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Accuracy of proposers' beliefs in an allocation-type game

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Abstract

In the context of an allocation game, this paper analyses the proposer's reported beliefs about the responder's willingness to accept (or reject) the proposed split of the pie. The proposer's beliefs are elicited via a quadratic scoring rule. An econometric model of the proposer's beliefs is estimated. The estimated proposer's beliefs are then compared with the actual responder's choices. As a result of this comparison, we observe that the proposer tends to underestimate the empirical acceptance probability, especially when the slice of the pie allocated to the proposer is large.

JEL classification: C51; C52; C72; D84

Keywords: Model construction and estimation; Allocation game; Beliefs elicitation and evaluation.

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1 Introduction

In the context of ultimatum games (Güth et al., 1982), a crucial role is played by the proposer's beliefs.

Researchers analysing data from contexts such as this clearly cannot rely on data on the proposer's allocation decisions alone, because these are insufficient to identify the proposer's decision-making process. It, in fact, depends on his/her beliefs about the responder's willingness to accept the proposed split (Manski, 2002).

In similar frameworks, researchers may resort to rational expectations theory. Essentially, they may assume that proposers are acquainted with the relevant data and that they form their beliefs accordingly, so that the probability a proposer assigns to the responder's willingness to accept a certain split coincides with the empirical responders' acceptance rate.

Another possibility is that of asking the proposer about his/her beliefs directly.[1] This approach raises the issue of which rule to adopt to incentivise beliefs.[2]

Inspired by the referred literature, in this paper, we verify the accuracy of proposer's beliefs, elicited via a quadratic scoring rule (Selten, 1998). We develop an econometric model of their formation, and we compare the estimation results to the empirical responders' acceptance rate. The accuracy of proposers' beliefs is also evaluated by an indicator which does not rely on any econometric model.

The paper is organised as follows: Section 2 delineates the main features of the laboratory experiment; Section 3 is dedicated to the econometric modelling of the proposer's beliefs; the estimation results of such model are presented and discussed in Section 4; Section 5 concludes.

2 The experiment

The experiment is composed of different parts. We analyse data only from one part of the experiment. Therefore, here we draw out only the characteristics that are crucial for

[1] See Manski (2004) for a review.

[2] The literature on the matter is quite rich. See, for example, a recent study by Armantier and Treich (2013) and all references therein.

understanding the econometric models constructed in a later section of the paper.[3]

In the experiment, there are two roles, the proposer and the responder, and a pie made of 100 tokens to split. There are five pre-determined possible split of the pie:

$$(10, 90) \qquad (30, 70) \qquad (50, 50) \qquad (70, 10) \qquad (90, 10)$$

where the first value in the brackets represents the number of tokens allocated to the proposer and the second value, instead, is the number of tokens allocated to the responder.

The allocation are presented to both proposers and responders in a random order.[4]

Those who are assigned the role of the proposer are asked, for each allocation (in a random order), which is the likelihood they assign to the facts that the responder will "approve" or "not approve" that allocation. In order to report their beliefs, they are given 100 tokens and asked to allocate these tokens between the two possible responders' actions in a way that reflects their convictions. The elicitation of the proposer's beliefs is incentivised by a quadratic scoring rule. Each token obtained from the beliefs elicitation part is converted according to the rate 1 token = €0.10.

Those who are assigned the role of the responder are asked, in a random order for each of the five allocations, whether they are willing to "approve" or "not approve" the proposed split of the pie.

Both proposers and responders are made clear that, if the proposed split is approved by the responder, then they will get the number of tokens indicated therein. If, instead, the responder does not approve the proposed split of the pie, they both get 0 tokens. Each token obtained from the split of the pie is converted according to the rate 1 token = €0.20.

Neither proposers nor responders receive any feedback about their partner's decision throughout the experiment.

We collected data from 174 proposers and 174 responders in the period Aug.-Dec. 2013. The experimental sessions were conducted in the laboratory of the Max Planck Institute of

[3]The experimental instructions and further detail about the experimental procedures are available from the authors on request.

[4]The roles of proposers and responders are assigned to the participants, at random, at the beginning of the experiment and kept until the end. There is an equal number of proposers and responders in each sitting.

Economics, directed by Professor Werner Güth.

3 The model of proposer's beliefs

Let us imagine that proposer i's beliefs about the responder's willingness to accept the proposed split of the pie may be represented by an urn containing azure balls (a) and red balls (r) in a proportion that reflects her conviction. We can also imagine that, when the proposer is asked to report her beliefs by allocating tokens to the two alternatives "the responder will accept the split" and "the responder will reject the split", she samples with replacement 100 balls from this urn (one ball for each token to be allocated). Then, a number of tokens equal to the number of azure balls so sampled is allocated to the first alternative, the remainder of the 100 tokens (which corresponds to the number of red balls sampled) is instead allocated to the second alternative.

Let us denote the probability that proposer i draws an azure ball from her urn (that is the probability she assigns to the responder's willingness to accept) as $p_{a,i}$ and let us assume that it follows a beta distribution of parameters α and β:

$$p_{a,i} \sim \text{Beta}(\alpha, \beta), \tag{1}$$

with $\alpha > 0$ and $\beta > 0$.

Given the properties of the beta distribution, the expected probability of the responder accepting the split is

$$\text{E}(p_a) = \frac{\alpha}{\alpha + \beta}, \tag{2}$$

and its variance is

$$\text{Var}(p_a) = \frac{\alpha \times \beta}{(\alpha + \beta)^2 \times (\alpha + \beta + 1)}. \tag{3}$$

Since for each proposer i, $i = 1, \ldots, N$, we observe $t = 100$ draws from the same urn (one for each available token to allocate), we can thing of each draw as a random variable, X_i, following a binomial distribution of parameters t and $p_{a,i}$,

$$X_i \sim \text{Bin}\left(t, p_{a,i}\right), \tag{4}$$

so that the probability for proposer i of drawing k azure balls in 100 trials conditional on $p_{a,i}$ is

$$P\left(X_i = k_i | p_{a,i}, t\right) = \binom{t}{k_i} p_{a,i}^{k_i} \left(1 - p_{a,i}\right)^{t-k_i}. \tag{5}$$

Integrating with respect to p_a and using the result in Mosimann (1962)[5], we obtain that the probability that proposer i allocates k_i out of 100 tokens on the option "the responder will accept the split" follows a beta-binomial distribution,

$$\begin{aligned} f\left(k_i|t,\alpha,\beta\right) &= \binom{t}{k_i}\frac{\mathrm{B}\left(k_i+\alpha, t-k_i+\beta\right)}{\mathrm{B}\left(\alpha,\beta\right)} \\ &= \frac{\Gamma\left(t+1\right)\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha+\beta+t\right)} \times \frac{\Gamma\left(\alpha+k_i\right)}{\Gamma\left(\alpha\right)\Gamma\left(k_i+1\right)} \times \frac{\Gamma\left(\beta+k_i\right)}{\Gamma\left(\beta\right)\Gamma\left(t-k_i+1\right)}, \end{aligned} \tag{6}$$

where B and Γ are the beta and the gamma functions, respectively.

The full sample log-likelihood for the set of n proposers is

$$LogL\left(\alpha,\beta\right) = \sum_{i=1}^{n} logf\left(k_i|t,\alpha,\beta\right). \tag{7}$$

4 Estimation results

The model described in the previous section is estimated using proposers' beliefs data. Our sample consists of 174 subjects. Each proposer is observed 5 times, one for each possible split of the pie. The model is estimated separately for each split of the pie. To estimate the model, we use the method of Maximum Likelihood. The program is written in Stata 13 and is available from the authors on request.

[5]He demonstrates that the beta distribution is a prior conjugate of the multinomial distribution.

Table 1 displays the estimation results. It is organised in 5 columns, one for each proposed split of the pie. The estimated values of α and β are reported in the first two rows of the table. Based on such values, the table also shows an estimate of the expected probability of the responder accepting the split, $\mathrm{E}(p_a)$ and its variance, $\mathrm{Var}(p_a)$, according to eqq. (2) and (3), respectively. The reported standard errors are obtained by the delta method.

Table 2 displays the empirical probabilities that the responder accepts the split, termed $\tilde{p}_a$, based on responders' data for each proposed split. These values are calculated as the proportion of times the responders in our samples accepted the proposed split of the pie. We observe that such empirical probabilities increase from 90% when the proposed split allocates 10 tokens to the proposer and 90 to the responder to almost 100% when the pie is equally divided. Such probabilites decrease to 76% and 46% when the proposed split assigns 70 and 10 tokens to the proposer and, consequently, 30 and 90 tokens to the responder, respectively.

In a graphical format, Figure 1 combines the results from Table 1 and Table 2. It displays 5 charts, one for each proposed split of the pie, of $\mathrm{Beta}(\alpha, \beta)$, which represents the distribution over the population of the probability the proposer assigns to the responder's willingness to accept the proposed split of the pie (or, equivalently, the probability that proposer i draws an azure ball from her urn). Each beta distribution is drawn according to the parameters' estimates from Table 1. Each chart displays two vertical lines, one red and one green. The red line represents the estimated expected probability of the responder accepting the split, $\mathrm{E}(p_a)$, calculated by eq. (2) and reported in Table 1. The green line, instead, represents the empirical probabilities that the responder accepts the proposed split, calculated from the responders' data as explained in the previous paragraph (see Table 2). The charts point out very clearly that the proposers tend to underestimate, on average, the responder's acceptance probability. The gap between the proposers' expected acceptance probability and the responders' actual acceptance probability, which can be appreciated as the distance between the red and the green vertical lines, is particularly sizable for pie's splits $(70, 30)$ and $(90, 10)$, that is when the slice assigned to the proposer is larger than that assigned to the responder.

We also derive a measure of the accuracy of the proposer's beliefs, separately for each split of the pie. It is computed as the quadratic distance between the proposer's beliefs and $\tilde{p}_a$, the

	proposed split of the pie				
	(10,90)	(30,70)	(50,50)	(70,30)	(90,10)
α	0.4996	1.1167	1.6318	0.9422	0.4218
	(0.0909)	(0.1965)	(0.3473)	(0.1032)	(0.0438)
β	0.0739	0.1448	0.1111	0.6802	0.7971
	(0.0109)	(0.0189)	(0.0166)	(0.0737)	(0.0966)
$\mathrm{E}(p_a)$	0.8719	0.8852	0.9362	0.5807	0.3461
	(0.0187)	(0.0155)	(0.0109)	(0.0224)	(0.0233)
$\mathrm{Var}(p_a)$	0.0713	0.0449	0.0218	0.0928	0.1020
	(0.0119)	(0.0085)	(0.0058)	(0.0060)	(0.0076)
LogL	-391.00	-480.37	-377.16	-792.93	-748.68
observations	174	174	174	174	174

Table 1: Maximum likelihood estimates of model (7)'s parameters. The model is estimated from the proposers' data for each split of the pie, separately. The table also reports estimates of the expected probability of the responder accepting the split, $\mathrm{E}(p_a)$, for each proposed split based on the estimates of α and β, and its variance, $\mathrm{Var}(p_a)$.

	proposed split of the pie				
	(10,90)	(30,70)	(50,50)	(70,30)	(90,10)
$\tilde{p}_a$	0.9023	0.9195	0.9885	0.7644	0.4598
observations	174	174	174	174	174

Table 2: Empirical probabilities that the responder accepts the split, $\tilde{p}_a$, based on responders' data for each proposed split, calculated as the proportion of times the responders in our samples accepted the proposed split of the pie.

empirical probabilities that the responder accepts the split as computed from the responder's data and reported in Tab. 2:

$$\delta = \sum_{i=1}^{n} \left[\frac{k_i}{100} - \tilde{p}_a \right]^2 \Big/ n \tag{8}$$

The results of this computation are displayed in Tab. 3. They show that the accuracy of beliefs improves when the slice of the pie designated for the proposer increases from 10 to 50. When the slice designated for the proposer is 70, instead, her beliefs reach their worst accuracy level, which only marginally improves when 90% of the pie is allocated to the proposer. The results of this "crude" indicator of the accuracy of the proposer's beliefs reflect very closely those obtained by the econometric model described in Section 3.[6] This justifies our use of

[6]The adjective "crude" is used with the meaning that no prior hypothesis on the proposer's beliefs formation process is made.

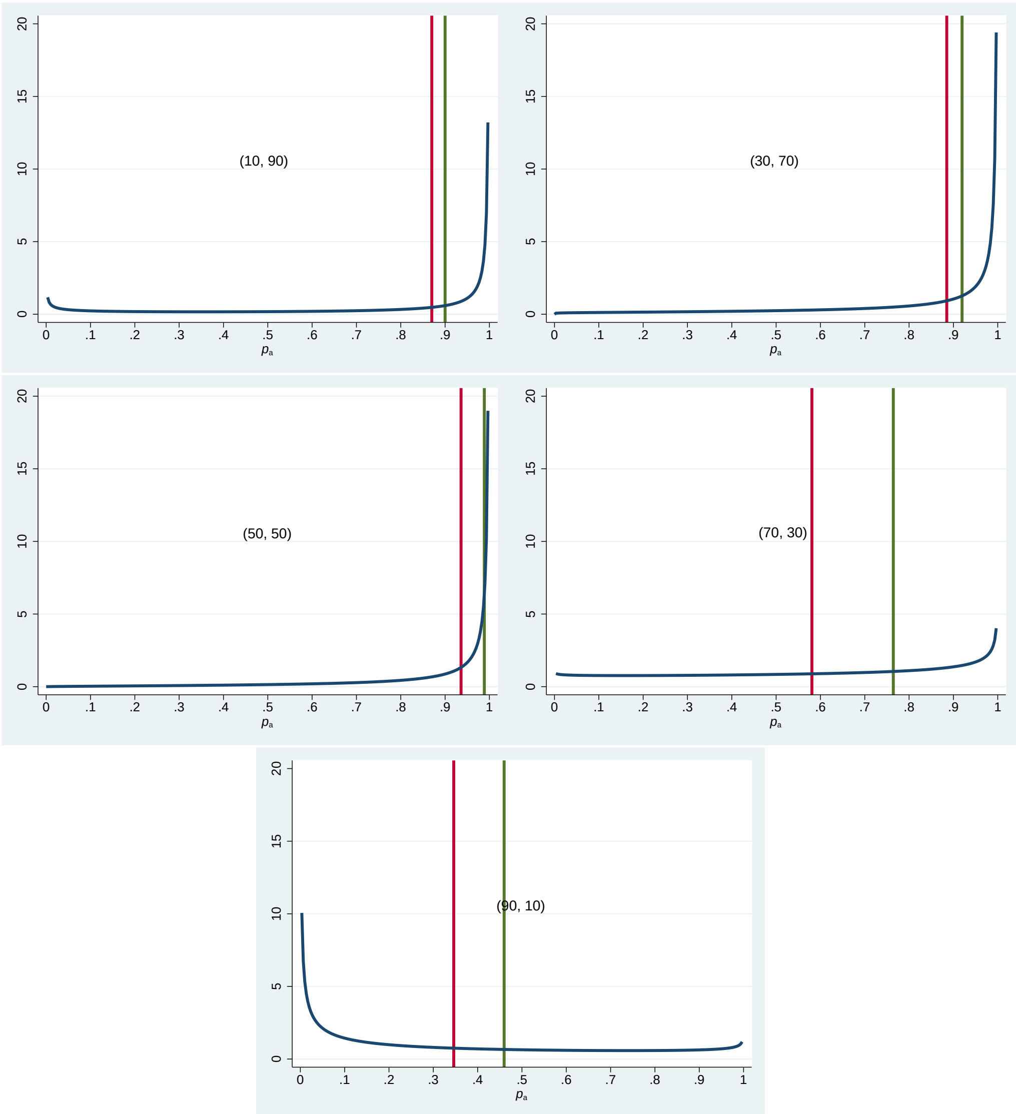

Figure 1: Distribution of the probability that the proposer assigns to the responder's willingness to accept, p_a, based on the estimation results in Table 1 per proposed split of the pie. The red vertical line indicates the estimated expected probabilities of the responder accepting the split, $\mathrm{E}(p_a)$. The green vertical line indicates the empirical probability that the responder accepts the split based on responders' data. The corresponding allocation is indicated on each graph.

such an econometric modelling of beliefs data.

	proposed split of the pie				
	(10,90)	(30,70)	(50,50)	(70,30)	(90,10)
δ	0.0694	0.0394	0.0217	0.1363	0.1172
	(0.0139)	(0.0089)	(0.0058)	(0.0111)	(0.0060)
observations	174	174	174	174	174

Table 3: Accuracy on the proposer's beliefs that the responder accepts the split, δ, computed from the data on proposers' beliefs and responders' acceptance rates according to Eq. (8).

5 Conclusion

In this paper, we have compared the proposer's reported beliefs about the responder's action and the responder's actual decision concerning the split of a pie, in the context of an allocation game. An econometric model of the proposer's beliefs has been developed.

We have shown how the proposer, on the basis of the estimation results of the econometric model, tends to underestimate the responder's acceptance probability. The acceptance probability has been computed from the responders' data.

The downward bias in the proposer's expected acceptance probability is particularly severe when the proportion of the pie assigned to the proposer exceeds the equal split. It appears that proposers' tend to underestimate the proportion of rational responders in the population (a rational responder should accept any positive slice of the pie, that is he should always accept the split, in our experiment). This happens for any proposed split of the pie, but it is not so prominent when the split of the pie is in favour of the responder.

Our findings might have at least two alternative consequences in terms of the analysis of allocation games data. We have to consider two cases: 1) the reported beliefs are an accurate description of the proposer's beliefs; 2) the reported beliefs are not trustworthy.

In the first case, the consequence is that the rational expectations hypothesis should not be used when data are analysed. This implies that the analyst cannot assume that the acceptance probability of the responder is known to the proposer and that it corresponds to the empirical probability, calculated as the proportion of times, in the sample, responders have accepted the proposed split of the pie. Since we have demonstrated that the empirical probability does not coincide with the expected probability, following that approach would introduce a severe bias in the estimates of the proposer's behaviour.

If the second case holds, then its immediate consequence is that the quadratic scoring rule is not the appropriate incentive mechanism to elicit subjects' beliefs and that more appropriate incentive schemes should be researched.

Our data do not enable us to establish which of the two cases holds. This will be object of future analyses.

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