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Taking Technology Seriously**

by

**Zuoquan Zhao
Max Planck Institute of Economics**

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For editorial correspondence,
please contact: egppapers@econ.mpg.de

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Max Planck Institute of Economics
Group Entrepreneurship, Growth and
Public Policy
Kahlaische Str. 10
07745 Jena, Germany
Fax: ++49-3641-686710

A Spatial Model of Growth: Taking Technology Seriously¹

Zuoquan Zhao

Max Planck Institute of Economics, Jena, Germany

zhao@econ.mpg.de

Abstract

This paper attempts to develop a spatial model of economic growth in which technology and externalities are assumed to be accountable for production in geographical space. Linking externalities to the extent of intensity of production across locations in continuous space, we introduce spatial range into the production function for technological, human, and physical capitals. Our model argues that the long-run growth rate of an economy is determined not just by the growth rates of the three factors of production but by their rates of change in spatial range over the territory of the economy. In other words, spatial intensity and accumulation matter for growth. Our model is consistent with studies on knowledge spillovers, geographical agglomeration, urban and regional growth, and trade. The primary policy implication of our model is the significance of establishing efficient mechanisms or channels that promote innovation, diffusion, trade, and factor mobility over the territory of an economy.

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It is not as if we always have it everywhere, but there is a process in which knowledge is being created all the time in different places, and is then being diffused. This evolving distribution should be reflected in a model of production, if it is to describe an entire economy in which different people know different things. As a consequence, the idea of an aggregate production function becomes very dubious, unless a new variable is introduced, representing the distribution and diffusion of new knowledge.

Kenneth Arrow (1994)

1. Introduction

Technology is the engine for economic growth. As widely documented, technological innovation and diffusion make significant contributions to the growth of nations, industries, and firms (see Archibugi & Michie, 1998). The role of technology is recognized as technical change in Solow's neoclassical growth theory and revealed as *R&D* and other resources devoted to innovation in endogenous growth theory, while the role of diffusion is linked to knowledge spillovers in the new growth theory (Fine, 2000). Especially, the significance of externalities due to spillovers from technology, human capital, and/or physical capital is addressed in the new growth paradigm (Barro, 1999). However, technology and related externalities are not directly treated in the two growth theories. The endogenous growth theory stresses efforts leading to innovation, e.g. expenditures on *R&D* and the number of researchers rather than the number of innovations and patents, not to speak patent citations and technological imitations that are related to innovation diffusion under market mechanism. In addition, the source of externalities is not specified in the aggregate production function.

This paper attempts to develop a spatial model of economic growth in which technology (innovated, imitated, bought, imported, or learned) and externalities are assumed to be

accountable for production in geographical space. We believe that, as shown in Gort and Konakayama (1982) and Johnston (1966), it is more straightforward to take into account the number of technologies used by firms regardless of its origin or source. For example, individual technology needs to be counted as many times as it is used by different firms at the same time. We also believe that, as demonstrated in Papageorgiou & Smith (1983) and Papageorgiou (1978), the externalities of production are closely related to its distribution across locations in space; this applies not only to technology but to the two other factors of production: human and physical capitals. The use of spatial distribution as an indicator of externalities is based on the observation that the distribution of production in space would be random (or uniform) if there is no externality for technology and factor inputs; therefore, any agglomeration of production indicates the extent of externality for the distribution because it represents a departure from spatial randomness.

To incorporate spatial externalities (or agglomeration economies) into a growth model, three issues need to be resolved. They include 1) how to measure the extent of intensity for a spatial distribution; 2) how to introduce the element of spatial intensity into the aggregate production function; and 3) how to extend agglomeration economies to the entire geographical space of an economy. In our paper, we first present a spatial intensity index by using the ratio of the amount of economic activity to the geographical range of the activity in space. Here spatial range is measured by the mean distance, indicating the extent of dispersion of the activity from its center of gravity. Unlike many other agglomeration indices (Sweeney & Feser, 2004), our index measures the extent of intensity in continuous space by counting every location and every amount of the activity involved. Especially, our index differentiates a change in geographical range from that in spatial intensity, both of which are widely thought as opposite in the literature.

Moreover, we introduce spatial range into the aggregate production function and relate the extent of spatial intensity to aggregate growth. The spatial production function indicates that output is a function of both factor inputs and their spatial ranges. In addition, we argue that the spatial division of labor is determined by the spatial extent of market. As market expands spatially due to either population dispersion or decreasing transport costs, increasing spatial division of labor promotes competition among locations e.g. cities and fosters aggregate productivity growth. The extension of agglomeration economies to the entire geographical space of an economy, which may be better termed economies of networks as discussed in Johansson & Quigley (2004), are also related to cross-location or -regional spillovers due to trade, factor mobility and technological diffusion (Feldman, 1999). In the literature, the geographical scope of agglomeration economies is limited to regions because of spatial specialization (Kim, 1995), and to cities or smaller areas due to urban specialization and diversification (Glaeser *et al.* 1992).

The link of externalities with spatial distribution in general and geographical intensity in particular is consistent with and may reconcile several different lines of research on growth. They include knowledge spillovers, space and geography, urban and regional growth, and trade. Feldman (1999) and Audretsch and Feldman (1996) believe that knowledge spillovers go hand in hand with innovation wherever there is larger extent of spatial agglomeration. Fujita and Thisse (2003) and Martin and Ottaviano (2001) speculate that geographical agglomeration of economic activity promotes aggregate growth. Many scholars, e.g. Henderson (2003), Nijkamp and Poot (1998) and Richardson (1973) show that agglomeration economies and factor mobility account for the growth of cities and regions. Rossi-Hansberg (2005) and Krugman (1991) argue that agglomeration enhances growth in trade.

The rest of the paper is organized as follows. Section 2 introduces the spatial model of growth. Its policy implications are briefly discussed in Section 3. Section 4 concludes.

2. A Spatial Model

2.1 The Spatial Intensity Index

Denote w a factor of production distributed across a variety of locations with amount w_j at location j (x_j, z_j) in the geographical space of an economy S ; I the extent of geographical intensity, D the geographical range, and C the center of gravity of w .

Here the spatial intensity index (simply the I index),

$$I = \frac{W}{\pi D^2}. \quad (1)$$

where W is the total of w_j ; and D (2-dimensional) represents the geographical range of w . I increases with W if D is constant, and decreases if D declines and W is constant.

The geographical range D is identified using the mean distance,

$$D = \frac{\sum w_j D_j}{W}. \quad (2)$$

where D_j denotes the distance between location j and the center of gravity C .

The center of gravity C is represented by a pair of coordinates,

$$x = \frac{\sum w_j x_j}{W}; z = \frac{\sum w_j z_j}{W}. \quad (3)$$

where x_j and z_j are the coordinates of location j .

The spatial distribution of w has one of the following three characteristics: 1) if w has the same geographical range as that of the space of S , it is most likely a random distribution without externality; 2) if w has a larger geographical range than that of the space of S , it will be skewed to the periphery of S with more externalities in the periphery than in the interior; 3) if w has a smaller geographical range than that of the space of S , it will be skewed to the interior of S with more externalities in the interior than in the periphery.

2.2 The Spatial Production Function

Denote Y_i output, A_i quantity of technology, H_i quantity of human capital, and K_i quantity of physical capital for firm i ; D_{Ai} , D_{Hi} , and D_{Ki} the geographical range for the distribution of the three factors of production within the place of the firm,

The production function of the firm

$$Y_i = F\left(\frac{A_i}{D_{Ai}^2}, \frac{H_i}{D_{Hi}^2}, \frac{K_i}{D_{Ki}^2}\right). \quad (4)$$

Following the Cobb-Douglas format, we have

$$Y_i = B_i A_i^\alpha H_i^\beta K_i^\gamma D_{Ai}^{-2\xi} D_{Hi}^{-2\psi} D_{Ki}^{-2\zeta}. \quad (5)$$

Here B_i is the residual. This spatial production function suggests growth and productivity of the firm depend not only on the quantity of the three factors of production but also on the place of the firm and how they are organized and distributed within that place. In a high density area where agglomeration externalities are high, the firm needs to keep these factors as dense as possible to lower the transaction costs (Coase, 1937) and to save land rent; meanwhile it needs advanced technology and high quality space-saving equipment in general and more skillful labor in particular for gaining as more externalities as possible from the nearby surroundings and environment, offsetting high wages for the labor and high rents and costs for the physical and technological capitals. The story would be different for the firm located at a low density place.

Denote Y aggregate output, A total quantity of technology, H total quantity of human capital, and K total quantity of physical capital for all firms in the economy; and D_A , D_H , and D_K the gross geographical range for the distribution of the three factors of production over the territory of the economy respectively; assume A spills over or stays for higher profits, H moves or stays for higher wages, K flows or stays for higher returns across the territory of the economy.

The production function for the economy

$$Y = F\left(\frac{A}{D_A^2}, \frac{H}{D_H^2}, \frac{K}{D_K^2}\right). \quad (6)$$

The Cobb-Douglas format

$$Y = BA^\alpha H^\beta K^\gamma D_A^{-2\xi} D_H^{-2\psi} D_K^{-2\zeta}. \quad (7)$$

The dynamics of agglomeration of A , H , and K rests upon the balance between centripetal and centrifugal forces over the space of the economy. The centripetal forces (F_p) includes Marshallian externalities, technological innovations, and home market effect, while the centrifugal forces force (F_f) consists of international market effect, comparative advantage, transport costs, agglomeration diseconomies (such as high land rents, high wages, and high congestion costs), and national security (such as spatial equity). The interaction between F_p and F_f may lead to different paths of agglomeration. For the three factors of production, I increases while D decreases if $F_p \gg F_f$; both I and D goes up if $F_p > F_f$ and W grows faster than I ; I and D are constant if $F_p = F_f$; both I and D decline if $F_p < F_f$ and W grows slower than I ; and I drops and D expands if $F_p \ll F_f$.

At aggregate spatial equilibrium these three factors are well matched at each location over the space of the economy and no factor gains more benefits from moving to any other location, thus

$$D_A = D_H = D_K = D_e.$$

Then from (7)

$$Y = BA^\alpha H^\beta K^\gamma D_e^{-2(\xi+\psi+\zeta)}. \quad (8)$$

Increase returns appear when $(\alpha + \beta + \gamma) - 2(\xi + \psi + \zeta) > 1$.

For the long term spatial steady state, $\alpha - 2\xi = \text{constant}$, $\beta - 2\psi = \text{constant}$, and $\gamma - 2\zeta = \text{constant}$.

From (4) and (1), we have

$$Y = F(I_A, I_H, I_K), \quad (9)$$

Differentiate (7) divided by Y and rearrange terms, we have

$$\frac{\dot{Y}}{Y} = \frac{I_A}{Y} \frac{\partial F}{\partial I_A} \frac{\dot{I}_A}{I_A} + \frac{I_H}{Y} \frac{\partial F}{\partial I_H} \frac{\dot{I}_H}{I_H} + \frac{I_K}{Y} \frac{\partial F}{\partial I_K} \frac{\dot{I}_K}{I_K},$$

(10)

Simplifying,

$$\frac{\dot{Y}}{Y} = \frac{I_A}{Y} M_{I_A} i_A + \frac{I_H}{Y} M_{I_H} i_H + \frac{I_K}{Y} M_{I_K} i_K.$$

(11)

where M_{I_H} , M_{I_K} , and M_{I_A} are the social marginal product of the spatial intensity of A , H , and K respectively; i_A , i_H , and i_K are the growth rate of the spatial intensity of A , H , and K respectively.

From (11), we have the rate of growth of output per labor

$$\frac{\dot{y}}{y} = \frac{hI_A}{YL} M_{I_A} i_A + \frac{hI_H}{YL} M_{I_H} i_H + \frac{hI_K}{YL} M_{I_K} i_K.$$

(12)

where $y = Y/L$ and $h = H/L$ (human capital per labor). Functions (11) and (12) illustrate that the extent of spatial intensity matters for the growth of an economy. The long-run growth rate of the economy is determined by the growth rates of spatial intensity of technological, human, and physical capitals over the geographical space of the economy. The rate of spatial intensity of A , H , and K can grow in an economy of any size if technological innovations continue to grow and spread out. In other words, increasing returns are possible for the spatial intensity of A , H , and K if social institutions and governmental policies can maintain a favorable environment in which technological innovations, human capital, and physical capital reinforce one another. Notably, declining transport and trade costs enhance the sharing

of human, physical, and technology capitals among cities over a large geographical scope of an economy due to rising mobility of these factors.

Notably, our spatial growth model is in fact a more methodological than a theoretical framework. Interestingly, it outlines growth as a complex spatial accumulative process in which externalities play a significant role. It endogenizes externalities by taking location and spatial distribution into account. It relaxes various assumptions of the neoclassical growth theory. For example, perfect competition, complete information, and zero externalities are not necessary for modeling growth in space.

3. Policy Implications

Innovation, diffusion and spatial externalities are important for economic growth. Efficient mechanisms or channels need to be established to promote innovation, diffusion, and factor mobility over the territory of an economy. The focus of growth-oriented policy should be on taking advantage of agglomeration economies among a few large cities and metropolitan areas with considerable effort devoted to induce high interdependence among them and to keep agglomeration diseconomies in check within each of them.

4. Conclusion

We presented a model of economic growth in a spatial context in which technology and externalities are considered accountable. Externalities were linked with the extent of spatial intensity of factors of production and their spatial ranges were introduced to the production function. Our model showed that both the factors of production especially technology and their spatial distributions contribute to the long run growth of an economy. Much more microeconomic foundations and empirical tests are to be made for our spatial growth model.

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