Buying Versus Hiring
– An Indirect Evolutionary Approach –

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Abstract

On an otherwise symmetric oligopoly market with stochastic demands for heterogeneous products firms can either hire an employee or partner or buy the required labor input on the labor market. Whereas the wage of hired labor does not depend on the realization of stochastic demand, the price of bought labor input reacts positively to product demand. We first solve the market by deriving the equilibrium price vector. We then assume that the number of hiring firms will tend to increase when hiring firms make higher profits than non-hiring firms. We explicitly derive the stationary distribution of the thus defined stochastic adaptation process.

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1 Introduction

Institutions can either be selected by anticipating their likely implications in the form of institutional design (order policy, mechanism design, etc.) or just evolve based on some kind of competition for survival. Examples of the first approach are the fiction of a social contract behind the veil of ignorance (e.g. Harsanyi, 1953 and 1955, and Rawls, 1971) or the decision for specific rules of democratic voting (e.g. direct vs. indirect democracy, minimum voting shares of parties to be presented in parliament[1]). On the other hand, it is rather likely that the basic principles of modern legal systems, e.g. *in dubio pro reo* or who has the *burden of proof*, have evolved over time and been shaped by previous experience. Even in the case of money where one might see a case of institutional design (by declarative acts, e.g., when introducing the euro), evolutionary explanations seem to be crucial (see, e.g., Wärneryd, 1989).

Although our formal setup allows for the usual optimal design approach[2], we favor the second approach of letting the institutional composition of markets evolve rather than being rationally deliberated. More specifically, we consider a simple homogeneous oligopoly market where production requires labor which can be hired (the institution of hired hands which, in some way, looks like partial slavery) or bought (the more symmetric situation of trade). Since demand levels and wages are stochastic, hiring is safer than buying.

Such an institutional analysis is crucial since modern life depends on a division of labor to a degree which is probably beyond what could reasonably be expected in the past. Obtaining labor input can rely on trade (the trading partners specialize in the production of particular commodities which are exchanged on the product market) or in forming larger production units or firms where the partners can perform special tasks[3]. The question of when to organize a firm's division of labor by trade or by production is one of the basic economic problems. It was discussed earlier by Coase (1937) and is still the subject of lively debate (Jensen, 2000, Lewin and Phelan, 2002).

Here we will not discuss all aspects that matter like transaction costs, the incompleteness of delivery and employment contracts, etc. Rather than assuming that the way of organizing a division of labor is rationally chosen, we will allow it to evolve according to past success of alternative forms of division of labor. More

[1]It is surprising that such "barriers to entry" are consciously introduced and even celebrated (as guaranteeing political stability) when designing democracies, but apparently considered an evil in economic markets (see "Barriers to New Competition", Bain, 1956).

[2]Since it is open for polymorphisms, it might even turn out to be a more interesting exercise since what is optimal might depend on the institutional choice of others. In any case, our results will be rather informative as to what to expect from such an analysis.

[3]In view of the many ways to guarantee labor supply, which firms use in modern economies, it is, of course, a simplification to distinguish only these two cases and neglect all aspects of codetermination, incentive payment, etc.

specifically, we study a standard oligopoly market which we modify by assuming that demand levels are stochastic. In order to produce, firms need labor input which they can obtain by either hiring an employee (or partner) or by buying labor input on a labor market whose price depends on stochastic product demand. In the tradition of indirect evolution,

- we first determine the market results for all possible numbers n of hiring firms among the altogether s firms on the otherwise symmetric market and
- then define a stochastic process by assuming that the number of hiring (and non-hiring) firms adapts monotonically to the difference in the profits of hiring and non-hiring firms.

The transition matrix of this process can be explicitly described. Therefore, we are able to calculate the limit stationary distribution. In section 2, we describe and solve the market model. These results allow us to determine, in section 3, the stochastic adaptation process. It is stochastic due to the uncertainty of demand and, possibly, to an adaptation rate smaller than 1 and a mutation rate larger than 0.

2 The market model

Suppose there are s firms on the market who sell a homogeneous product. The only costs of selling are due to labor input requirements. Market demand for the product is supposed to be stochastic. Uncertainty in market demand is caused by fluctuations in macroeconomic variables. Before uncertainty in demand is resolved, the firms have to decide either to hire labor force at a fixed wage rate or to wait for the resolution of uncertainty and to buy the amount of labor in the labor market, necessary to meet product demand.

A hiring firm can hire an employee at a constant wage rate $\bar{w}$ regardless of the realization of demand. All hiring firms will be indexed by i and their number denoted by n where, of course, $0 \leqq n \leqq s$. Instead of hiring an employee, one may buy as much labor as needed on the labor market after the random shock has been realized. However, buying firms face a fluctuating wage rate w. A buying firm will be indexed by j and their number denoted by m with $0 \leqq m \leqq s$ where, of course, $m + n = s$.

Let x_h denote firm $h's$ amount of sales and X denote total amount of sales. The market price is fluctuating and given by

$$p(X) = z - X, \tag{1}$$

where z is a random variable with probability distribution $\phi(\cdot)$ whose mass is concentrated on the compact interval $[\underline{z}, \bar{z}] \subset I\!R_{++}$ with $0 < \underline{z} < \bar{z}$.

To keep the model as simple as possible, the labor unit is chosen such that each unit of output requires one labor unit. Let N (and resp. M) denote the set of hiring (and resp. buying) firms. The profit functions can then be defined as follows:

$$For \;\; i \in N \;\; : \qquad \pi_i(z) := (z - X)x_i - k, \tag{2}$$

$$for \;\; j \in M \;\; : \qquad \pi_j(z) := (z - X)x_j - w(z)x_j = (z - X - w(z))x_j, \tag{3}$$

where $k := \bar{w}x_i^*(\bar{z})$ denotes the fixed labor costs of a hiring firm which are generated by a fixed salary $\bar{w}$ and the firm's plan to provide maximal equilibrium demand $x_i^*(\bar{z})$ of firm i. The wage rate $\bar{w}$ of a hired employee will be fixed such that a risk-neutral worker is indifferent to working at a firm i or at a firm j.

For any given z, firms calculate their profit maximizing amount of sales and their profits, which implies that

$$For \;\; i \in N \;\; : \qquad x_i^*(z) = \frac{z + mw(z)}{s+1}, \tag{4}$$

$$for \;\; j \in M \;\; : \qquad x_j^*(z) = \frac{z - (n+1)w(z)}{s+1}, \tag{5}$$

$$for \;\; i \in N \;\; : \qquad \pi_i^*(z) = \frac{(z + mw(z))^2}{(s+1)^2} - k, \tag{6}$$

$$for \;\; j \in M \;\; : \qquad \pi_j^*(z) = \frac{(z - (n+1)w(z))^2}{(s+1)^2}. \tag{7}$$

Since an employee should be indifferent to being hired by a firm in N or M, we require

$$\bar{w} = \int w(z) d\phi(z).$$

For our evolutionary analysis we assume that expected profits are never negative whenever defined. From (6) and (7) it is obvious that this can be easily achieved by low enough wages $w(z)$ in the various states z of nature.

3 Model Dynamics

In the previous section, the one period optimal amounts of sales were derived for any given and commonly known number n of hiring firms and m of buying firms. In this section, we drop the assumption of a fixed number of hiring and buying firms. We suppose that the number of firms pursuing a particular employment strategy may evolve in an evolutionary process. More precisely, we determine the number n of hiring firms at a given point t in time endogenously. Typically, in such a process the number of hiring firms may either increase or decrease by 1 during a sufficiently small time interval $[t, t+h]$. The tendency of the number of hiring firms to increase or decrease will be governed by the expected payoff advantage of hiring firms over buying firms. Since the total number of firms is supposed to remain constant (equal to s), we are able to model the evolution of the number of hiring firms as a bounded birth and death process whose state variable n is constrained to the set $\mathcal{S} = \{0, 1, \ldots, s\}$.

The evolutionary forces driving this process are essentially based on expected profit differences between the two types of employing firms. More precisely, we define

$$\Delta(n) := [E_i^*(n) - E_j^*(n)],$$

where $E_i^*(n) := E(\pi_i^*(z))$ and $E_j^*(n) := E(\pi_j^*(z))$ denote the expected profit of a hiring (or resp. buying) firm. If $n = s$ (or resp. $n = 0$), that is, if there is no buying (or resp. hiring) firm in the market, the expected profit $E_j^*(n)$ (and resp. $E_i^*(n)$) is set equal to zero.

That expected success determines the evolutionary drift can be justified by assuming infinitely many interactions before an adaptation of n. Clearly, under such assumptions the variance of i- and resp. j-earnings and therefore of their difference becomes 0. Thus, the expected or average success differential $\Delta(n)$ decides whether the average composition parameter n (of all infinitely many markets) will likely increase or decrease.

Another assumption is that for all composition parameters n all s competitors decide rationally. In other words, we let n evolve, that is, no firm decides rationally whether to buy or to hire, although in sales competition firms are perfect profit maximizers. We view such dichotomy as an advantage of the indirect evolutionary approach (see Berninghaus, Güth and Kliemt, 2002, for a discussion how this is related and in fact generalizes the evolutionary approach). Nevertheless, it must, of course, be justified why n evolves and other market behavior is rationally deliberated.

One possibility to justify the dichotomy between institutional and sales choices is that the latter are far more frequent so that rationality in market decision may actually be justified as a rest point of appropriate learning[4] (the so-called 'as if' justification of rationality, see Friedman, 1953). Another argument could be to view hiring versus buying later as a cultural trait like, for instance, the cultural differences in allowing or condemning the business of money lenders. After all, hiring labor looks like partial slavery (during worktime the employer can dictate what to do). It seems at least worthwhile to explore more thoroughly how such inherited traits will (culturally) evolve over time.

[4]In view of the infinitely many encounters before an adaptation of n, any initial phase of finitely many encounters to learn rationality would not matter.

A birth and death process is completely characterized by its birth and death rates $\{\lambda_n, \mu_n\}_n$ $(n = 0, 1, \ldots, s)$. For example, λ_n is approximately equal to the probability that n will increase by 1 during a sufficiently small time interval (birth rate), while μ_n is the probability that n will decrease by 1 (death rate). In our model we assume that the birth and death rates are determined by expected profit differences $\Delta(\cdot)$ together with a mutation rate $\epsilon > 0$ which describes additional but less important evolutionary forces (noise).[5]

Definition 1 *For $n \in \{1, \ldots, s-1\}$ and $\beta > 0$:*

$$\lambda_n := \beta[\Delta(n)]^+ + \epsilon, \tag{8}$$
$$\mu_n := \beta[\Delta(n)]^- + \epsilon, \tag{9}$$

for $n = 0$

$$\lambda_0 := \beta[\Delta(0)]^+ + \epsilon, \tag{10}$$
$$\mu_0 := 0, \tag{11}$$

for $n = s$

$$\lambda_s := 0, \tag{12}$$
$$\mu_s := \beta[\Delta(s)]^- + \epsilon. \tag{13}$$

β is a positive parameter which measures the impact of payoff differences on the intensity of the birth and death process. By varying this parameter, one can vary how fast firms switch their hiring strategy. The number of hiring firms cannot grow anymore if all firms employ the hiring strategy, and it cannot decrease if all firms employ the buying strategy. Therefore, we set $\mu_0 = 0$ and $\lambda_s = 0$. Since we have $[\Delta(0)]^+ = 0$, definition (10) becomes $\lambda_0 = \epsilon$. Since all firms make nonnegative profits, we have $[\Delta(s)]^- = 0$ (because of $E_j^*(s) = 0$ and $E_i^*(s) \geqq 0$) which implies $\mu_s = \epsilon$ (see definition (13)).

An important point of the adaptive dynamics specified above is that it concerns the market composition parameter n and not an individual tendency in each of the s firms. This can be justified by a short enough time interval allowing only one firm to adjust within such a time interval, for instance a randomly selected $i(j)$-firm if $\Delta(n)$ is positive (negative). The advantage is that one avoids the possibility that all firms switch all of a sudden from one institution to another, which is possible in case of individually independent adaptation dynamics (e.g. Kandori, Mailath, and Rob, 1993).

[5]Note that we use the following convention: Let $a \in I\!R$, then a^+ is equal to a if $a > 0$ and equal to 0 if $a \leqq 0$. Analogously, we set a^- equal to $-a$ if $a < 0$ and equal to 0 if $a \geqq 0$.

4 Dynamic convergence

It is a well-known fact (e.g. van Doorn, 1980) from the theory of birth and death processes that the process, specified in definition 1, admits a unique ergodic limit distribution for n, provided the birth and death rates (except for μ_0 and λ_s) are strictly positive.

Theorem 1 *(v. Doorn, 1980, p. 91) Suppose $\epsilon > 0$, then there exists a unique ergodic distribution ν^ϵ for the evolutionary process which is explicitly defined by*

$$\nu_n^\epsilon = \frac{\xi_n}{\sum_{k=0}^{s} \xi_k} \tag{14}$$

with $\xi_0 = 1$ *and* $\xi_k = \frac{\lambda_0 \lambda_1 \cdots \lambda_{k-1}}{\mu_1 \mu_2 \cdots \mu_k}$.

Now we want to analyze the limit distribution ν^ϵ in more detail. The crucial term in the analysis is the profit difference $\Delta(n)$ which can be explicitly written (after some lengthy calculations, see Appendix I) as

$$\begin{aligned} \Delta(n) \quad &= \quad \frac{2}{s+1} \int z w(z) d\phi(z) + \frac{(s-1-2n)}{s+1} \int w^2(z) d\phi(z) - \\ & \quad\quad \frac{\bar{z} + (s-n) w(\bar{z})}{s+1} \int w(z) d\phi(z). \end{aligned} \tag{15}$$

It is easy to see from (15) that $\Delta(n)$ is a linear function in n whose slope[6] is given by

$$\Delta'(n) = \frac{1}{s+1} [w(\bar{z}) \int w(z) d\phi(z) - 2 \int w^2(z) d\phi(z)]. \tag{16}$$

Therefore, $\Delta(n)$ is either monotonically increasing or decreasing in n. Consequently, there might exist a critical number $n^* \in I\!R$ such that λ_n (μ_n) is equal to ϵ for $n \leqq n^*$ ($n > n^*$) or vice versa. With this number we could predict the long-run behavior of the evolutionary process in question. In order to learn more about the critical number n^*, we make the following simplifying assumptions:

A1) $\phi(\cdot)$ is the uniform distribution over the nonempty interval $[\underline{z}, \bar{z}]$,

A2) $w(\cdot)$ is linearly increasing in z, that is, $w(z) := az$ (for $a > 0$).

Monotonicity of $w(\cdot)$ in z is a reasonable assumption when high z-values are supposed to be associated with good macroeconomic performance and high employment levels, because rather inelastic labor supply equilibrium wage rates

[6] For the sake of an easy description, we will interpret n as a real number $n \in [0, s]$ rather than an integer in that range whenever this is convenient.

increase when labor demand increases as in an economic boom. By assumption A2) we postulate a very simple form of monotonicity. Our results would not change substantially for an arbitrary increasing function $w(\cdot)$. A1) is a simplifying assumption allowing us to deduce rather easily some explicit facts about $\Delta(n)$.

Theorem 2 *Suppose assumptions A1) and A2) hold, then function* $\Delta(\cdot)$ *is decreasing in* n.

Proof: By computing[7] the integrals in (16), we obtain the following expression for the slope of $\Delta(n)$

$$\begin{aligned}\Delta'(n) &= \frac{a^2}{(s+1)(\bar{z}-\underline{z})}[\frac{\bar{z}}{2}(\bar{z}^2-\underline{z}^2)-\frac{2}{3}(\bar{z}^3-\underline{z}^3)] = \\ & \frac{a^2}{6(s+1)}[-4\underline{z}^2-\bar{z}^2-\underline{z}\bar{z})] < 0\end{aligned}$$

due to $(\bar{z}^2-\underline{z}^2)=(\bar{z}-\underline{z})(\bar{z}+\underline{z})$ and $(\bar{z}^3-\underline{z}^3)=(\bar{z}-\underline{z})(\bar{z}^2+\bar{z}\underline{z}+\underline{z}^2)$.

q.e.d.

It follows from theorem 2 that the critical number n^* may be determined as follows:

1. $\Delta(\cdot)$ intersects the interval $[0,s]$ from above,
2. $\Delta(\cdot)$ intersects the nonnegative real line at a point $n^* > s$,
3. $\Delta(\cdot)$ has no intersection with the nonnegative real line.

Case 1 is the most interesting one since both types of firms may survive in the long run. Note that one could easily impose constraints for $\underline{z}$ and $\bar{z}$ such that only case 1 is possible. Let us nevertheless also consider the borderline cases 2 and 3. Here one type of firm will disappear from the market or will only have a small survival probability (either all birth rates or all death rates are equal to ϵ). In the following theorem, it is demonstrated that case 3 will not occur for some plausible assumptions concerning the parameters of our model.

A3) The inequalities $s \geq 3$ and $a > 0.2$ hold.

A4) $(1-a(s+2))(\bar{z}^2+\bar{z}\underline{z})+2\underline{z}^2(2+a(s-1)) > 0$.

Assumption A3) is only a mild restriction, given that 1) we want to consider more general markets than duopoly markets and that 2) we require a sufficiently

[7] For more detailed calculations, see Appendix II.

strong dependence between the states z of macroeconomic performance and the market wage rate $w(z)$. It will be shown below that assumption A3) guarantees that $\Delta(n)$ intersects the nonnegative real line at a point $n^* < s$. A4) is a purely technical assumption which implies $\Delta(0) > 0$ and therefore excludes case 3. In the following theorem, we analyze the relevance of our assumptions for the position of n^* on the real line.

Theorem 3 *Suppose assumptions A1)-A4) hold, then we are in case 1, i.e.* $\Delta(n)$ *intersects the interval* $[0, s^*]$ *from above in its interior implying* $0 < n* < s$.

Proof: We express the difference function $\Delta(\cdot)$ as the difference of a constant and the function

$$\Delta(n) = \frac{1}{6(s+1)}(A - nB)$$

which is linear in n where

$$A := a[(1 - a(s+2))(\bar{z}^2 + \bar{z}\underline{z}) + 2\underline{z}^2(2 + a(s-1))]$$

and

$$B := a^2(4\underline{z}^2 + \bar{z}^2 + \bar{z}\underline{z}).$$

In Appendix III, we give a more detailed derivation of A. B has already been calculated in the proof of theorem 2, where we showed that $B = -\Delta'(n)(6(s+1))$.

Obviously, the statement in the theorem is true if it can be shown that a) $\Delta(0) = \frac{1}{6(s+1)}A > 0$ and b) $\Delta(s) = \frac{A-sB}{6(s+1)} < 0$.

$A > 0$ follows immediately from A4). We can express $\Delta(s)$ explicitly by[8]

$$\Delta(s) = \frac{a}{6(s+1)}[(1 - 2a(s+1))(\bar{z}^2 + \bar{z}\underline{z}) + 2\underline{z}^2(2 - a(3s+1))].$$

It follows from A3) that $(1 - 2a(s+1)) < 0$ and $(2 - a(3s+1)) < 0$ which implies $\Delta(s) < 0$.

q.e.d.

One has to take care in handling assumptions A3) and A4) together. In A3) we postulate lower bounds for the parameters s and a, while in A4) there are implicitly given upper bounds for these parameters, which are, moreover, interrelated in a rather complicated way. Assumption A4) only assures $\Delta(s) < 0$ which is compatible with a situation where either $\Delta(0) > 0$ or $\Delta(0) < 0$. In the former case, a bimorphism results, that is, both types of firms (hiring and buying) survive in the market. In the latter case, only the buying firms will survive.

[8]For an explicit derivation, see Appendix IV.

Looking more closely at A4), we can see that the left-hand side of the inequality A4) is composed of two parts, namely the term $(1-a(s+2))(\bar{z}^2+\bar{z}\underline{z})$, which may become negative if a and/or s is large enough. This effect of large parameters a and s may be compensated by the second term $2\underline{z}^2(2+a(s-1))$ which is always positive according to A3). However, one has to consider the different weights of both expressions. Since the relation $\bar{z} > \underline{z}$ holds, the weight $(\bar{z}^2+\bar{z}\underline{z})$ of the negative part on the left-hand side of A.4) is larger than the weight $\underline{z}^2$ of the positive part: this difference in weights mainly depends on the difference $(\bar{z}-\underline{z})$ which (in case of uniform distributions) captures the volatility of demand. We will set this difference appropriately such that inequality A4) is satisfied. A simple numerical example will illustrate the generic (due to using strict inequalities) feasibility of our assumptions in the next section.

5 A numerical illustration

First, we will discuss assumption A4) in more detail. Denote by

$$p(a,s) := (1-a(s+2))(\bar{z}^2+\bar{z}\underline{z}) + 2\underline{z}^2(2+a(s-1))$$

the left-hand side of A4), which has to be positive in order to guarantee that a bimorphism evolves (provided A3) is also satisfied). Obviously, the upper bound $\bar{z}$ of the set of states of macroeconomic performance can be simply expressed as a function of the lower bound $\underline{z}$ by $\bar{z} = b\underline{z}$ (with $b > 1$). Therefore, we can substitute the model parameters and obtain the expression

$$\begin{aligned} p(a,s) &= (1-a(s+2))(b^2\underline{z}^2+b\underline{z}^2) + 2\underline{z}^2(2+a(s-1)) \\ &= \underline{z}^2[(b^2+b)(1-a(s+2)) + 2(2+a(s-1))] \\ &= \underline{z}^2 f(a,s) \end{aligned}$$

with

$$f(a,s) := (b^2+b)(1-a(s+2)) + 2(2+a(s-1)). \tag{17}$$

Now we have reformulated the problem of finding the parameter constellations guaranteeing assumption A4) into the problem of finding parameter constellations (a,s) for which function $f(\cdot)$ is strictly positive. From the definition of $f(\cdot)$ it is easy to see why the size of the difference $(\bar{z}-\underline{z})$ is relevant for the results, which has already been discussed in the previous section. The size of this difference is measured by the parameter $b(>1)$. In the graphical plots of Figure 1, the graph of $f(\cdot)$ over the a,s-plane is illustrated for selected values of b.

For the plots we assumed lower bounds $a = 0.3$ and $s = 3$ in light of assumption A3). The influence of b is clearly pointed out in Figure 1: with increasing b, the values of a and s have to be smaller in order to guarantee the positivity

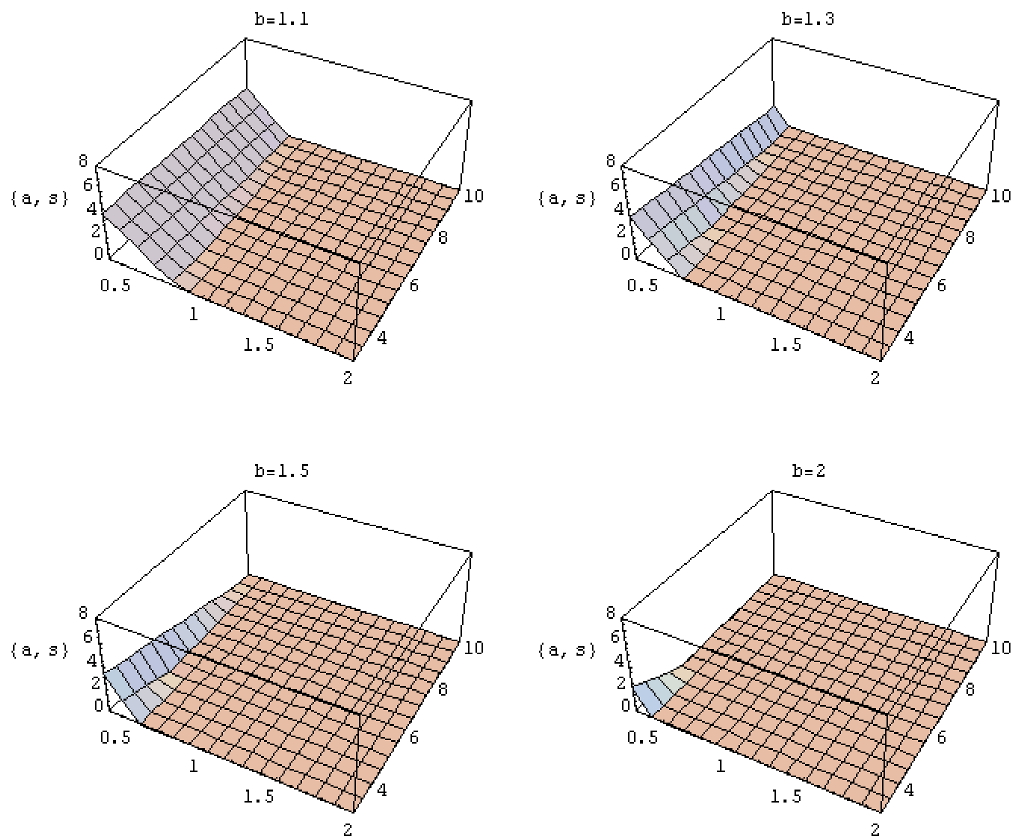

Figure 1: Graph of $f(\cdot)$ for $b = 1.1;\ 1.3;\ 1.5;\ 2.0$ as functions of a and s

of $f(\cdot)$. In other words, for large values of b (e.g. $b = 2$) one has to expect that hiring firms will not survive if the market is rather large. Only in small markets with small wage reactivity parameter (small a) will a bimorphism evolve.

Furthermore, it is interesting to conclude from the plots in Figure 1 that there is a negative correlation between a and s. For each a we can find a parameter s^* such that $f(a, s^*) = 0$ but that s^* decreases with increasing a. If the wage reactivity, measured by a, increases sufficiently, then A4) is not satisfied anymore, meaning that the hiring firms will not survive in the long run. This seems plausible from an economic point of view since high wage volatility makes it more attractive to buy labor.[9]

We conclude this section by reviewing a numerical example of a bimorphic

[9]Since hiring firms face the same labor costs regardless of z,, their profit volatility exceeds by far that of buying firms whose labor costs rely on a built-in flexibility (in the sense of high (low) costs in case of high (low) revenues).

population. It is easy to check that the parameter values

$$a = 0.25, \quad \underline{z} = 8, \quad \bar{z} = 10, \quad s = 8$$

satisfy assumptions A3) and A4) due to $f(0.25, 8) = 3.28125 > 0$. Note that we set b equal to $b = 1.25$. In order to determine the probabilities of the random variable n, the number of hiring firms, we use the result of theorem 1. According to this theorem, we can calculate the probabilities ν_k $(k = 0, 1, \ldots, s)$ of the possible values of n. It is crucial for the calculation of these probabilities to know the explicit birth and death rates of the stochastic evolutionary process λ_s, μ_s. These rates are based on the expected payoff difference $\Delta(n)$. By using our results at the beginning of this section, we can express the expected payoff difference as depending on $a, b, \underline{z}$ and s as follows:

$$\Delta(n) = \frac{a\underline{z}}{6(s+1)}[4 + b + b^2 - a(2 + 4n - 2s + b(2 + n + s) + b^2(2 + n + s))].$$

The birth rate, for example, is given by $\lambda_n = \beta\Delta(n)^+ + \epsilon$, where β is a parameter which represents the impact of the economic profit differences on the stochastic adaptation process and ϵ represents a mutation term which represents stochastic noise of the adaptation process. For the sake of specificity set $\beta = 2$ and $\epsilon = 0.1$. Table 5 presents the stationary probability limit distribution of the evolutionary process

ν_0	ν_1	ν_2	ν_3	ν_4
0.00650	0.13295	0.79060	0.06681	0.00305

ν_5	ν_6	ν_7	ν_8	
1-4 0.00009	≈ 0	≈ 0	≈ 0	

which is illustrated in Figure 2.

A short glance at $\Delta(n)$ shows that it hits the nonnegative real axis at $n^* \approx 2$. The limit probability distribution of n is a simple unimodal curve with maximum near 2. The values v_k for $k \geq 4$ are too small to be distinguished from zero. It can be shown that the curve will exhibit an increasing tendency toward the maximum $n = 2$ when ϵ decreases to zero. One can also experiment with differing values of β to obtain different pictures of the stationary distribution. However, this would not question the qualitative features of Figure 2.

6 Conclusions

Our attempt to explore in detail the tendency to hire or buy labor shows that even a rather simple model allows for the three basic results, namely

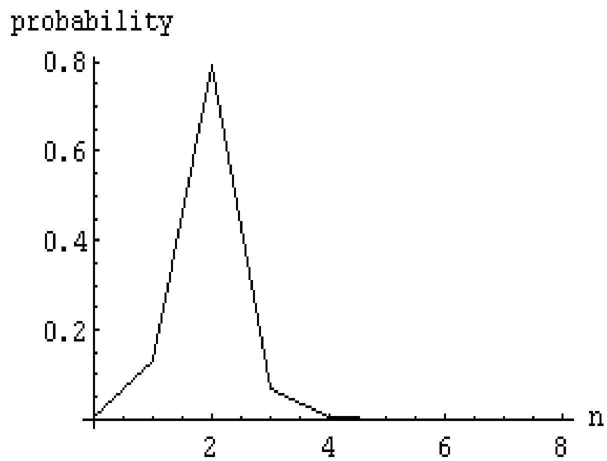

Figure 2: Stationary limit distribution ν of the evolutionary process

- $n^* = 0$, i.e. the pure trade world where labor is supplied on spot markets like industrial inputs,
- $n^* \geq s$, i.e. where labor is hired only, and
- $0 < n^* < s$, i.e. a bimorphism of both.

From our analysis in sections 4 and 5 we see that for many parameter constellations case $n^* = 0$ will typically evolve. This is true especially for large markets and/or for high reactivity of wage rates on macroeconomic data. On the other hand, these results also depend on the difference of "good" and "bad" macroeconomic states (measured by $\bar{z}$ and $\underline{z}$), that is, on the volatility of the macroeconomic influence on this particular market. Large differences in these variables render the result $n^* = 0$ more probable.

Whereas market behavior is decided rationally, n^* has been derived as the rest point of an adaptation process formulated directly for the market composition parameter n, rather than deriving the evolutionary dynamics of n via individual adaptation of all s interacting firms. An advantage of this approach is obviously that evolution can only gradually change the market composition (the assumption that a market might switch from $n = 0$ to $n = s$ in just one round, e.g. Kandori, Mailath, and Rob, 1993, may be technically desirable for certain purposes but is extremely unrealistic).

Note, however, that n^* is also the likely candidate for the solution of the game where firms rationally decide to hire or buy in the form of an institutional design choice first before determining their sales behavior. In case of an integer solution n^* with $0 < n^* < s$, multiple strict equilibria would, of course exist with exactly n^* hiring firms each. This problem of equilibrium coordination is avoided by our evolutionary approach. This approach would be an exercise in individual institutional design in a situation where one's own best institution may depend on the design choices of others/ made by others.

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Appendix

Appendix I: Derivation of relation (15)

By inserting $x_i(\bar{z}) = \frac{\bar{z}+mw(\bar{z})}{s+1}$ and $\bar{w} = \int w(z)d\phi(z)$, we obtain

$$\begin{aligned}\Delta(n) &= \int \frac{(z+mw(z))^2}{(s+1)^2}d\phi(z) - \frac{\bar{z}+mw(\bar{z})}{s+1}\int w(z)d\phi(z) - \int \frac{(z-(n+1)w(z))^2}{(s+1)^2} \\ &= \frac{1}{(s+1)^2}\int [(z+mw(z))^2 - (z-(n+1)w(z))^2]d\phi(z) - \frac{\bar{z}+mw(\bar{z})}{s+1}\int w(z)d\phi(z).\end{aligned}$$

The term in "[ ]" brackets can be simplified as follows:

$$[zw(z)(2m+2(n+1)) - w^2(z)(n^2+2n+1-m^2)] = [2zw(z)(s+1)) + w^2(z)(s+1)(s-1-2n)].$$

Therefore, we conclude

$$\Delta(n) = \frac{2}{s+1}\int zw(z)d\phi(z) + \frac{s-1-2n}{s+1}\int w^2(z)d\phi(z) - \frac{\bar{z}+(s-n)w(\bar{z})}{s+1}\int w(z)d\phi(z).$$

Appendix II: Proof of Theorem 2

Since the density function $\phi(\cdot)$ is given by

$$\phi(z) = \begin{cases} \frac{1}{\bar{z}-\underline{z}} & \text{for} \quad z \in [\underline{z} - \bar{z}] \\ 0 & \textit{otherwise} \end{cases}$$

and $w(z) = az$, the expression (16) can be transformed as follows:

$$\begin{aligned}\Delta'(n) &= \frac{1}{(\bar{z}-\underline{z})(s+1)}[a^2\bar{z}\int_{\underline{z}}^{\bar{z}} zdz - 2a^2\int_{\underline{z}}^{\bar{z}} z^2dz] \\ &= \frac{a^2}{(\bar{z}-\underline{z})(s+1)}[\bar{z}(\frac{\bar{z}^2}{2} - \frac{\underline{z}^2}{2}) - 2(\frac{\bar{z}^3}{3} - \frac{\underline{z}^3}{3})].\end{aligned}$$

Appendix III: Determination of A (proof of theorem 3)

Determining the part of $\Delta(n)$ which is not dependent on n, we obtain

$$\frac{1}{(s+1)}[2\int_{\underline{z}}^{\bar{z}} zw(z)d\phi(z) + (s-1)\int_{\underline{z}}^{\bar{z}} w^2(z)d\phi(z) - (\bar{z}+sw(\bar{z}))\int_{\underline{z}}^{\bar{z}} w(z)d\phi(z)]$$

which can be transformed into

$$\frac{1}{(s+1)(\bar{z}-\underline{z})}[2a\int_{\underline{z}}^{\bar{z}} z^2dz + (s-1)a^2\int_{\underline{z}}^{\bar{z}} z^2dz - (\bar{z}+sa\bar{z})a\int_{\underline{z}}^{\bar{z}} zdz]$$

$$= \frac{a}{(s+1)(\bar{z}-\underline{z})}[(2+(s-1)a)(\frac{\bar{z}^3}{3}-\frac{\underline{z}^3}{3}) - \bar{z}(1+as)(\frac{\bar{z}^2}{2}-\frac{\underline{z}^2}{2})]$$

$$= \frac{a}{6(s+1)}[2(\bar{z}^2+\bar{z}\underline{z}+\underline{z}^2)(2+a(s-1)) - 3\bar{z}(1+as)(\bar{z}+\underline{z})]$$

$$= \frac{a}{6(s+1)}[(1-a(s+2))(\bar{z}^2+\bar{z}\underline{z}) + 2\underline{z}^2(2+a(s-1))]$$

$$= \frac{1}{6(s+1)}A.$$

Appendix IV: Determination of $\Delta(s)$ (proof of theorem 3)

By calculating the expression $\frac{1}{6(s+1)}(A-nB)$ explicitly and inserting $n=s$, we obtain

$$\Delta(s) = \frac{1}{6(s+1)}[a(1-a(s+2)-sa)(\bar{z}^2+\bar{z}\underline{z}) + 2a\underline{z}^2(2-a(3s+1))]$$
