NOW OR LATER ?
–An Analysis of the Timing of Threats in Bargaining

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Abstract

The variable threat-bargaining model of Nash (1953) assumes that threats in the sense of binding commitments as to what one will do if bargaining ends in conflict, are chosen before bargaining. By comparison, late threats to be chosen after bargaining end in conflict, appear more natural and would be self-enforcing, i.e., require no commitment power. Instead of exogenously imposing the timing of threats, they are derived endogenously as in indirect evolution or endogenous timing. Based on a duopoly market, we first derive the equilibrium for all possible constellations regarding the timing of threats. From these results we can show that the evolutionary analysis surprisingly justifies the early timing of threats as suggested by Nash (1953).

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1 Introduction

Negotiations or bargaining usually assume that parties can commit themselves to any feasible course of action, i.e., if an agreement on what bargaining should accomplish is reached, it can be implemented by a contract binding all parties involved. Now such commitment power may not only be used to implement an agreement, but also to (pre)commit to an action when negotiations end in conflict. This is exactly what happens in the celebrated bargaining model of Nash (1953) where

- first, parties (pre)commit to what they will do in case of conflict, i.e., to their so-called threat strategies
- before bargaining in the form of simultaneously stating demands.[1]

Often bargaining models assume a given feasible set of payoff vectors as originally suggested by Nash (1950 and 1953). To allow for an institutional underpinning of such a frame, this will be derived rather than assumed by considering a market contest. In spite of the enormous popularity of the bargaining model suggested by Nash (1950 and 1953), his variable threat model is rarely applied (see Mayberry, Nash, and Shubik, 1954, for an early application and illustration). One reason might be that the idea to commit to threats before bargaining seems counterintuitive. Knowing how the other party will hurt one in case of no agreement may kill the goodwill for reaching a fair agreement.[2] A more intuitive alternative procedure would be to first bargain, and to act noncooperatively only if there is failure to reach an agreement.

Here we do not restrict ourselves to those cases where either both parties threaten before or after negotiating, but allow for independent timing dispositions (early and late threat) of both parties and thus also for the two bimorphic constellations where one party threatens early and the other late. The results of this more complete analysis will enable us to determine the constellation of individual timing dispositions which is either evolutionarily stable or rationally chosen. In the first case, our analysis would be one of indirect evolution where timing dispositions evolve (in the shadow of the past) and bargaining moves are rationally deliberated (in the shadow of the future). The latter assumption renders our analysis as a study of endogenous timing which assumes that not only bargaining moves but also timing dispositions are rationally deliberated.[3]

In our view, such studies help to reduce the variety of bargaining models which makes it so difficult to predict bargaining outcomes. If not all four constellations of timing dispositions are possible but only the stable ones, this troublesome ambiguity of bargaining outcomes can be greatly reduced. Furthermore, such an analysis can answer the question whether early

[1]Although Nash assumes only one round of simultaneous demands, nothing much changes if parties confront finitely many rounds with no costs of delay.

[2]To illustrate the strange timing of decisions assumed by Nash (1953), consider the case of human marriage. Here such timing would mean deciding about the divorce settlement before having started the partnership.

[3]In principle, indirect evolution can be seen as an approach to providing a bridge between the extreme cases of direct evolution (denying any deliberation) and perfect rationality (denying any evolution); see Berninghaus, Güth and Kliemt, 2001, for a discussion and illustration.

or late (or even bimorphic timing of) threats should be expected. In view of the intuitive argument above, it may be rather surprising that the timing of threats, as suggested by Nash (1953), is the likely result.

In the following sections, we first describe the market environment (section 2), the pie whose distribution is negotiated (section 3) and the mutant space of timing dispositions (section 4) before deriving the bargaining outcome for the possible constellations of timing dispositions (sections 5 to 7). Section 8 is devoted to the analysis of indirect evolution or endogenous timing for particular parameter constellations, and section 9 concludes.

2 The duopoly market

Usually, bargaining models just specify the set of feasible payoff vectors and the vector of conflict payoffs (see, in addition to Nash (1950 and 1953), also Kalai and Smorodinsky, 1975). Here it does not matter much whether an agreement either specifies the agreed-upon payoffs directly or indirectly by the corresponding market behavior. This, however, does not hold any longer when the behavior in case of conflict, i.e., when failing to reach an agreement, can be chosen strategically. It is, therefore, necessary to provide an institutional backing for the bargaining model, for example, a market context providing some chances to gain from cooperation.

Here we will restrict ourselves to just two parties, X and Y, who are sellers on a heterogeneous market. To allow for (mostly) analytic results, we have to make rather strong assumptions regarding the functional form of how profits depend on prices. This is no substantial restriction of our approach, but possibly of our specific results. In general, one may find a market underpinning for any reasonable (bounded, strictly concave) feasible set and threat point and perform a similar analysis. This might sometimes challenge our results, but we consider this as rather unlikely since our market model seems to be a good approximation of most other such models.

Let there be two firms denoted by X and Y whose sales prices are denoted by p_x and p_y. Their linear demand functions specify their sales amounts $X(p_x, p_y)$ and $Y(p_x, p_y)$ for all possible price constellations (p_x, p_y) with $p_x, p_y \geqq 0$ in the form of

$$\begin{aligned} X(p_x, p_y) &= \alpha_x - \beta_x p_x - \gamma_x p_y, \\ Y(p_x, p_y) &= \alpha_y - \beta_y p_y - \gamma_y p_x. \end{aligned}$$

Production costs are assumed to be linear as well so that profits become

$$\begin{aligned} \Pi_x(p_x, p_y) &= (p_x - c_x)X(p_x, p_y) \\ \Pi_y(p_x, p_y) &= (p_y - c_y)Y(p_x, p_y) \end{aligned}$$

with $\alpha_x > \beta_x c_x + \gamma_x c_y \geqq 0$, and $\alpha_y > \beta_y c_y + \gamma_y c_x \geqq 0$. Here c_i denotes the constant unit production cost $(i = X, Y)$.

With the help of the profits x, or, respectively, y per unit of sales, i.e., $x := (p_x - c_x)$ and $y := (p_y - c_y)$, the profit functions can then be defined as depending on x and y :

$$\begin{aligned} \Pi_x(x, y) &= x(\alpha_x - \beta_x c_x - \gamma_x c_y - \beta_x x - \gamma_x y), \\ \Pi_y(x, y) &= y(\alpha_y - \beta_y c_y - \gamma_y c_x - \beta_y y - \gamma_y x). \end{aligned}$$

Now sales amounts can be chosen without loss of generality such that

$$\alpha_x - \beta_x c_x - \gamma_x c_y = 1 = \alpha_y - \beta_y c_y - \gamma_y c_x.$$

Similarly, we can set the monetary unit in such a way that $\beta_x = 1$. Simplifying our notation by setting

$$\gamma := \gamma_x, \quad \beta := \beta_y, \quad \text{and} \quad \delta := \gamma_y$$

finally leads to

$$\begin{aligned} \Pi_x(x,y) &= x(1 - x - \gamma y), \\ \Pi_y(x,y) &= y(1 - \beta y - \delta x). \end{aligned}$$

Since a common price increase by the same amount should decrease both sales amounts, we require that the following basic restrictions apply for the demand function parameters

Assumption 1

$$\gamma > -1, \quad \delta > -\beta.$$

In later sections, we need some further restrictions for the demand function parameters in order to guarantee results which are reasonable from an economic point of view.

Similar to seller X, who sells less if x increases, a price increase of seller Y alone should decrease Y's sales amount so that, furthermore, we require

Assumption 2

$$\beta > 0.$$

In the tradition of Nash (1950 and 1953), the negotiation game is assumed to consist of just one round of payoff demands d_i for $i = x, y$ with $d_i = \Pi_i(x, y)$ for some possible price vector (x, y). If the vector (d_x, d_y) is feasible in the sense of

$$d_x = \Pi_x(x,y) \qquad \text{and} \qquad d_y = \Pi_y(x,y)$$

for one possible price vector (x, y), this demand vector (d_x, d_y) is implemented by a binding contract specifying these price choices. Otherwise, conflict would result.

This classic bargaining game is enriched by the (timing of) choices specifying what to do in case of conflict. Here we allow for the following cases:

- the Nash case, where both parties choose their threats befor negotiating,
- the more intuitive case, where only after conflict in negotiations parties choose their sales prices independently, and
- the two bimorphic cases, where one party (pre)commits before negotiating, whereas the other decides what to do in case of conflict only after failing to reach an agreement.

Based on the analysis of all these cases, we will then be able to determine whether parties want to commit to threats (the prices in case of conflict) before or after negotiating, where, of course, one party might prefer the earlier and the other the later timing. If timing is chosen strategically before negotiating or (pre)committing to threats, we speak of endogenous timing. Here the game starts by both parties choosing whether to threaten "before" or "after" negotiating. Knowing those timing constellations, they then bargain according to the appropriate case described above.

Another interpretation of our results is that timing dispositions are inherited traits which evolve over time according to their relative fitness. Actually, we will determine the evolutionarily stable timing dispositions by applying the concept of evolutionarily stable strategies (see Maynard Smith and Price, 1973). According to the latter interpretation, our study is one of indirect evolution (see Güth and Kliemt,1998, for a conceptual discussion and Berninghaus et al. 2002, for how this is related to direct evolution) since it combines rational deliberation (when choosing threats and demands) with behavioral adaptation (when to threaten).

In section 8, we study the problem of timing dispositions under special assumptions. Thus, we will consider the special case of complete symmetry which requires

$$\beta = 1 \quad \text{and} \quad \gamma = \delta. \tag{1}$$

Nonsymmetric situations with fewer parameters can be explored by setting either

$$\gamma = \delta \quad \text{and} \quad \beta \neq 1 \tag{2}$$

or

$$\beta = 1 \quad \text{and} \quad \gamma \neq \delta. \tag{3}$$

3 The pie

If different timing of threats induces different threats, this could also influence the sum of the agreement payoffs when side payments are excluded. Here we want to rule out the latter effect by concentrating on the distributional aspect of the timing of threats. The efficiency issue will be avoided by allowing for side payments, requiring that the payoff of both firms is linear in money. In that case, efficiency means to maximize the sum of both profits

$$\Pi(x, y) = \Pi_x(x, y) + \Pi_y(x, y).$$

From

$$\frac{\partial \Pi(x, y)}{\partial x} = 0 \quad \text{and} \quad \frac{\partial \Pi(x, y)}{\partial y} = 0$$

and the concavity of $\Pi(x, y)$ one obtains

$$x^* = \frac{2\beta - \gamma - \delta}{4\beta - (\gamma + \delta)^2}$$

$$y^* = \frac{2 - \gamma - \delta}{4\beta - (\gamma + \delta)^2}.$$

Inserting theses values into total profits $\Pi(x,y)$, one obtains

$$\Pi^* := \Pi(x^*, y^*) = \frac{1+\beta-\gamma-\delta}{4\beta-(\gamma+\delta)^2} \tag{4}$$

which represents the pie in the bargaining situation.

4 The mutant space

For $i = X, Y$ the two possible timing dispositions are

- E_i meaning that party i chooses his or her threat before bargaining and
- L_i meaning that party i only threatens after conflict in bargaining.

It is important to note that early and late threats require quite different degrees of self-commitment. In the case of late threats, the chance to reach an agreement is lost. Thus, parties choose threats which are self-serving. In other words: late threats are self-enforcing and do not require any self-commitment. Actually, late threats do not have to be explicitly announced since they are rationally anticipated: In case of human marriage you do not have to tell your spouse what you will do in case of divorce. If she entertains rational expectations, she will know what will happen in such a case.

Early threats would, however, be usually (very)self-detrimental if they had to be carried out. Thus, without self-commitment early threats are just cheap talk announcements which nobody will view as a reliable intention. Since our analysis, following Nash (1953), views early threats as definite intentions, it assumes - as far as early threats are concerned - unrestricted self-commitment power.[4]

Since timing dispositions can evolve independently for both parties, one has to consider the two monomorphisms

- $E = (E_x, E_y)$, where both threaten before bargaining as assumed by Nash (1953) and
- $L = (L_x, L_y)$, where both threaten only after not reaching an agreement, and

the two bimorphisms

- (E_x, L_y) and
- (L_x, E_y).

In the following we will first determine the bargaining outcome for these four constellations and then analyze which of the four constellations will finally evolve when solution payoffs measure reproductive success.

[4]One may wonder why, in case of commitment power, we do not directly commit to payoff demands. In our view, this could reflect procedural requirements: A party who commits to a payoff demand before negotiating would automatically induce conflict regardless of the demand made.

Timing dispositions for actual market decisions are endogenously derived by Spencer and Brander (1992), Hamilton and Slutzky (1990), Sadanand and Sadanand (1996), Hurkens and Van Damme (1997), and Güth and Güth (2001) where it is partly assumed, as we did, that timing dispositions are commonly known and partly not known. What distinguishes our approach is that we do not derive the timing of actual market decisions. These are the choices x^* and y^* which maximize the pie Π and whose timing is irrelevant since they are specified in the binding bargaining agreement. Rather, we endogenously determine the timing of threats which are never realized since conflict is always avoided (unlike in bargaining with incomplete information, see Harsanyi and Selten, 1972). Nevertheless, we will see that the timing of threats is crucial for the final agreement. Moreover, it will be shown that early threats, as assumed by Nash (1953), are the most likely timing dispositions, although they appear counterintuitive (see our discussion above).

Timing dispositions in an abstract bargaining situation are endogenously derived by Güth and Ritzberger (1999) where, however, this concerns the timing of the actual demands which may be determined simultaneously, as supposed by Nash (1950, 1953), or sequentially,[5] as in ultimatum bargaining (Güth, 1976). Güth and Ritzberger, furthermore, assume that the size of the pie is uncertain and randomly determined after an early but before a late demand. By an early demand one may thus exploit the other party as in ultimatum bargaining, but also risk conflict if the pie turns out to be rather small.

Our approach does not rule out a random pie but only certain cases when this random event takes place. If the random event precedes, for instance, even an early threat, nothing at all is changed, and even risk attitudes would not matter since parties know their bargaining environment for sure. On the other hand, if even late threats precede the random realization of the pie and if both parties are risk neutral, all our analyses apply, too. One only has to interpret profits as average profits. Thus, what we essentially rule out is a random pie which is determined after early and before late threats.

More basically, our specific bargaining environment illustrates that bargaining for a given or randomly determined pie is a rather special assumption. Even when the amount which the parties can share, is (stochastically) known, the agreement surplus (the pie minus the conflict payoffs) depends on the timing of threats. Random events influencing the size of the pie will most likely also influence the conflict payoffs for all constellations of timing threats and thus the agreement surplus. More specifically, the agreement surplus will usually be different for different timings of threats. This illustrates a basic advantage of studying bargaining problems in a natural environment like markets, allowing for variable threats and idiosyncratic risks which may affect the size of the pie and the agreement surplus rather differently.

5 Early threats: The case of $E = (E_x, E_y)$

This case corresponds to variable threat-bargaining games as ingeneously analyzed by Nash (1953). Denote for $i = X, Y$ by d_i the payoff level of party i in case of conflict. Now, given

[5] Attempts aiming at preemptive payoff demands are excluded since they violate procedural rules.

a constant pie Π^*, the optimal [6] demands u_x^* and u_y^* require

$$u_x^* - d_x = u_y^* - d_y \quad \text{and} \quad u_x^* + u_y^* = \Pi^*,$$

i.e., equal agreement dividends $u_i^* - d_i$ for both parties $i = x, y$.

By choosing d_x party x therefore wants to maximize

$$u_x^* = \frac{\Pi^* + d_x - d_y}{2}$$

or

$$u_x^* = u_x(x, y) = \frac{\Pi^* + \Pi_x(x, y) - \Pi_y(x, y)}{2}$$

by choosing threat x. Similarly, party y maximizes

$$u_y^* = u_y(x, y) = \frac{\Pi^* + \Pi_y(x, y) - \Pi_x(x, y)}{2}$$

with respect to y. The saddle point solution $(x^*(E), y^*(E))$ of these mutually best response attempts is given by

$$\begin{aligned} x^*(E) &= \frac{2\beta - \gamma + \delta}{4\beta + (\delta - \gamma)^2}, \\ y^*(E) &= \frac{2 + \gamma - \delta}{4\beta + (\delta - \gamma)^2}. \end{aligned}$$

Inserting these decisions into the definition u_x^* and u_y^* of agreement payoffs yields

$$u_x^*(E) = \frac{4\beta^2 + \delta^2 - \delta^3 - 4\beta\gamma - 2\beta\gamma\delta + \gamma^2 + \gamma^2\delta}{16\beta(\beta - \delta\gamma) - (\delta^2 - \gamma^2)^2}, \tag{5}$$

$$u_y^*(E) = \frac{\gamma\delta^2 - 2\gamma\delta - \gamma^3 + 4\beta - 4\beta\delta + \beta\delta^2 + \beta\gamma^2}{16\beta(\beta - \delta\gamma) - (\delta^2 - \gamma^2)^2} \tag{6}$$

6 Late threats: The case of $L = (L_x, L_y)$

When threats are late, they are not chosen to trigger optimal agreement payoffs but rather to induce optimal conflict payoffs as such. Thus, the constellation of optimal threats $x^*(L)$ and $y^*(L)$ in case of late threats is nothing else than the familiar duopoly equilibrium originally suggested by Cournot (1838) which, for the case at hand, is given by

$$\begin{aligned} x^*(L) &= \frac{2\beta - \gamma}{4\beta - \gamma\delta}, \\ y^*(L) &= \frac{2 - \delta}{4\beta - \gamma\delta}. \end{aligned}$$

[6] "Optimality" here is defined by the axioms formulated by Nash, or some other set of axioms implying the Nash-bargaining solution, e.g., Lensberg (1988).

The agreement outcomes u_x^*, u_y^* with $u_x^* - d_x^* = u_y^* - d_y^*$ and $u_x^* + u_y^* = \Pi^*$ anticipating the optimal conflict payoffs

$$\begin{aligned} d_x^* &= \frac{(2\beta-\gamma)^2}{(4\beta-\gamma\delta)^2}, \\ d_y^* &= \frac{\beta(2-\delta)^2}{(4\beta-\gamma\delta)^2} \end{aligned}$$

are

$$u_x^*(L) := u_x(x^*(L), y^*(L)) = \frac{A_{LL}(\beta,\gamma,\delta)}{2(\gamma\delta-4\beta)^2(4\beta-(\gamma+\delta)^2)}, \tag{7}$$

$$u_y^*(L) := u_y(x^*(L), y^*(L)) = \frac{B_{LL}(\beta,\gamma,\delta)}{2(\gamma\delta-4\beta)^2(4\beta-(\gamma+\delta)^2)} \tag{8}$$

with

$$A_{LL}(\beta,\gamma,\delta) := 32\beta^3 - \gamma^2(\delta^3+2\gamma\delta+\gamma\delta^2+\gamma^2) - 4\beta^2(2\delta^2+4\gamma\delta+\gamma(8+\gamma)) +$$
$$\beta(\delta^4+2\delta^3(-2+\gamma)+12\gamma^2\delta+4\gamma^2(2+\gamma)+2\delta^2(2+2\gamma+\gamma^2))$$

and

$$B_{LL}(\beta,\gamma,\delta) := 4\beta^2(8-8\delta+2\delta^2+\gamma^2) + \gamma^2(-\delta^3-\delta^2(-2+\gamma)+2\gamma\delta+\gamma^2) -$$
$$\beta(\delta^4+\delta^2(4-12\gamma)+2\delta^3(-2+\gamma)-4\delta(-4+\gamma)\gamma+4\gamma^2(2+\gamma)).$$

7 Bimorphic threats

Let us first consider the constellation (E_x, L_y). Since firm Y chooses its threat after failing to reach an agreement it simply maximizes $\Pi_y(x,y)$ with respect to y where x is the given and known threat of party X. We therefore obtain the optimal threat for Y as a function

$$y^*(x) = \frac{1-\delta x}{2\beta}$$

of X's earlier threat x. Since this leads to the agreement payoff

$$u_x^* = \frac{\Pi^* + \Pi_x(x, y^*(x)) - \Pi_y(x, y^*(x))}{2}$$

for party X, the optimal threat of party X must maximize u_x^*, implying

$$x^*(E_x, L_y) = \frac{2\beta-\gamma+\delta}{4\beta+\delta^2-2\delta\gamma}$$

and thus

$$y^*(E_x, L_y) = y^*(x^*(E_x, L_y)) = \frac{2\beta(2-\delta)-\gamma\delta}{2\beta(4\beta+\delta^2-2\gamma\delta)}.$$

The agreement payoffs resulting from these threats are

$$u_x^*(E_x, L_y) = \frac{A_{EL}(\beta,\gamma,\delta)}{8\beta(4\beta+\delta^2-2\gamma\delta)(4\beta-(\gamma+\delta)^2)}, \tag{9}$$

$$u_y^*(E_x, L_y) = \frac{B_{EL}(\beta,\gamma,\delta)}{8\beta(4\beta+\delta^2-2\gamma\delta)(4\beta-(\gamma+\delta)^2)} \tag{10}$$

with

$$A_{EL}(\beta,\gamma,\delta) = 32\beta^3 - \gamma^2(\gamma+\delta)^2 - 4\beta^2\gamma(8+4\delta+\gamma) + 4\beta(2\delta^2-2\delta^3+3\gamma^2\delta+\gamma^2(2+\gamma))$$

and

$$B_{EL}(\beta,\gamma,\delta) = \gamma^2(\gamma+\delta)^2 + 4\beta^2(8-8\delta+2\delta^2+\gamma^2) - 4\beta\gamma(-2\delta^2-\delta(\gamma-4)+\gamma(2+\gamma))$$

The analogous results for (L_x, E_y) are

$$x^*(y) = \frac{1-\gamma y}{2}$$

$$y^*(L_x, E_y) = \frac{2+\gamma-\delta}{4\beta-2\gamma\delta+\gamma^2}$$

$$x^*(L_x, E_y) = x(y^*(L_x, E_y)) = \frac{4\beta-(2+\delta)\gamma}{2(4\beta-2\gamma\delta+\gamma^2)}$$

and

$$u_x^*(L_y, E_y) = \frac{A_{LE}(\beta,\gamma,\delta)}{8(4\beta-2\gamma\delta+\gamma^2)(4\beta-(\gamma+\delta)^2)} \tag{11}$$

$$u_y^*(L_x, E_y) = \frac{B_{LE}(\beta,\gamma,\delta)}{8(4\beta-2\gamma\delta+\gamma^2)(4\beta-(\gamma+\delta)^2)} \tag{12}$$

with

$$A_{LE}(\beta,\gamma,\delta) := 32\beta^2 + \delta^4 + 2\delta^3(\gamma-2) + 8\gamma^2 + 8\gamma^2\delta + \delta^2(2+\gamma)^2 - 8\beta(\delta^2+4\gamma+2\gamma\delta)$$

and

$$B_{LE}(\beta,\gamma,\delta) := -\delta^4 - 2\delta^3(\gamma-2) - 16\gamma\delta - 8\gamma^3 + 8\beta(4-4\delta+\delta^2+\gamma^2) - \delta^2(4-12\gamma+\gamma^2).$$

8 A simple 2×2 game of timing

Now we combine the results of the previous sections. Either both players (simultaneously) determine the threat point before bargaining (strategy "E(arly)" in an appropriately formulated noncooperative 2-person game), and, respectively, after bargaining (strategy "L(ate)"), or one player determines his threat point early, while the other player determines her threat point after bargaining. The payoff matrix of this 2-person game is given as follows.

		player 2	
		E	L
player 1	E	$u_x^*(E)$, $u_y^*(E)$	$u_x^*(E_x, L_y)$, $u_y^*(E_x, L_y)$
	L	$u_x^*(L_x, E_y)$, $u_y^*(L_x, E_y)$	$u_x^*(L)$, $u_y^*(L)$

In order to determine the equilibria of this noncooperative game, we have to compare the payoffs of each strategy configuration separately. Since the expressions are dependent on three parameters, it would be rather difficult to compare the resulting payoffs in full generality. Therefore, we start our analysis by reducing the number of independently varying parameters and proceed from the simplest case to the general case of three independently varying parameter values.

8.1 The completely symmetric case

In this section, we concentrate on the parameter constellation

$$\beta = 1 \quad \text{and} \quad \gamma = \delta.$$

In principle, it is easy to derive the relevant payoffs of the general 2×2 game from the explicitly calculated values in the previous sections. One has only to insert the parameter values $\beta = 1$ and $\gamma = \delta$ into the general payoff functions developed so far. However, first we have to make sure that all endogenously determined decision variables, in our model the prices (in the sense of unit profits), are nonnegative.[7] Note that our general restrictions imply $\gamma = \delta > -1$ (see assumption 1), guaranteeing positive unit profits for both monomorphisms, i.e., for $E = (E_x, E_y)$ and $L = (L_x, L_y)$. Bimorphic dispositions, e.g., (E_x, L_y), require

$$x < \frac{1}{\delta} = \frac{1}{\gamma} \tag{13}$$

to guarantee $y^*(x) > 0$ for positive x. For $x^*(E_x, L_y) > 0$ we need the additional parameter restriction $|\gamma| < 2$ and $\gamma < \sqrt{5} - 1$, i.e., altogether with (1) we have the more restrictive condition

Assumption 3

$$-1 < \gamma < 1.23607$$

which is supposed to hold for the rest of this subsection.

Under these additional constraints, we can express all payoffs in the 2×2 game as dependent on one parameter, denoted by γ. More concretely, the payoff table takes the following form

[7] In case of positive production costs per unit, even negative unit profits could rely on positive sales prices. We, however, prefer to rely on positive unit profits for our evolutionary analysis which implicitly requires a long- run perspective.

		player 2	
		E	L
player 1	E	$\frac{1}{4(1+\gamma)}$, $\frac{1}{4(1+\gamma)}$	$\frac{\gamma^2-8-\gamma^3}{8(1+\gamma)(-4+\gamma^2)}$, $\frac{3\gamma^2+\gamma^3-8}{8(1+\gamma)(-4+\gamma^2)}$
	L	$\frac{3\gamma^2+\gamma^3-8}{8(1+\gamma)(-4+\gamma^2)}$, $\frac{\gamma^2-8-\gamma^3}{8(1+\gamma)(-4+\gamma^2)}$	$\frac{1}{4(1+\gamma)}$, $\frac{1}{4(1+\gamma)}$

This is the payoff table of a symmetric 2×2 game. In order to determine the equilibria of this game, we need consider only one player (because of symmetry). Let us denote by $D_{EL}(\gamma) := u_x^*(E) - u_x^*(L_x, E_y)$ the payoff difference for player 1 generated by deviating from strategy configuration (E, E) to (L, E). If this difference is positive, deviation from (E, E) is not profitable. We obtain the difference function explicitly by

$$D_{EL}(\gamma) = \frac{\gamma^2}{32 - 8\gamma^2}. \tag{14}$$

As shown in Figure 1, the difference is always positive in the range of admissible values of γ except for the special case $\gamma = 0$ of monopolistic competition (without strategic interaction).[8] Therefore, we conclude that deviating from configuration (E, E) is not profitable for player

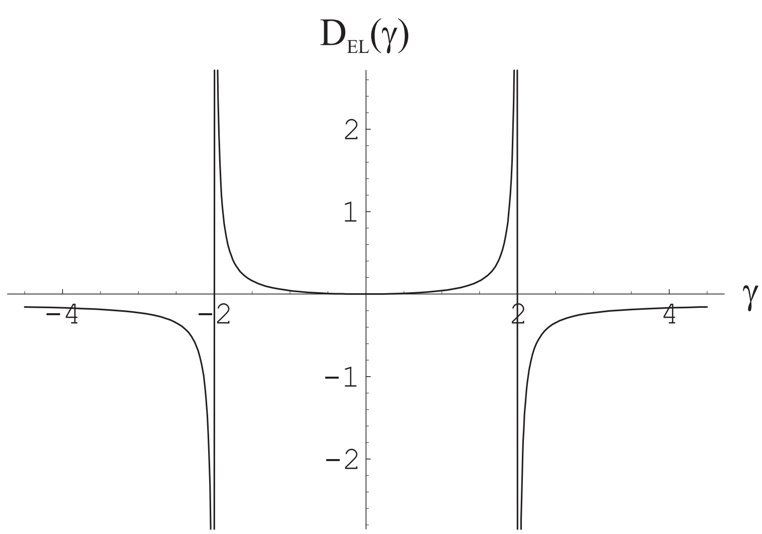

Figure 1: Deviaton from (E, E)

1 and, consequently, not for player 2, either.

[8]If one does not exclude negative unit profits, one has to consider the difference function for larger values of γ. By plotting the difference from -1 to 5, we see that for values $\gamma < 2$ deviation is not profitable. For $\gamma = 2$ the difference is not defined. However, for larger values of γ it seems as if the configuration (E, E) can be destabilized. A symmetric argument holds for values $\gamma \in (-5, -1)$.

We repeat this "exercise" for the strategy configuration (L, L). The difference function is defined here as $D_{LE}(\gamma) := u_x^*(L) - u_x^*(E_x, L_y)$. By some straightforward calculations one can show that the relation

$$D_{LE}(\cdot) \equiv -D_{EL}(\cdot)$$

holds which states/indicates that the payoff difference generated by deviating from (L, L) is the negative of the difference function D_{EL}. This result is illustrated by Figure 2. For admissible parameter values $\gamma \in (-1, 1.23607)$ the pair (L, L) cannot be stabilized because positive values for $D_{EL}(\cdot)$ at the same time imply negative values of $D_{LE}(\cdot)$. We summarize

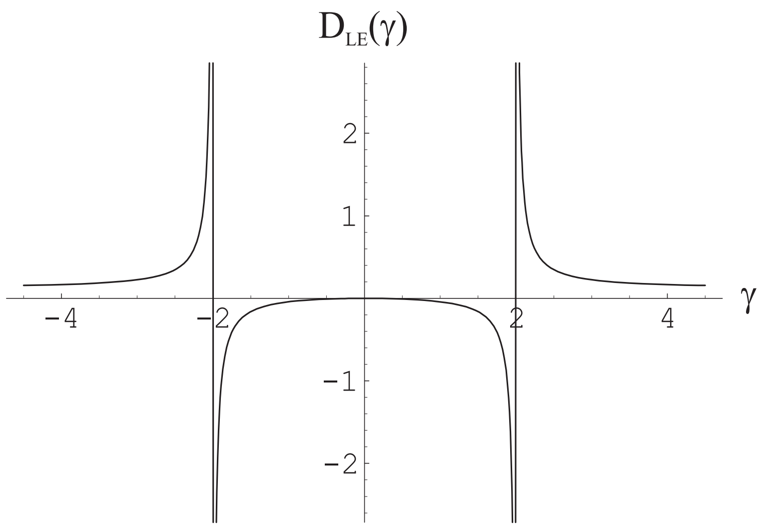

Figure 2: Deviaton from (L, L)

our results in the fact below.

Fact 1 *Given the symmetric timing game where assumption 3 holds, the strictly dominant timing disposition, i.e., the unique evolutionarily stable bargaining procedure is to commit before negotiations as suggested by Nash (1953).*

Proof: Because of $D_{LE}(\cdot) = -D_{EL}(\cdot)$, we need only consider the function $D_{EL}(\gamma) = \frac{\gamma^2}{32-8\gamma^2}$ which is well defined for values $\gamma \neq 0$ and $\gamma \neq |2|$.

Obviously we have $D_{EL}(0) = 0$. The first derivative

$$D'_{EL}(\gamma) = \frac{64\gamma}{(32 - 8\gamma^2)^2}$$

is strictly positive for $\gamma > 0$ and strictly negative for $\gamma < 0$. Therefore, $D_{EL}(\cdot)$ is strictly positive for $\gamma \in (-2, 2)$, which implies the desired result.

q.e.d.

Before moving on to discuss less special situations, let us briefly interpret Fact 1. It states that an early commitment to behavior in case of conflict is better regardless of whether or not the other party is preempting. This holds even though threats are never carried out. As in ritual fighting or in purely aggressive attitudes or in pax atomica among superpowers, threats are purely counterfactual, telling the other what one would do otherwise. Committing to such counterfactual threats early is always better if done independently of when the other party commits to his or her threat.

8.2 The asymmetric two-parameter cases

8.2.1 The case $\beta \neq 1$ and $\gamma = \delta$

Again we start the analysis by guaranteeing positive unit profits x and y for all four constellations of timing dispositions which can be guaranteed by the assumption

Assumption 4

$$|\gamma| < \min\{1, \beta\}.$$

Now the payoff table of the 2×2 timing game depends on the two parameter β and γ. More concretely, the pie and the individual payoffs are given as follows

$$\Pi^* = \frac{1+\beta-2\gamma}{4\beta-4\gamma^2}, \tag{15}$$

$$u_x^*(E) = \frac{2\beta^2+\gamma^2-\beta\gamma(2+\gamma)}{8\beta(\beta-\gamma^2)}, \tag{16}$$

$$u_y^*(E) = \frac{\beta(2-2\gamma+\gamma^2)-\gamma^2}{8\beta(\beta-\gamma^2)}, \tag{17}$$

$$u_x^*(E_x, L_y) = \frac{8\beta^2-\gamma^4+2\beta\gamma^2(2+\gamma)-\beta^2\gamma(8+5\gamma)}{8\beta(\beta-\gamma^2)(4\beta-\gamma^2)}, \tag{18}$$

$$u_y^*(E_x, L_y) = \frac{2\beta(\gamma-3)\gamma^2+\gamma^4+\beta^2(8-8\gamma+3\gamma^2)}{8\beta(\beta-\gamma^2)(4\beta-\gamma^2)}, \tag{19}$$

$$u_x^*(L_x, E_y) = \frac{8\beta^2-2\beta\gamma(4+3\gamma)+\gamma^2(3+2\gamma+\gamma^2)}{8(4\beta^2-5\beta\gamma^2+\gamma^4)}, \tag{20}$$

$$u_y^*(L_x, E_y) = \frac{4\beta(2-2\gamma+\gamma^2)-\gamma^2(5-2\gamma+\gamma^2)}{8(4\beta^2-5\beta\gamma^2+\gamma^4)}, \tag{21}$$

$$u_x^*(L) = \frac{8\beta^2+\gamma^2(3+2\gamma)-\beta\gamma(8+5\gamma)}{8(4\beta^2-5\beta\gamma^2+\gamma^4)}, \tag{22}$$

$$u_y^*(L) = \frac{\gamma^2(2\gamma-5)+\beta(8-8\gamma+3\gamma^2)}{8(4\beta^2-5\beta\gamma^2+\gamma^4)}. \tag{23}$$

At first glance, these payoffs look more complicated than in the symmetric case. However, we will see in a moment that the equilibrium analysis follows the same lines as in the completely symmetric case. We consider the difference functions $D_{EL}^x(\beta,\gamma) := u_x^*(E) - u_x^*(L_x, E_y)$ (resp. $D_{EL}^y := u_y^*(E) - u_y^*(E_x, L_y)$ for player 2) in order to see whether the combination (E, E) can be stabilized as an equilibrium. These difference functions are explicitly given by

$$D_{EL}^x(\beta,\gamma) = \frac{\gamma^2}{32\beta^2-8\beta\gamma^2}$$

and

$$D^y_{EL}(\beta,\gamma) = \frac{\gamma^2}{32\beta - 8\gamma^2}.$$

Because of $D^x_{EL}(\beta,\gamma) = \frac{1}{\beta} D^y_{EL}(\beta,\gamma)$ and $\beta > 0$, we need to check only the profitability of deviation from (E, E) for one of the players. To be definitive, we analyze the deviation profitability of player 1, i.e., the difference function $D^x_{EL}(\cdot)$ which in this subsection is a function of two parameters. In order to obtain more information about the behavior of this function, we plot it as functionally dependent on γ alone for two representative values of β in Figures 3 and 4.

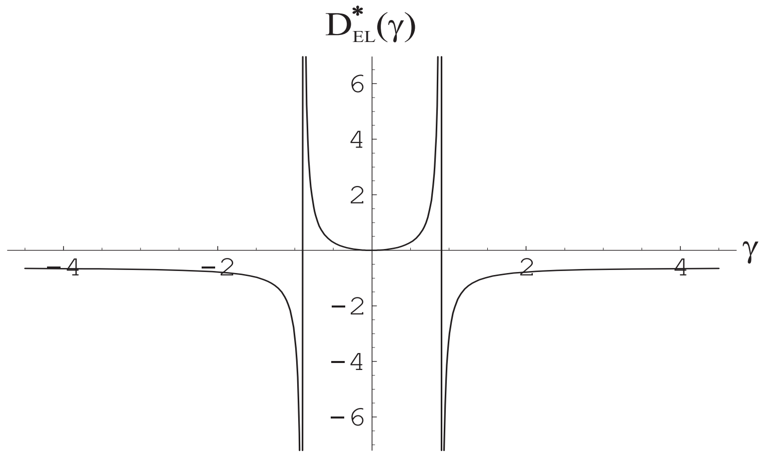

Figure 3: Graph of $D^x_{EL}(\beta,\cdot)$ for $\beta = 0.2$

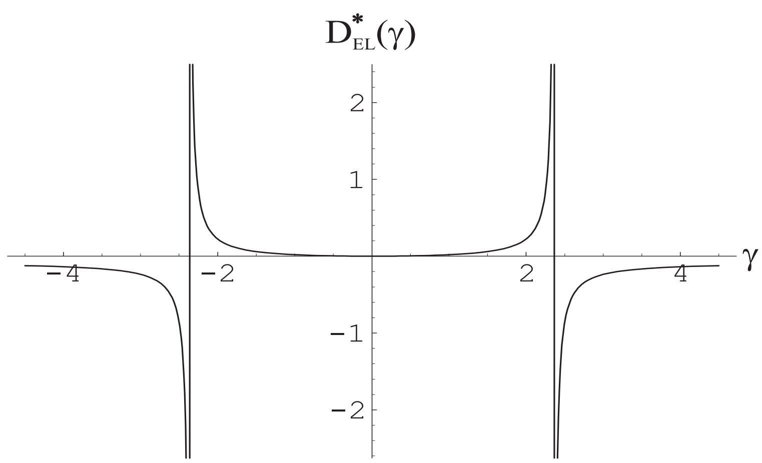

Figure 4: Graph of $D^x_{EL}(\beta,\cdot)$ for $\beta = 1.4$

We can see from these graphs and the definition of $D^x_{EL}(\cdot)$ that the difference function is not defined for values $\gamma = \pm 2\sqrt{\beta}$. When β is large enough so that the critical value $|2\sqrt{\beta}|$ lies outside the interval,[9] $(-1, 1)$ the function $D^x_{EL}(\beta,\cdot)$ is a symmetric parabola with positive

[9] It is easy to see that this will happen for $\beta > 0.25$.

values in this interval except for $\gamma = 0$ (see Figure 4), i.e., for admissible values $|\gamma| < 1$ the difference function is strictly positive. If β is small enough the critical value is in the interval $(-1, 1)$. However, assumption 4 implies $|\gamma| < \beta < 2\sqrt{\beta}$ (for $\beta < 0.25$). Therefore, the admissible parameter values γ generate positive values of the difference function, implying that deviation from (E, E) is not profitable.

We check the equilibrium property of constellation (L, L) by the same method. It turns out that this is an easy task since the difference functions $D^x_{LE}(\cdot)$ and $D^y_{LE}(\cdot)$ are the negative of the difference functions for deviation from (E, E). More concretely, we have

$$D^x_{LE}(\beta, \gamma) = \frac{\gamma^2}{-32\beta^2 + 8\beta\gamma^2} = -D^x_{EL}(\beta, \gamma)$$

or, respectively,

$$D^y_{LE}(\beta, \gamma) = \frac{\gamma^2}{-32\beta + 8\gamma^2} = -D^y_{EL}(\beta, \gamma).$$

This implies that (L, L) cannot be stabilized as an equilibrium of the 2×2 timing game.

Fact 2 *Suppose assumption 4 holds, then fact 1 is a robust result in the sense that it extends to the range of asymmetric situations in the forms $\gamma = \delta$ and $\beta \neq 1$.*

Proof: It suffices to consider the deviation of one player from (E, E). From a formal point of view, it is slightly easier to consider the difference function $D^y_{EL} = \frac{\gamma^2}{32\beta - 8\gamma^2}$.

The derivative of this difference function is equal to

$$\frac{64\beta\gamma}{(32\beta - 8\gamma^2)^2}.$$

Because of $D^y_{EL}(\beta, 0) = 0$ and $\beta > 0$, we can apply the arguments of fact 1.

q.e.d.

8.2.2 The cases of $\beta = 1$ and $\gamma \neq \delta$

In the second case of one-dimensional asymmetry, we begin our analysis by imposing appropriate parameter restrictions to guarantee positive unit profits x and y for all four constellations of timing dispositions. Altogether, with assumption 1 we require that the following restrictions hold in this subsection:

Assumption 5

$$\min\{\gamma,\ \delta\} > -1, \quad \max\{\gamma,\ \delta\} < 1.23607 (= \sqrt{5} - 1), \quad |\delta - \gamma| < 2.$$

The payoff table of the 2×2 timing game now depends on the parameters δ and γ. More concretely, the pie and the individual payoffs are given as follows

$$\Pi^* = \frac{1}{2+\delta+\gamma}, \tag{24}$$

$$u_x^*(E) = \frac{2+\delta+\delta^2-\gamma-\gamma\delta}{(2+\delta+\gamma)(4+\delta^2-2\delta\gamma+\gamma^2)}, \tag{25}$$

$$u_y^*(E) = \frac{2+\gamma+\gamma^2-\delta(1+\gamma)}{(2+\delta+\gamma)(4+\delta^2-2\delta\gamma+\gamma^2)}, \tag{26}$$

$$u_x^*(E_x,L_y) = \frac{16+8\delta^2-8\gamma-2\gamma^2+\gamma^3+\delta(8-8\gamma+\gamma^2)}{8(2+\delta+\gamma)(4+\delta^2-2\delta\gamma)}, \tag{27}$$

$$u_y^*(E_x,L_y) = \frac{16+8\gamma+2\gamma^2-\gamma^3-\delta(8+8\gamma+\gamma^2)}{8(2+\delta+\gamma)(4+\delta^2-2\delta\gamma)}, \tag{28}$$

$$u_x^*(L_x,E_y) = \frac{\delta^3+8(-2+\gamma)+\delta^2(-2+\gamma)+8\delta(-1+\gamma)}{8(2+\delta+\gamma)(-4+2\delta\gamma-\gamma^2)}, \tag{29}$$

$$u_y^*(L_x,E_y) = \frac{8\delta(1+\gamma)-8(2+\gamma+\gamma^2)-\delta^2(-2+\gamma)-\delta^3}{8(2+\delta+\gamma)(-4+2\delta\gamma-\gamma^2)}, \tag{30}$$

$$u_x^*(L) = \frac{16-\delta^3-8\gamma-2\gamma^2+\gamma^3+\delta(8-8\gamma+\gamma^2)+\delta^2(2-\gamma+\gamma^2)}{2(2+\delta+\gamma)(-4+\delta\gamma)^2}, \tag{31}$$

$$u_y^*(L) = \frac{16+\delta^3+8\gamma+2\gamma^2-\gamma^3+\delta^2(-2+\gamma+\gamma^2)-\delta(8+8\gamma+\gamma^2)}{2(2+\delta+\gamma)(-4+\delta\gamma)^2}. \tag{32}$$

Again we consider the difference functions $D^x_{EL}(\gamma,\delta) := u_x^*(E)-u_x^*(L_x,E_y)$ (resp. $D^y_{EL}(\gamma,\delta) := u_y^*(E)-u_y^*(E_x,L_y)$ for player 2) in order to see whether the combination (E,E) can be stabilized as an equilibrium. By explicit calculations we find

$$D^x_{EL}(\gamma,\delta) = \frac{\delta^2(2-\delta+\gamma)^2}{8(4-2\delta\gamma+\gamma^2)(4+\delta^2-2\delta\gamma+\gamma^2)}$$

and

$$D^y_{EL}(\gamma,\delta) = \frac{\gamma^2(2+\delta-\gamma)}{8(4+\delta^2-2\delta\gamma)(4+\delta^2-2\delta\gamma+\gamma^2)}.$$

In order to check the stability of constellation (L,L), we consider difference functions $D^x_{LE}(\gamma,\delta) := u_x^*(L)-u_x^*(E_x,L_y)$ and $D^y_{LE}(\gamma,\delta) := u_y^*(L)-u_y^*(L_x,E_y)$ which can be calculated explicitly as follows

$$D^x_{LE}(\gamma,\delta) = \frac{-\delta^2(-4+2\delta-2\gamma+\gamma^2)^2}{8(4+\delta^2-2\delta\gamma)(-4+\delta\gamma)^2},$$

$$D^y_{LE}(\gamma,\delta) = \frac{-\gamma^2(-4-2\delta+\delta^2+2\gamma)^2}{8(-4+\delta\gamma)^2(4-2\delta\gamma+\gamma^2)}.$$

Since the graphs of the difference functions show much variation when the parameters γ and δ are varied, it is not worthwhile to present graphical illustrations. We will proceed directly to the numerical evaluation instead.

Fact 3 *Suppose assumption 5 holds, then fact 1 is a robust result in the sense that it extends to the range of asymmetric situations where $\beta = 1$ and γ and δ vary independently from each other.*

Proof: We have to check each of the difference functions separately.

a) First, we consider

$$D^x_{EL}(\gamma, \delta) = \frac{\delta^2(2 - \delta + \gamma)^2}{8(4 - 2\delta\gamma + \gamma^2)(4 + (\delta - \gamma)^2)}.$$

It is easy to check that, because of assumption 5, all bracket expressions in the denominator are strictly positive. The numerator is strictly positive except for $\delta = 0$ where it is equal to zero.

Similar conclusions hold for $D^y_{EL}(\cdot)$. This function is strictly positive over the domain of admissible parameter constellations except for $\gamma = 0$.

Summarizing, the constellation (E, E) is generically an equilibrium. It is a weak equilibrium if the price effect on the respective competitor is zero.

b) Next, we consider

$$D^x_{LE}(\gamma, \delta) = \frac{-\delta^2(4 + 2(\gamma - \delta) - \gamma^2)^2}{8(4 + \delta^2 - 2\delta\gamma)(-4 + \delta\gamma)^2}.$$

Again the bracket expressions in the denominator are strictly positive for all admissible parameter values. The denominator is strictly negative except for $\delta = 0$ and for the degenerate $(\gamma_x(\delta), \delta)$ pairs with $\gamma_x(\delta) := 1 - \sqrt{5 - 2\delta}$. For these pairs the bracket expression $(4 + 2(\gamma - \delta) - \gamma^2)$ is equal to zero. By straightforward calculations one can show that only pairs $(\gamma_x(\delta), \delta)$ with $\delta \in (0.5, 1.23607)$ satisfy assumption 5.

Similar conclusions hold for $D^y_{LE}(\cdot)$. This function is strictly negative except for $\gamma = 0$ and for $(\gamma_y(\delta), \delta)$ pairs with $\gamma_y(\delta) := \frac{2\delta + \delta^2 - 4}{2}$. For these pairs the bracket expression $(4 + 2(\delta - \gamma) - \delta^2)$ is equal to zero. One can show that only pairs with $\delta \in (0.732051, 1.23607)$ satisfy assumption 5.

It is interesting to see whether there exists an admissible pair $(\gamma*, \delta^*)$ so that $D^x_{LE}(\gamma^*, \delta^*) = D^y_{LE}(\gamma^*, \delta^*) = 0$ holds. For this parameter constellation (L, L) could be stabilized as an equilibrium, but not as a strict equilibrium. By intersecting the functions $\gamma_x(\cdot)$ and $\gamma_y(\cdot)$, one finds $\delta^* = 0.828427$.

Summarizing, the constellation (L, L) is generically not an equilibrium. For $\delta^* = \gamma^* = 0$ and $\gamma^* = -0.828427$, $\delta^* = 0.828427$ both players are indifferent between choosing L or E. For the remaining parameter constellations deviation from (L, L) is profitable.

q.e.d.

8.2.3 The 3-dimensional case

It has been illustrated above that Fact 1 is no pathology of Symmetry, but extends to nonsymmetric situations as well, regardless of which one of the two demand parameters is assumed to differ (see Facts 2 and 3). For the full 3-dimensional case an analysis similar to the above would be very difficult. We therefore rely on numerical analysis when analyzing it.

More specifically, we successfully used the following algorithm.

(*i*) Randomly select[10] triples (β, γ, δ) satisfying assumptions 1 and 2 !

(*ii*) Compute for each (β, γ, δ) the unit profits so that the triple (β, γ, δ) is discarded (meaning that one starts again with (i)) when these are negative, whereas the triple (β, γ, δ) is kept otherwise!

(*iii*) Determine the 2×2-bimatrix as in section 8 for the kept triple (β, γ, δ) and its pure strategy equilibria.

Altogether, we explored 1 million triples (β, γ, δ) of which 768170 were discarded. Of the 231830 kept triples (β, γ, δ), 231828 had the unique equilibrium (E, E), similar to Facts 1, 2 and 3, while one had the unique equilibrium (E, L) or, respectively, (L, E). This clearly demonstrates that our main conclusion that in generic stationary or more or less changing market environments early threats will evolve or strategically be chosen is a very robust result.

9 Conclusions

Early threats require unconstrained self-commitment. Otherwise, they would be just cheap talk announcements which nobody takes seriously. Without self-commitment, therefore, threats have to be self-enforcing and could only be justified as early announced late threats. In this sense, our initial wondering that late rather than early threats are more general can be still maintained and justified. If there is no self-commitment, one only will determine one's threat when bargaining has failed. In the case of human marriage, this means that if one does not sign a marriage contract, conflict behavior will be determined only after terminating the joint venture.

If self-commitment is possible, however, our analysis supports early threats, as suggested by Nash (1953), as the likely result. This holds for all symmetric and easily calculable asymmetric situations as well as for a large random sample of full-dimensional parameter vectors. Although we cannot claim that early threats are universally dominant, we never found a generic case where this was not true. In our view, this provides a novel and hopefully illuminating justification of Nash's (1953) variable threat-bargaining model.

One may, of course, object that in bargaining under complete information threats are just counterfactuals. Nevertheless, or even because of this, it is a surprising result that early threats are robust outcomes of evolving or rationally chosen timing dispositions. For an analysis with incomplete information, where threats have to be carried with positive probability to prove one's relative strength, one might rely on the generalized Nash-bargaining solution (see Harsanyi and Selten, 1972).

[10]More specifically, we first selected independently and randomly $\gamma, \delta \in (-1, 4)$ and $\beta \in (0, 4)$ under the additional restriction $\delta > -\beta$ by using a random number generator (of *Free Pascal*).

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