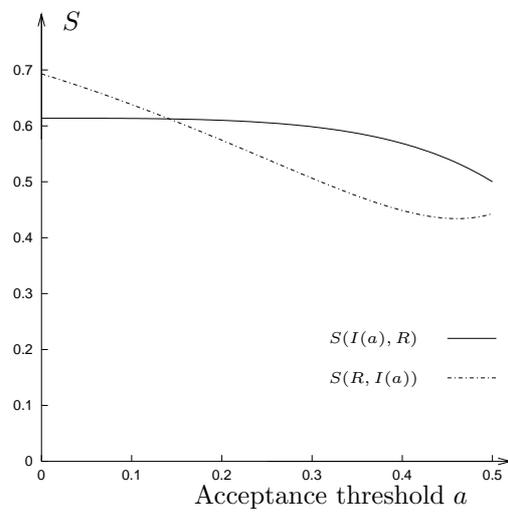
List of Figures

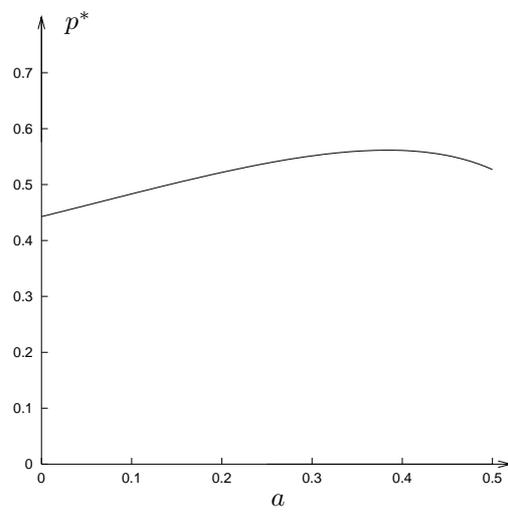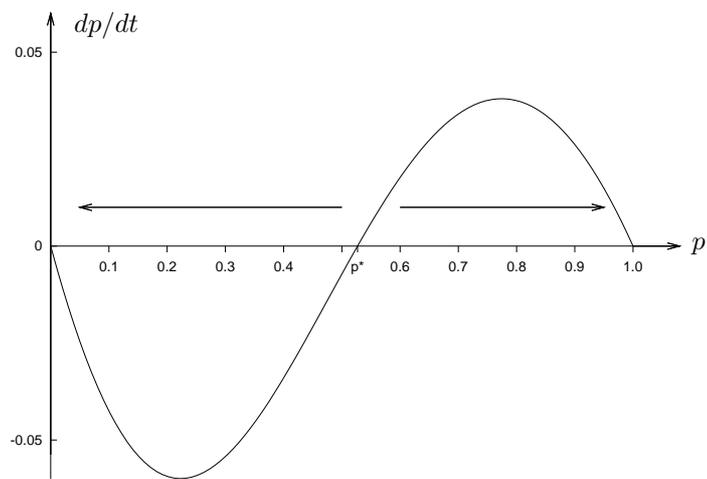Figure 1: $S(R, I(a))$ plotted against $S(I(a), R)$.

14

Figure 2: $p^\star$ as a function of a

Figure 3: Replicator dynamics for $a = \frac{1}{2}$