JENA ECONOMIC RESEARCH PAPERS

2013 – 048

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www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

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D-07743 Jena
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Equilibrium Selection under Limited Control - An Experimental Study of the Network Hawk-Dove Game

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November 2013

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Abstract

For games of simultaneous action selection and network formation, game-theoretic behavior and experimental observations are not in line: While theory typically predicts inefficient outcomes for (anti-)coordination games, experiments show that subjects tend to play efficient (non Nash) strategy profiles. A reason for this discrepancy is the tendency to model corresponding games as one-shot and derive predictions. In this paper, we calculate the equilibria for a finitely repeated version of the Hawk-Dove game with endogenous network formation and show that the repetition leads to additional equilibria, namely the efficient ones played by human subjects. We confirm our results by an experimental study. In addition, we show both theoretically and experimentally that the equilibria reached crucially depend on the order in which subjects adjust their strategy. Subjects only reach efficient outcomes if they first adapt their action and then their network. If they choose their network first, they do not reach efficient outcomes.

Keywords: Network games, Hawk/Dove games, finitely repeated game

JEL Codes: D85, C72, C73, C92

1 Introduction

In recent years, online social networks such as Facebook or Twitter have attracted several millions of users worldwide. The motivation for the participants has been empirically studied in recent years. Users on Twitter, for example, have four main reasons: daily chatter, conversation, sharing information and reporting news (Java et al., 2007); users on Facebook (Joinson, 2008) are driven by interest in building social capital, communication, social network surfing and new contents, i.e., photos, profiles, etc. Similarly, online social networks in firms have three benefits for participants: personal sharing, career advancement, and promotion of projects (DiMicco et al., 2008). In sum, all motivations identified in these empirical studies resemble each other and either relate to (1) communication among participants or (2) influencing other participants by distributing information. However, it is still unclear how and why these motivations enjoy wide acceptance and attract high numbers of participants. In this paper, we suggest a behavioral economic approach to analyze online social networks.

In these networks, participants choose both their network, i.e., the participants they are linked to, and their action, i.e., the information they share and the frequency with which they update them. Based on the empirical studies of different motivations in online social networks, we believe there are basically two actions: one aggressive (hawk) and one defensive (dove) action. Given that links connect two different participants, the latter play an anti-coordination game: If both try to influence each other, each participant can unilaterally increase his payoff by deviating to a defensive communication strategy. On the other hand, if both follow a defensive communication strategy, they can unilaterally increase their payoffs by switching to a more aggressive action. Given that all linked participants see the same profile of a user, we expect our subjects to resort to the same action toward all of them.

Corresponding games, i.e., games with simultaneous action selection and network formation, have been studied in the past. In coordination games (e.g., Hojman and Szeidl, 2006), players choose their action in a $2x2$ coordination game and play this with all other players they are linked to. Here unilateral linking leads to complete networks, in which all players resort to the same equilibrium of the one-shot game, namely the payoff dominant or risk dominant equilibrium (Goyal and Vega-Redondo, 2005). This changes if bilateral linking is allowed, i.e., if both linked players have to agree on its establishment. Here different network structures emerge and the strategies played depend on the network structures generated, e.g., in circles only risk dominant actions are chosen, while in complete networks payoff and risk dominant equilibria are possible (Jackson and Watts, 2002). However, all theoretic analyses confirm that the risk dominant equilibria are more likely. In contrast, subjects in behavioral experiments (Corten and Buskens, 2010, Corbae and Duffy, 2008) play the payoff dominant equilibrium more often.

The tension between theory and experiment observed in networked coordination games also occurs in anti-coordination games, namely Hawk-Dove games with endogenous network formation. For unilateral linking, one-shot game-theoretic models predict for several parameter combinations that in equilibrium more hawks than doves (Bramoulle et al., 2004, Berninghaus and Vogt, 2006) exist in the network, with overall payoff decreasing as the number of hawks increases. In behavioral experiments, however, subjects tend to play strategy profiles, with higher numbers of doves staying closer to efficient strategy profiles of the one-shot game (Berninghaus at al., 2012). In addition, subjects in experiments on bilateral linking seem to favor fair outcomes and even coordinate on strategy profiles in which they alternate their behavior in each period to gain equal payoffs for all players (Tsvetkova and Buskens, 2012).

In this paper, we aim to extend the existing literature in several directions: First, we analyze Hawk-Dove games with endogenous network formation, hereafter called the Network Hawk-Dove game, as a finitely repeated instead of one-shot game. We show that by applying simple trigger strategies, the strategies observed experimentally are theoretically plausible and can be predicted. Second, we show that by limiting the strategy set for some periods, the theoretical predictions change. Namely, players reach lower payoffs if they can only choose their action in some periods, while they reach efficient outcomes if links are fixed or they are free to choose links and actions throughout the game. Third, we confirm our theoretical predictions by an experimental test of the Network Hawk-Dove game.

Our results help us to better understand online social networks: Successful online social networks allow the participant to easily change his links, whereas changing his action, i.e., modifying his profile is difficult. Given our results, this design principle might be one of the reasons guaranteeing the success of online social networks. Besides, we believe our approach enriches the existing literature. In addition to the existing Nash equilibria, Berninghaus et al (2012) suggest a new (one-shot) concept to predict the observed behavior. We show that the existing Nash concept suffices. In addition, Berninghaus et al. (2012) derive their results from an experiment in continuous time. The motivation for this approach is convincing: By letting subjects continuously choose their strategies, one can expect them to adapt their strategies faster to the strategies of others. As our results show, and as we extensively discuss in Section 5, this is not necessary.

2 The Network Hawk-Dove game

Each player $i \in \{1, \ldots, n\}$ in a Network Hawk-Dove game participates in a Hawk-Dove game with all players j she is linked to, using a Network game. We first introduce the Hawk-Dove game and the Network game separately before combining them into the Network Hawk-Dove game.

In the Network game $G^N := \{\Sigma^N, \Pi^N(\cdot)\}$, player i decides with whom she wants to interact.[1] Hence, each strategy in the Network game σ_i^N is a subset of all players $\sigma_i^N \subseteq \Sigma^N \setminus \{i\}$ with $\Sigma^N := \{1, \ldots, n\}$. Establishing links to other players is not costless but infers a cost $k > 0$ per established link. The payoff of the Network game is $\Pi^N(\sigma_i^N, \sigma_{-i}^N) := -k \cdot |\sigma_i^N|$. Each strategy profile $\sigma^N = (\sigma_1^N, \ldots, \sigma_n^N)$ implies a directed graph $g(\sigma^N) = (V(\sigma^N), E(\sigma^N))$ with edges $E(\sigma^N)$ and vertices $V(\sigma^N)$. Each vertex $v_i \in V(\sigma^N)$ corresponds to 1 player $i \in \{1, \ldots, n\}$. For each link player i establishes to player j, i.e, if $j \in \sigma_i^N$, an edge (i, j) from vertex v_i to v_j exists in $E(\sigma^N)$. Each player i has a set of contacts $N_i(\sigma^N) := \sigma_i^N \cup \{j | (j, i) \in E(\sigma^N)\}$, consisting of the players to whom she establishes a link, σ_i^N, and those who establish a link to her, $\{j | (j, i) \in E(\sigma^N)\}$.

The Hawk-Dove game is a symmetric 2×2 normal form game $G^B := \{\Sigma^B, \Pi^B(\cdot)\}$. Player i in the Hawk-Dove game can choose from two strategies, hawk (H) and dove (D), i.e., $\Sigma^B := \{H, D\}$. One can interpret hawk as an aggressive and dove as a defensive strategy. If two hawks interact, their payoff is minimal. All other strategy profiles lead to pareto-optimal outcomes ([**?**]), although only (H,D) and (D,H) are Nash equilibria. Table 1 summarizes the payoff matrix of $\Pi^B(\cdot)$.

The non-cooperative Network Hawk-Dove game $\Gamma := \{S; P\}$ is a combination of Hawk-Dove game G^B and Network game G^N. Hereafter, we call strategies in the Hawk-Dove game actions and strategies in the Network game links to distinguish them from the strategies in the Network Hawk-Dove game. In the Network Hawk-Dove game, each player i chooses her action σ_i^B

[1] We model the Network game as a non-cooperative game (Bala and Goyal, 2000).

Table 1: Payoff Matrix Hawk-Dove game (with $a > b > c > d > 0$)

	Hawk (H)	Dove (D)
Hawk (H)	d,d	a,c
Dove (D)	c,a	b,b

and her links σ_i^N. Hence, the strategy set $S := \Sigma^B \times \Sigma^N$ in the Network Hawk-Dove game is a Cartesian product of the strategy sets in G^B and G^N. Notice that, although each player i chooses whether to establish a link to every other player j, she only chooses her action once. Each player i, pays for her links and participates in a Hawk-Dove game G^B with every linked player. Clearly, the payoff of player i using strategy $s_i = (\sigma_i^B, \sigma_i^N) \in S$ depends on the number of contacts playing hawk $n_H^i(s) = \sum_{j \in N_i(\sigma^N)} 1_{\{\sigma_j^B = H\}}$ and dove $n_D^i(s) = \sum_{j \in N_i(\sigma^N)} 1_{\{\sigma_j^B = D\}}$. The number of contacts in the neighborhood is $n^i(s) := n_H^i(s) + n_D^i(s)$. Given the payoff function $P : S \to \mathcal{R}$, the payoff of player i is:

$$P_i(s_{-i}, \{\sigma_i^B = H; \sigma_i^N\}) := d \cdot n_H^i(s) + a \cdot n_D^i(s) - k\,|\sigma_i^N|$$

$$P_i(s_{-i}, \{\sigma_i^B = D; \sigma_i^N\}) := c \cdot n_H^i(s) + b \cdot n_D^i(s) - k\,|\sigma_i^N|$$

Given this definition of the Network Hawk-Dove game, k can be any real number. However, we limit our analysis to (1) $k \geq 2d$, (2) $a > b > k > c > d$ and (3) $n \geq 3$. (1) $k > 2d$ follows the standard interpretation of Hawk-Dove games, where d represents injuries from aggressive behavior and typically is 0 (Neugebauer et al., 2008) or negative (Smith and Price, 1973). Hence, linking two hawks is not beneficial, even if both alternate in paying for the link. If $a > b > k > c > d$ holds (2), links from hawks to doves pay, while they do not pay between hawks, and links from doves to hawks do not pay, while they pay between doves, offering all players a variety of link decisions. This changes for all other parameter combinations: If $k > a$, no links are established. If $a > k > b$, only links from hawks to doves pay and each hawk will link to all doves. If $c > k > d$, only links between hawks do not pay. Hence, doves benefit from links to all others, turning their link decision trivial again. Due to our first restriction, $d > k$ cannot hold. (3) We assume that more than 2 players, i.e., $n \geq 3$, participate in our Network Hawk-Dove game to investigate real networks and not only Hawk-Dove games with an outside option.

2.1 Efficiency

We believe that investigating Network Hawk-Dove games experimentally is especially worthwhile as several equilibria of the one-shot Network Hawk-Dove game are not efficient. In this section, we first characterize the parameter combinations that lead to efficient strategy profiles before deriving the Nash equilibria of the one-shot game and comparing the resulting predictions.

To characterize efficient strategy profiles, we basically proceed in two steps. First, we show under which conditions links are efficient before using this information to derive efficiency conditions for the whole network.

Theorem 1. *In efficient strategy profiles of Network Hawk-Dove games*

a) all links are unilateral,

b) all hawks are linked to all doves but not to any hawks, and

c) all doves are linked to all other doves.

Proof. Condition (a) is a consequence of the game design. Players benefit from all other players they are linked to. Redundant links do not increase payoff.

Conditions (b) and (c) follow from $a > b > k$ and $k > 2d$. □

Definition 1 (Link efficiency)**.** *A strategy profile is link efficient if all links are in line with Theorem 1.*

Using the definition of link efficiency, we can now derive the number of hawks in an efficient strategy profile.

Theorem 2. *In efficient strategy profiles of Network Hawk-Dove games, all links are link efficient and the number of hawks $\overline{n_H}$ is*

a) 0 *if* $a + c \leq b + \frac{1}{2}k$ *holds, or*

b) $\lfloor z \rfloor$[2] *and/or* $\lceil z \rceil$ *with* $z = \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k}(n-1))$ *if* $a + c > b + \frac{1}{2}k$.

[2] $\lfloor z \rfloor$ is the largest integer smaller or equal to z, and $\lceil z \rceil$ is the smallest integer number higher or equal to z.

Proof. Let n_H be the number of hawks and $n_D = n - n_H$ the number of doves in the network. In link efficient strategy profiles, the aggregated payoff of all players is

$$P(\cdot) = n_H \cdot (n - n_H) \cdot (a + c - k) + 0.5(n - n_H) \cdot (n - n_H - 1) \cdot (2b - k)$$

with $n_H \cdot (n - n_H)$ being the number of links between hawks and doves yielding $a + c - k$ per link and $0.5(n - n_H) \cdot (n - n_H - 1)$ being the number of links between doves yielding $2b - k$. To identify the maximum of $P(\cdot)$, we differentiate $P(\cdot)$ twice:[3]

$$\partial P(\cdot)/\partial n_H = (n - 2n_H) \cdot (a + c - k) + (2n_H - 2n + 1) \cdot (b - 0.5k)$$

and

$$\partial^2 P(\cdot)/\partial n_H^2 = -2(a + c - b - 0.5k).$$

Condition (a) follows from $\partial^2 P(\cdot)/\partial n_H^2 \geq 0$: If $\partial^2 P(\cdot)/\partial n_H^2 = 0$, then $\partial P(\cdot)/\partial n_H = (1 - n) \cdot (b - \frac{1}{2}k) < 0$ as $n \geq 3$ and $b > k$. Hence, $P(\cdot)$ is maximal if $\overline{n_H} = 0$. The same holds for $\partial^2 P(\cdot)/\partial n_H^2 > 0$: Here a minimum lies at $\partial P(\cdot)/\partial n_H = 0$ dissolving to $n_H = \frac{1}{2}(n + \frac{b-0.5k}{b+0.5k-a-c} \cdot (n-1))$. With the observation that $a + c > k$ for the minimum $n_H > n - \frac{1}{2}$ holds. Hence, the maximal payoff is reached with $\overline{n_H} = 0$.

Condition (b) is the consequence of $\partial^2 P(\cdot)/\partial n_H^2 < 0$. Here a maximum lies at $\partial P(\cdot)/\partial n_H = 0$ being equivalent to $\overline{n_H} = \frac{1}{2}(n + \frac{b-0.5k}{b+0.5k-a-c} \cdot (n-1))$. □

Example 1. *As a running example in the paper, we use the parameters summarized in Table 2. Specifically, in a network of* $n = 6$ *players, establishing links costs* $k = 50$*. A hawk being linked to another hawk earns* $d = 20$*, while he earns* $a = 80$ *when linked to a dove. Doves earn* $b = 60$ *for links to other doves and* $c = 40$ *for links to hawks. We also use these parameters in our behavioral experiment described in Section 4.1. Notice that for these parameters all conditions (1) to (3) for network Network Hawk-Dove games motivated in the previous subsection are fulfilled (1)* $k = 50 \geq 40 = 2d$*, (2)* $a = 80 > b = 60 > k = 50 > c = 40 > d = 20$ *and (3)* $n = 6 \geq 3$*.*

For this set of parameters $a + c = 120 > 85 = b + \frac{1}{2}k$ *holds. Hence, an efficient strategy profile can be established if* $\overline{n_H} = \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k}(n-1)) =$

[3] Notice that our analysis is a simplification as we search the optimum of $P(\cdot)$ under the condition that $0 \leq n_h \leq n$. However, we ignore this condition when differentiating and manually check for the borders of the condition later.

Table 2: Running example of Network Hawk-Dove game with $k = 50$ and $n = 6$

	Hawk (H)	Dove (D)
Hawk (H)	$d = 20, d = 20$	$a = 80, c = 40$
Dove (D)	$c = 40, a = 80$	$b = 60, b = 60$

0.5 *hawks face* $n_D = n - n_H = 5.5$ *doves. In this case, the maximal payoff in the population is* $\overline{n_H} \cdot (n - \overline{n_H}) \cdot (a + c - k) + \frac{1}{2}(n - \overline{n_H}) \cdot (n - \overline{n_H} - 1) \cdot (2b - k) =$ 1058.75*. If the number of hawks is an integer, the maximal payoff in the group is 1050. This payoff is reached if both* $\overline{n_H} = 0$ *and* $\overline{n_H} = 1$.

2.2 One-shot Nash equilibria

To compare efficient strategy profiles and Nash strategy profiles, we derive conditions for the Nash equilibria in the one-shot Network Hawk-Dove game Γ. Definition 2, Theorem 4, and the corresponding proof follow the idea introduced by Berninghaus and Vogt (2006) and are repeated here to simplify understanding of the subsections and subsubsections below. Although the arguments in these proofs differ from Bramoulle et al. (2004), all predictions in the present subsection are equivalent to their results.

In a Nash equilibrium, no player has an incentive to deviate by either changing his links $\sigma_i^{N^*} \in \Sigma^N$ or actions $\sigma_i^{B^*} \in \Sigma^B$ unilaterally. Hence, the definition of a Nash equilibrium in Network Hawk-Dove games is as follows:

Definition 2 (Nash condition)**.** *Each strategy profile* $s^* = (\sigma^{N^*}, \sigma^{B^*})$ *in* Γ *is a Nash equilibrium if*

$$\forall i: \quad P_i(s^*_{-i}, s^*_i) \geq P_i(s^*_{-i}, s_i) \text{ for } s_i \in S_i.$$

Using the Nash condition, we first characterize conditions for links in a Nash equilibrium. We then use these to derive the Nash equilibrium for the whole network.

Theorem 3. *In Nash equilibria of Network Hawk-Dove games*

a) all links are unilateral,

b) all hawks establish links to all doves but not to any hawks, and

c) all doves establish links to every other dove.

Proof. Condition (a) and its proof are equivalent to Condition (a) of Theorem 1.

Conditions (b) and (c) follow directly from $a > b > k > c > d$. □

Definition 3 (Link balance)**.** *A strategy profile is link balanced if all links are in line with Theorem 3.*

The conditions for links are similar in link balanced and link efficient strategy profiles. In both configurations, only links involving at least one dove exist. However, the link balance is more specific. While the direction of links between hawks and doves is not specified according to link efficiency, hawks have to pay for these links in link balanced strategy profiles.

Theorem 4. *In Nash equilibria of Network Hawk-Dove games, all links are link balanced and the number of hawks n_H^* satisfies*

$$n_H^* \geq \frac{a-b}{a-b+c-d}(n-1) > 0.$$

Proof. In link balanced strategy profiles, unilateral deviation from hawk to dove is beneficial if the following inequality holds: $P_i(s_{-i}^*, \{H; \sigma_i^{N^*}\}) = n_D^* a - k|\sigma_i^{N^*}| < n_D^* b - k|\sigma_i^{N^*}| = P_i(s_{-i}^*, \{D; \sigma_i^{N^*}\})$. This inequality is always false as $a > b$ holds. Each dove player deviates from dove to hawk if $P_i(s_{-i}^*, \{D; \sigma_i^{N^*}\}) = n_H^* c + (n_D^* - 1)b - k|\sigma_i^{N^*}| < n_H^* d + (n_D^* - 1)a - k|\sigma_i^{N*}| = P_i(s_{-i}^*, \{H; \sigma_i^{N^*}\})$ holds. This inequality is false if the inequality of the theorem is met. □

Example 2. *We apply the parameters from Table 2 to Theorem 4. The number of hawks n_H^* is above $\frac{a-b}{a-b+c-d}(n-1) = 2.5$ in Nash equilibrium. This clearly differentiates efficient strategy profiles from Nash equilibria: In the former only* 0 *or* 1 *hawks exist, while in the latter* 3 *or more hawks participate.*

Nash equilibria and efficient strategy profiles do not only differ in our running example. If $a+c \leq b+\frac{1}{2}k$, the number of hawks in an efficient strategy profile is $\overline{n_H} = 0$ (see Theorem 2). This always differs from the equilibrium prediction, which needs n_H^* to be greater than 0 (see Theorem 4). If $a+c > b+\frac{1}{2}k$, the upper bound for the number of hawks is $\overline{n_H} = \frac{n}{2} \geq \frac{1}{2}(n - \frac{b-0.5k}{a+c-b-0.5k}(n-1))$ in efficient strategy profiles, while the Nash equilibrium prescribes a lower bound for the number of hawks. Hence, this also allows for several parameter combinations in which efficient outcome and Nash equilibria differ.

Notice that the Nash equilibrium with the lowest overall payoff is always the strategy profile in which all players resort to hawk and link to no other players.

Lemma 1. *The Nash equilibrium of the Network Hawk-Dove game, yielding the lowest payoff for all players, is characterized by only hawk players ($n_H = n$) and no links.*

Proof. According to Theorem 4, the described strategy is a Nash equilibrium as all links are link balanced and $n = n_H^* \geq n - 1 \geq \frac{a-b}{a-b+c-d}(n-1)$ holds if $c > d$, which is a property of the Hawk-Dove game. As no links exist, the payoff in this equilibrium is 0. Any other Nash equilibrium yields higher payoffs for at least two players and no lower payoffs for any of the players since in all other Nash equilibria at least one player resorts to dove and at least one link from another player to this dove exists, yielding positive payoffs for both players, given the links are link balanced. □

2.3 Finitely repeated game Nash equilibria

In the introduction to this paper, we discuss that in anti-coordination games with endogenous network formation subjects often resort to strategy profiles that yield higher payoffs than the payoffs reachable when resorting to one-shot Nash equilibria. Repeatedly playing the game, even for a finite number of periods, can help to reach efficient outcomes. Here trigger strategies can be used to punish players who deviate from the desired strategy profile (Friedman, 1985, Benoit and Krishna 1985). In this subsection, we first present such trigger strategies before we apply these trigger strategies to the Network Hawk-Dove game.

2.3.1 Trigger strategy equilibria for games with finite horizon

Solution 1 summarizes a trigger strategy (as proposed by Friedman (1985), Benoit and Krishna (1985)) to establish desired strategy profiles that are no equilibria of the stage game. The basic idea of the trigger strategy is that subjects begin playing the desired strategy profile in the first period and keep playing this strategy. If one player deviates, all others punish her by switching to the Nash equilibrium yielding the lowest payoff for the deviator. If there is no deviation, all players switch to the Nash equilibrium yielding the highest overall payoff for the last $\{t^*+1, ..., T\}$ periods.

Solution 1 (Trigger strategy for games with finite horizon)**.** *To establish a desired strategy profile in a finitely repeated game, the following trigger strategy can be used:*

a) In $t = 1$ play the desired strategy profile.

b) In $t \in \{2, ..., t^*\}$ keep playing the desired strategy profile if no player deviated. Otherwise, resort to the Nash equilibrium of the stage game yielding the lowest payoff for the deviator.

c) In $t \in \{t^*+1, ..., T\}$ play the Nash equilibrium of the stage game, yielding the highest overall payoff if no player deviated. Otherwise, resort to the Nash equilibrium of the stage game yielding the lowest payoff for the deviator.

Using this trigger strategy results in additional sub game perfect equilibria, in which players resort to a strategy profile yielding higher payoffs than the payoff maximal Nash equilibrium, if the following conditions for the game hold. First, players need to resort to a Nash equilibrium in the terminal period. Otherwise, at least one player could deviate from the strategy profile played. Second, during periods $\{t^*+1, ..., T\}$ two Nash equilibria, one to punish deviators and one to reward non deviators, are required, or else a player deviating in period t^* could not be punished in subsequent periods. Third, for the punishment to be effective, the sum of payoffs in the Nash equilibrium to reward, minus the payoffs in the Nash Equilibrium to punish in periods $\{t^*+1, ..., T\}$, has to exceed the payoff gain for deviating from the desired strategy in period t^*. Otherwise, a player would deviate in t^*. Notice that this condition is sufficient for periods $\{1, ..., t^*-1\}$ to prevent any deviations because deviations in earlier periods would increase the number of periods other players could use to punish and, hence, could not lead to a payoff increase. As the described trigger strategy does not allow for beneficial deviations in any period, the resulting Nash equilibrium is sub game perfect.

2.3.2 Trigger strategies equilibria in the Network Hawk-Dove game

Our analysis of the one-shot game shows that efficient strategy profiles and Nash equilibria in the Network Hawk-Dove game differ in the number of hawks in the network. Hence, (1) the desired strategy profile played in period $t = 1$ should ensure a certain number of hawks n_H. All links should be link balanced (see Definition 3) to ensure that payoffs are equally distributed. (2) The best punishment for deviations from the desired strategy profile is to play the Nash equilibrium (see Lemma 1), with $n_H = n$ hawks having no links. This ensures that all payoffs are 0. (3) As the desired strategy profile in periods 1 to t^* is no equilibrium in the one-shot game, players have an incentive to deviate. To overcome this, we choose to play the Nash equilibrium of the one-shot game yielding the highest payoff during the last periods. The following definition summarizes this trigger strategy:

Definition 4 (Trigger strategy for the Network Hawk-Dove game)**.** *Let the trigger strategy in the Network Hawk-Dove game be as follows:*

a) In $t = 1$ play hawk or dove so that n_H hawks are in the network and ensure that the links are link balanced.[4]

b) In $t \in \{2, ..., t^*\}$ keep playing your strategy as in (a) if no player deviated. Otherwise, play the hawk strategy and remove all links.

c) In $t \in \{t^*, ..., T\}$ play the Nash equilibrium of the one-shot game yielding the highest overall payoff with n_H^* hawks if no player deviated from the desired strategy in $t \leq t^*$. Otherwise, play the hawk strategy and remove all links.

The described trigger strategy is rather generic as three parameters, i.e., n_H, n_H^*, and t^*, are not specified. We derive values for these parameters in the remainder of this section. As we have shown when deriving efficient strategy profiles (see Theorem 2) for several parameter combinations, the payoff increases when the number of hawks decreases. We now define the minimum number of hawks $\underline{n_H}$ in a Nash equilibrium of the one-shot Network Hawk-Dove game. Applying the trigger strategy from Definition 4 is only

[4]Notice that finding a corresponding strategy profile is a coordination problem. However, we do not discuss how to find this strategy profile. Deviations can be punished since, after seeing the strategies in the previous period, all players know whether it was found or not.

meaningful for $n_H < \underline{n_H}$ as for all $n_H \geq \underline{n_H}$ one of the payoff maximal strategy profiles is a Nash equilibrium. Here the links, which are a subset of all link efficient strategy profiles, have to be link balanced.

Definition 5 (Minimum number of hawks in one-shot Nash equilibrium)**.** *The minimum number of hawks in the Nash equilibrium of the one-shot Network Hawk-Dove game is*

$$\underline{n_H} = \lceil \frac{(n-1)(a-b)}{a-b+c-d} \rceil.$$

Using the minimum number of hawks in the Nash equilibrium of the one-shot Network Hawk-Dove game, we parameterize the trigger strategy to result in additional Nash equilibria of the T-period repeated Network Hawk-Dove game.

Theorem 5. *To reach Nash equilibria that are no Nash equilibria of the stage game in the T-period repeated Network Hawk-Dove game Γ, the trigger strategy for the Network Hawk-Dove game (Definition 4) has to be parameterized as follows:*

a) $n_H < \underline{n_H}$,

b) $n_H^* = \begin{cases} \underline{n_H} & \text{if} \quad \overline{n_H} < \underline{n_H} \\ \overline{n_H} & \text{if} \quad \overline{n_H} \geq \underline{n_H} \end{cases}$ *with $\overline{n_H}$ being the efficient number of hawks,*

c) $t^* \leq T - \frac{n_H \cdot (d-c) + (n-n_H-1)\cdot(a-b)}{min\{n_H^* \cdot c + (n-n_H^*-1)(b-k);(n-n_H^*-1)(a-k)\}}$.

Proof. Condition (a) follows directly from the observation that the desired strategy profiles are non Nash equilibria of the stage game.

Condition (b) characterizes the equilibrium played in periods $t^*+1, ..., T$ if no player deviated. In the equilibrium, all links are link balanced and the number of hawks is chosen such that the highest overall payoff for all players is reached. If one Nash equilibrium is efficient ($\overline{n_H} \geq \underline{n_H}$), this should be played. Therefore $n_H^* = \overline{n_H}$ has to hold. Otherwise, the Nash equilibrium with the highest overall payoff is characterized by $n_H^* = \underline{n_H}$ as the payoff in the one-shot game is monotonically decreasing if the number of hawks increases (see the proof of Theorem 2).

Condition (c) characterizes the number of periods to resort to the payoff maximal equilibrium. If one player deviated in $t \leq t^*$, the payoff of all players is 0 in all subsequent periods. Therefore each player not deviating earns a minimum of

$$min\{n_H^* \cdot c + (n - n_H^* - 1)(b - k); (n - n_H^* - 1)(a - k)\}$$

for each of the $T - t^*$ last periods. For this payoff we expect that all links are link balanced (which they are in equilibrium). Further, the left argument $(n_H^* \cdot c + (n - n_H^* - 1)(b - k))$ is the minimal payoff for resorting to dove, i.e., the payoff if paying for all links, and the right argument $((n - n_H^* - 1)(a - k))$ is the payoff for resorting to hawk. The gain for not deviating has to be at least as high as the gain from deviating in $t = t^*$. A player being a hawk in t^* will not deviate as, given the links are link balanced, she is linked to doves only, and switching from hawk to dove will reduce her payoff. A dove, on the other hand, might switch to hawk in t^* to increase her payoff by

$$n_H \cdot (d - c) + (n - n_H - 1) \cdot (a - b).$$

For the trigger strategy to reach an equilibrium $(T - t^*) \cdot min\{n_H^* \cdot c + (n - n_H^* - 1)(b - k); (n - n_H^* - 1)(a - k)\} \geq n_H \cdot (d - c) + (n - n_H - 1) \cdot (a - b)$ has to hold, which simplifies to condition (c). □

Example 3. *Let us consider our running example again. The minimum number of hawks in the Nash equilibrium of the one-shot Network Hawk-Dove game is* $\underline{n_H} = 3 = \lceil 2.5 \rceil$*, and the number of hawks yielding the maximal payoff in a Nash equilibrium of the one-shot version of the game is also* $n_H^* = \underline{n_H} = 3$ *as* $\underline{n_H} = 3 > 0.5 = \overline{n_H}$ *(see the intermediate results in all previous examples). If* $n_H \in \{0, 1, 2\}$*, then* $n_H < 3 = \underline{n_H}$ *holds and all* n_H *hawks link to all* $n - n_H$ *doves and all doves are unilaterally linked to each other during periods* 1 *to* t^**. In periods* $t^* + 1$ *to* T*, the players switch to the Nash equilibrium with* $\underline{n_H} = 3$ *hawks. Any deviation is punished by subsequently playing hawk and removing all links. The number of periods in which the Nash equilibrium of the stage game* $T - t^*$ *is played depends on* n_H*.* $T - t^*$ *is greater than or equal to* $2 > 1.67$ *for* $n_H = 0$*,* 1 *for* $n_H = 1$ *and* $1 > 0.33$ *for* $n_H = 2$*. In other words, the trigger strategies ensure for* $n_H = 1$ *that the efficient strategy profile is played except for one final period in a sub game perfect equilibrium.*

Clearly, besides the sub game perfect equilibria described in Theorem 5, other sub game perfect equilibria in the network exist. For example, any combination of n_H in periods 1 to t^* can be established as a sub game perfect equilibrium, at worst by increasing the number of periods for which the payoff maximal equilibrium is played. However, to describe the complete set of all equilibria is not the aim of this paper. Rather, we aim at showing that by playing the Network Hawk-Dove game repeatedly, strategy profiles with numbers of hawks can be established as Nash equilibrium that do not occur in the one-shot version of the game.

3 Fixed actions and links

A severe drawback of the trigger strategy introduced in Definition 5 is the need for coordination. Players have to coordinate on one equilibrium. Without any pre game communication this is difficult. Hence, in this section we extend our analysis to two variants with smaller subsets of strategies and evaluate whether these simpler variants of the game still yield efficient outcomes.

We have introduced the Network Hawk-Dove game as a combination of two isolated games, the Hawk-Dove game and the Network game. Using these two parts of the game, we now introduce two new versions of the Network Hawk-Dove game: (1) We derive the Fixed Action game $\Gamma^{\overline{B}}$ by reducing the strategy set of the players to $S^N = \Sigma^N$. Here we enforce a fixed action set $\overline{\sigma^B}$ for all players. Everything else remains unchanged. That is to say, each player can modify her set of links $s_i^N \in S^N$, although all actions $\overline{\sigma_i^B}$ in the Hawk-Dove game are fixed. (2) In the Fixed Link game $\Gamma^{\overline{N}}$, we reduce the strategy set of the players to $S^B = \Sigma^B$ and enforce a fixed set of links $\overline{\sigma^N}$ for all players. Everything else remains unchanged. That is to say, each player i faces costs for her links $|\overline{\sigma_i^N}| \cdot k$ and plays a Hawk-Dove game with all contacts.

3.1 One-shot Nash equilibria

Analogous to the one-shot Network Hawk-Dove game Γ, we derive Nash equilibria for both versions of the game, the Fixed Action game $\Gamma^{\overline{B}}$ and the Fixed Link game $\Gamma^{\overline{N}}$.

Theorem 6. *Given a Fixed Action game $\Gamma^{\overline{B}}$, a Nash equilibrium $s^* \in (\Sigma^B, S^N)$ is established if the strategy profile (σ^B, s^N) is link balanced.*

Proof. The proof is identical to the proof of Theorem 3. □

Notice that, if fixed actions $\overline{\sigma_i^B}$ are identical to actions in a Nash equilibrium of the one-shot Network Hawk-Dove game, the resulting Nash equilibrium in a Fixed Action game is equivalent to this equilibrium as for establishing links identical preconditions hold (see Theorems 4 and 6). Similarly, fixed actions $\overline{\sigma_i^B}$ that are identical to actions in an efficient strategy profile lead to links in the Fixed Action game that are compatible to the links in the efficient strategy profile. This follows from equivalent conditions for links in Theorem 2 and Theorem 6.
Having characterized the Fixed Action game, we now describe conditions for the Fixed Link game:

Theorem 7. *Given a Fixed Link game* $\Gamma^{\overline{N}}$*, a Nash equilibrium* s^* *in* (S^B, Σ^N) *is established if the following statement holds:*

a) Each player i uses the hawk action if in his neighborhood the following condition holds:

$$n_H^i(s^*) < \frac{a-b}{a-b+c-d} \cdot n^i(s^*),$$

with $n_H^i(s^*)$ $(n^i(s^*))$ *being the number of hawks in her neighborhood.*

b) Each player i uses the dove action otherwise.

Proof. Conditions (a) and (b) follow from the payoff function $P(\cdot)$ of the Network Hawk-Dove game. The payoff of the hawk action exceeds that of the dove action if the following inequality holds: $P_i(s^*_{-i}, \{H; \overline{\sigma_i^N}\}) = d \cdot n_H^i(s) + a \cdot n_D^i(s) - k\,|\overline{\sigma_i^N}| > c \cdot n_H^i(s) + b \cdot n_D^i(s) - k\,|\overline{\sigma_i^N}| = P_i(s^*_{-i}, \{D; \overline{\sigma_i^N}\})$. With $n_H^i(s) = n^i(S) - n_D^i(s)$, this inequality simplifies to the inequality of condition (a). A player resorts to the dove action if the inequality does not hold (condition (b)). □

To allow for trigger strategies in the finitely repeated version of the Fixed Link game, two equilibria are required in the stage game: one with a low and one with a high payoff. During the last periods, the equilibrium with the low payoff is played to punish deviations from desired strategies, while the one with high payoffs is applied to reward players for not deviating. Hence, we investigate whether at least two Nash equilibria exist in any Fixed Link game.[5]

[5]Notice that for reasons of understandability, we do not consider the costs for links in our analysis of the Fixed Link Game. Link costs are fixed in the Fixed Link Game and do not change behavior.

Lemma 2. *Each Nash equilibrium* (σ^B, σ^N) *of the one-shot Network Hawk-Dove game* Γ *implies the following Nash equilibria* (s^B, σ^N)[6] *in the Fixed Link Game* $\Gamma^{\overline{N}}$*:*

a) For all i (j) with $\sigma_i^B = H$ $(\sigma_j^B = D)$, $s_i^B = H$ $(s_j^B = D)$ *holds, or*

b) For all i (j) with $\sigma_i^B = H$ $(\sigma_j^B = D)$, $s_i^B = D$ $(s_j^B = H$ *or* $s_j^B = D$ *such that* $n_H < \frac{a-b}{a-b+c-d}(n-1))$ *holds.*

Proof. Condition (a) follows from the proof for equilibria in one-shot Network Hawk-Dove games.

Condition (b) is the result of a search for additional Nash equilibria: All players i with $\sigma_i^B = H$ have identical contacts. They are linked to all players j with $\sigma_j^B = D$. According to condition (a) of Theorem 7, they have to resort to the same strategy. Hence, $s_i^B = D$ holds in an additional equilibrium. All players j are linked to all other players in the network, and they choose their strategy s_j^B such that condition (a) of Theorem 7 holds for the whole network, i.e., $n_H < \frac{a-b}{a-b+c-d}(n-1)$. □

Example 4. *Our running example allows for a Nash equilibrium in the Network Hawk-Dove game with five hawks as* $5 \geq \underline{n_H} = 3$. *All hawks have exactly one link to the only dove in the network. Now let the links be fixed and let players play a Fixed Link game. If the players resort to the same strategies as in the Network Hawk-Dove game, the network is still in equilibrium. Let all* 5 *former hawks alter their action to dove. The former dove will choose to play hawk as in her neighborhood the number of hawks is* 0*. Now all doves have exactly one contact, namely a hawk being in equilibrium according to Theorem 7.*

Next, we analyze Fixed Link games derived from non Nash equilibrium strategy profiles in the Network Hawk-Dove games. In particular, we focus on link efficient strategy profiles. These strategy profiles are especially relevant in Fixed Link and Network Hawk-Dove games. As the cost for establishing a link to a hawk always exceeds the benefit and establishing a link to a dove is always beneficial, such strategy profiles can be expected to be frequent in these games. We show that, in analogy to Fixed Link games derived from

[6]Here and in the remainder, we call action decisons s^B and network decisions s^N in the derived game if they differ from the original game, while we call them σ^B and σ^N if they are identical to the original game.

Nash equilibria in the one-shot Network Hawk-Dove games, there are two types of equilibria in these games. Hence, all link efficient strategy profiles can continue to be played using the trigger strategies described above.

Lemma 3. *Every link efficient strategy profile* (σ^B, σ^N) *of the Network Hawk-Dove game* Γ *implies two Nash equilibria* (s^B, σ^N) *in the Fixed Link Game* $\Gamma^{\overline{N}}$*:*

a) For all i *with* $\sigma_i^B = H$, $s_i^B = H$ *and*

$$n_H + \delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < n_H + \delta_H$$

holds, or

b) For all i *with* $\sigma_i^B = H$, $s_i^B = D$ *and*

$$\delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < \delta_H$$

holds,

with n_H *being the number of players with* $\sigma_i^B = H$ *and* δ_H *being the number of players with* $\sigma_j^B = D$, $s_j^B = H$.

Proof. For all link efficient strategy profiles that are Nash equilibria in Γ this follows directly from Lemma 2. For all other link efficient strategy profiles in Γ $n_H < \frac{a-b}{a-b+c-d}(n-1)$ holds (see Theorem 4). Following the intuition of condition (b) in Lemma 2, all n_H players i with $\sigma_i^B = H$ have identical contacts. Hence, they all resort to the same action s_i^B. All $n_D = n - n_H$ other players either resort to $s_j^B = H$ or $s_k^B = D$.

Condition (a) analyzes the case of $s_i^B = H$. Whether an equilibrium is reached, depends on the fraction of players j with $s_j^B = H$ and $\sigma_j^B = D$ to players k with $s_k^B = D$ and $\sigma_k^B = D$. Let the number of players i be n_H, the number of players j be δ_H, and the number of players k be $n_D - \delta_H$ with $n = n_H + n_D$. In the neighborhood of each player i $n_H^i(s) < \frac{a-b}{a-b+c-d} \cdot n^i(s)$ has to hold for player i to resort to hawk (see Theorem 7). For players i this is equivalent to $\delta_H < \frac{a-b}{a-b+c-d} \cdot (\delta_H + n_D - \delta_H)$ as they are linked to players j and k only. All players j and k are linked to all players in the network. Hence, for them $n_H + \delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1)$ and $n_H + \delta_H > \frac{a-b}{a-b+c-d} \cdot (n-1)$, respectively, have to be fulfilled. The three inequalities are equivalent to

$$\delta_H < \frac{a-b}{a-b+c-d} \cdot (n - n_H) \tag{1}$$

and

$$n_H + \delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < n_H + \delta_H. \tag{2}$$

Inequality (1) is fulfilled if the left inequality of (2) is fulfilled.

Condition (b) focuses on all n_H players i resorting to $s_i^B = D$. Let the number of players j be δ_H with $s_j^B = H$ and the number of players k be $n_D - \delta_H$ with $s_k^B = D$. In the corresponding neighborhoods, the following inequalities have to hold: players i, $\delta_H > \frac{a-b}{a-b+c-d} \cdot (n - n_H)$; players j, $\delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1)$, and players k, $\delta_H > \frac{a-b}{a-b+c-d} \cdot (n-1)$. The three inequalities are equivalent to

$$\delta_H > \frac{a-b}{a-b+c-d} \cdot (n_D) \tag{3}$$

and

$$\delta_H - 1 < \frac{a-b}{a-b+c-d} \cdot (n-1) < \delta_H. \tag{4}$$

Inequality (3) is contained in the right-hand part of inequality (4) if $n_H \geq 1$, which is fulfilled for all non Nash equilibria of the one-shot Network Hawk-Dove game (see Theorem 4). □

Example 5. *We continue with our running example. Here three non Nash equilibrium strategy profiles with the described types of players are possible. For these strategy profiles the number of hawks n_H is $1, 2,$ or 3 (see our previous examples). We focus on the link efficient strategy profile with $n_H = 1$ players i with $\sigma_i^B = H$ and $n_D = 5$ players j with $\sigma_i^B = D$. In addition, $n_H + \delta_H - 1 < 2.5 = \frac{a-b}{a-b+c-d}(n-1) < n_H + \delta_H$ has to hold, i.e., $n_H + \delta_H = 3$. Now one equilibrium exists, with player i resorting to hawk, 2 other players resorting to hawk, and 3 other players resorting to dove. To be an equilibrium, players resort to hawk if $n_H^i(s) < \frac{a-b}{a-b+c-d} \cdot n^i(s) = \frac{1}{2}n^i(s)$ and to dove otherwise (see Theorem 7). In the neighborhood of the players with $\sigma_j^B = H$, this condition equals $2 < \frac{1}{2} \cdot 5 = 2.5$. As she resorts to hawk, she will not deviate. For the players with $\sigma_j^B = D$ resorting to dove (hawk) the condition is $3 > \frac{1}{2} \cdot 5 = 2.5$ ($2 < \frac{1}{2} \cdot 5 = 2.5$). Hence, no player has an incentive to unilaterally deviate. The game is in equilibrium. Now consider case (2). Here the player with $\sigma_j^B = H$ resorts to dove. $\delta_H - 1 < \frac{a-b}{a-b+c-d}(n-1) = 2.5 < \delta_H$ has to hold, i.e., $\delta_H = 3$. Therefore 3 players with $\sigma_j^B = D$ are hawks and 2 players with $\sigma_j^B = D$ are doves. The neighborhood of the players with $\sigma_j^B = D$ is equivalent to the neighborhood of the players with $\sigma_j^B = D$ in* *case 1. Hence, they will not unilaterally deviate. For the player with* $\sigma_j^B = H$ $n_H^i(s) = 3 > \frac{a-b}{a-b+c-d} \cdot n^i(s) = \frac{1}{2} \cdot 5 = 2.5$ *holds. Hence, he will not deviate either. Case (2) is also an equilibrium.*

With Lemma 2 and Lemma 3 we have shown link efficient strategy profiles with any number of hawks n_H exist implying only two types of Nash equilibria. In one Nash equilibrium all players with $\sigma_j^B = H$ are hawks, and in another Nash equilibrium all players with $\sigma_j^B = H$ are doves. This is a precondition for finitely repeated games to reach a Nash equilibrium in which, at least in some periods, strategy profiles are played that are not Nash equilibria of the stage game (see Section 2.3). This serves as a precondition for an analysis of the Nash equilibria in the finitely repeated version of the Fixed Link game.

3.2 Finitely repeated game Nash equilibria

In this subsection, we investigate what kind of equilibria result from finitely repeating Fixed Action and Fixed Link games. We let players explicitly choose the strategy parameters they could not choose in the first period of these games. That is to say, we let the players participate in a Network Hawk-Dove game in period $t = 1$ and let them play either the Fixed Action game or the Fixed Link game for $T - 1$ periods afterwards. We call the respective games T-period Fixed Action game and T-period Fixed Link game. We start our analysis with both Fixed Action and Fixed Link games, in which the players resort to a Nash equilibrium of the Network Hawk-Dove game in period $t = 1$.

Theorem 8. *In the* T*-period Fixed Action and Fixed Link game, playing a Nash equilibrium of the Network Hawk-Dove game in period* $t = 1$ *and all subsequent periods is a Nash equilibrium of both repeated games.*

Proof. The proof is obvious. The Nash equilibrium of the one-shot Network Hawk-Dove game in period $t = 1$ is a Nash equilibrium in all subsequent stage games since the conditions for Nash equilibria in the Fixed Action and the Fixed Link game are compatible to the conditions for Nash equilibria in the Network Hawk-Dove game. Hence, in every period a Nash equilibrium is played. □

In the remainder, we analyze whether other equilibria, yielding higher overall payoffs, can be established. As we have shown in Theorem 6 in the

Fixed Action game, only one equilibrium per action configuration exists in the one-shot game. Hence, only this Nash equilibrium can be played in the repeated version of the game. Therefore no punishment and no trigger strategy is possible.

Theorem 9. *In the T-period Fixed Action game, the following sub game perfect equilibria depending on the number of hawks in the network n_H exist:*

a) *All links are link balanced.*

b1) *If $T(a-b+c) \geq (T-1)k-d$ holds, the number of hawks n_H in the population has to satisfy the condition*

$$0 \leq n_H \leq \frac{T(a-b)}{T(a-b)+(T-1)c}n.$$

b2) *If $T(a-b+c) < (T-1)k-d$ holds, the number of hawks n_H in the population has to satisfy the condition*

$$\frac{T(a-b)-(T-1)k}{T(a-b+c)-(T-1)k-d}(n-1) \leq n_H \leq \frac{T(a-b)}{T(a-b)+(T-1)c}n.$$

Proof. Condition (a) follows directly from the conditions of equilibria in the Fixed Action game (see Theorem 6).

Condition (b1) follows from the gain a player, who is hawk in $t=1$, makes by switching to dove in $t=1$. Her payoff is $T(n-n_H)(a-k)$ for playing hawk. Switching to dove yields $T(n-n_H)(b-k)+(T-1)n_H \cdot c$ because she will still link to all doves, but in periods $t \in \{2,..,n\}$ all hawks will link to her to be in equilibrium. Playing hawk pays if

$$T(n-n_H)(a-k) > T(n-n_H)(b-k)+(T-1)n_H \cdot c,$$

which simplifies to $n_H < \frac{T\cdot(a-b)}{T\cdot(a-b+c)-c}n$, condition (b1).

Condition (b2) follows from the gain a player, who is dove in $t=1$, makes by switching to hawk in $t=1$. As dove, she receives $(n_H \cdot c+(n-n_H-1)b)T-\sum_{t=1}^{T}|\sigma_{i,t}^N|\cdot k$. When deviating to hawk in $t=1$, she receives $n_H \cdot d+(n-n_H-1)a-|\sigma_{i,1}^N|\cdot k+(T-1)(n-n_H-1)(a-k)$. Simplified, the payoff for a dove exceeds that for deviating to hawk if

$$n_H \cdot (T(a-b+c)-(T-1)k-d) < (n-1)\cdot(T(a-b)-(T-1)k)+k\sum_{t=2}^{T}|\sigma_{i,t}^N|.$$

This inequality corresponds to two conditions: (1) If $T(a-b+c)-(T-1)k-d<0$, $n_H > ((n-1)(T(a-b)-(T-1)k)+k\sum_{t=2}^{T}|\sigma_{i,t}^N|)/(T(a-b+c)-(T-1)k-d)$ has to hold, which is always fulfilled as $k\sum_{t=2}^{T}|\sigma_{i,t}^N| \leq k(T-1)(n-1)$ and $n_H \geq 0$ and corresponds to (b1). (2) If $T(a-b+c)-(T-1)k-d>0$, $n_H < ((n-1)(T(a-b)-(T-1)k)+k\sum_{t=2}^{T}|\sigma_{i,t}^N|)/(T(a-b+c)-(T-1)k-d)$. Here the right-hand side of the inequality is minimal if $k\sum_{t=2}^{T}|\sigma_{i,t}^N| = 0$. This corresponds to condition (b2).

The network is in equilibrium in periods 2 to T, while all conditions described here ensure that the overall game is in equilibrium. Hence, the described equilibrium is also sub game perfect. □

Example 6. *Consider again our running example. Here $60T \geq 50(T-1)-20$ holds for every $T>1$. Hence, $0 \leq n_H \leq \frac{20T}{70T-50}n$ has to hold. For $T=2$ this is equivalent to $n_H \leq 2 < \frac{8}{3}$, with the maximum n_H decreasing as T increases. A comparison of these results to the predictions of the one-shot Network Hawk-Dove game shows that for T-period Fixed Action games lasting for 2 periods or more, the efficient strategy profiles of the stage game can be established in equilibrium.*

The observation that the T-period Fixed Action game allows for the efficient strategy profiles of the one-shot Network Hawk-Dove game is not unique to our running example. As Theorem 9 postulates a lower bound for n_H and no upper bound (see Theorem 4), and the upper bound in the T-period Fixed Action game is below n (see Theorem 9), strategy profiles in the T-period Fixed Action game always exist, which cannot be reached in the Network Hawk-Dove game.

After deriving the Nash equilibria for the T-period Fixed Action game, we analyze the T-period Fixed Link game accordingly. Here several link configurations allow for two types of equilibria. Hence, we introduce a new trigger strategy. In the first period, players resort to the strategy profile with a desired number of hawks n_H. If one player deviates, the equilibrium strategy yielding the lowest payoff for the deviator is played until period T. If no player deviated until period t^*, the equilibrium yielding the highest payoff for all players is chosen.

Definition 6. *Let the trigger strategy in the T-period Fixed Link Game be as follows:*

a) In $t=1$ play hawk or dove so that n_H hawks are in the network. If *you are a hawk, link to all doves; if you are a dove, ensure that you are unilaterally linked to all other doves.*

b) *In $t \in \{2, ..., t^*\}$ keep playing your strategy as in (a) if no player deviated. Otherwise, play the equilibrium yielding the lowest payoff for the deviating player.*

c) *In $t \in \{t^*, ..., T\}$ keep playing your strategy if no player deviated from playing the equilibrium of the one-shot game yielding the highest overall payoff. Otherwise, play the equilibrium with the lowest payoff for the deviating player.*

In addition to the trigger strategy, we need to know whether the payoffs of the players, after switching from a certain strategy profile to a Nash equilibrium, are higher as hawk or dove. The following lemma investigates this aspect:

Lemma 4. *For players in the Fixed Link game $\Gamma^{\overline{N}} = \{(s^B, \sigma^N), P(\cdot)\}$ derived from a non Nash equilibrium, but from a link efficient strategy profile (σ^B, σ^N) in a Network Hawk-Dove Γ the following two observations hold:*

a) *Players with $\sigma^B = H$ reach higher payoffs with $s^B = H$ if $\delta_H < \frac{a-b}{a-b+c-d}(n - n_H)$ and with $s^B = D$ otherwise.*

b) *Players with $\sigma^B = D$ reach higher payoffs if $s^B = H$ than if $s^B = D$.*

Proof. Let n_H players i be the players with $\sigma^B = H$, δ_H players j be the players with $\sigma^B = D$, $s^B = H$, and $n_D - \delta_H$ players k be the players with $\sigma^B = D$, $s^B = D$.

Condition (a) follows from the equilibrium condition in Fixed Link games (see Theorem 7).

Condition (b) follows from considering the payoffs of the players with $\sigma^B = D$. We consider two cases: (1) all players i choose $s^B = H$ and (2) all players i choose $s^B = D$.

In case (1), each player j earns $(n_H + \delta_H - 1)d + (n_D - \delta_H)a$ and each player k earns $(n_H + \delta_H)c + (n_D - \delta_H - 1)b$. Hence, for $s^B = H$ yielding higher payoffs: $(n_H + \delta_H - 1)d + (n_D - \delta_H)a > (n_H + \delta_H)c + (n_D - \delta_H - 1)b$ has to hold. With $n_D = n - n_H$, this simplifies to

$$n_H + \delta_H - \frac{a-d}{a-b+c-d} < \frac{a-b}{a-b+c-d}(n-1). \tag{5}$$

As $\frac{a-d}{a-b+c-d} > 1$, this inequality is fulfilled for all link efficient strategy profiles that are no Nash equilbria (see Lemma 3, condition (a)).

In case (2), the condition to resort to hawk changes to $(\delta_H - 1)d + (n_H + n_D - \delta_H)a > (\delta_H)c + (n_H + n_D - \delta_H - 1)b$, which simplifies to

$$\delta_H - \frac{a-d}{a-b+c-d} < \frac{a-b}{a-b+c-d}(n-1). \qquad (6)$$

As $\frac{a-d}{a-b+c-d} > 1$, this inequality is fulfilled for all link efficient strategy profiles that are no Nash equilbria (see Lemma 3, condition (b)). □

Using the trigger strategy and Lemma 4, we can now derive the equilibria using the trigger strategy.

Theorem 10. *In the T-period Fixed Link game, following the trigger strategy yields a sub game perfect equilibrium if all players resort to the Nash equilibrium of the one-shot Network Hawk-Dove game for all periods T, but no other equilibria exist.*

Proof. We now consider whether the trigger strategy can yield additional equilibria in which more than $n_H = \frac{a-b}{a-b+c-d}(n-1)$ hawks persist. A dove following the trigger strategy in the worst case earns $t^*(n_H \cdot c + (n - n_H - 1)b) + (T - t^*) \cdot ((n_H + \delta_H) \cdot c + (n - (n_H + \delta_H) - 1)b)$, with δ_H being the number of additional hawks necessary to reach an equilibrium. Doves who switch to hawk in periods t^* to T earn more during these periods according to Lemma 4. Let us assume the dove deviates to hawk in t^* to ensure minimal punishment. Now her payoff is $(t^*-1)(n_H \cdot c+(n-n_H-1)b)+n_H d+(n-n_H-1)a+(T-t^*)\cdot((n_H+\delta_H)\cdot c+(n-(n_H+\delta_H)-1)b)$. To establish an equilibrium, $t^*(n_H \cdot c+(n-n_H-1)b)+(T-t^*)\cdot(n_H \cdot c+(n-(n_H+\delta_H)-1)b) > (t^*-1)(n_H \cdot c+(n-n_H-1)b)+n_H d+(n-n_H-1)a+(T-t^*)\cdot((n_H+\delta_H)\cdot c+(n-(n_H+\delta_H)-1)b)$ has to hold. This simplifies to $n_H > \frac{a-b}{a-b+c-d}(n-1)$, which is equivalent to condition (c) of Lemma 4, i.e., the Lemma describing the sub game perfect equilibria in the Network Hawk-Dove Game. As doves are always connected to all other players in the network, we can conclude that the Nash equilibria in the Fixed Link Game are a subset of the equilibria in the Network Hawk-Dove Game. □

Notice that in our formal analysis, we focus on T-period Fixed Link and T-period Fixed Action games. A further extension would be to investigate finitely repeated versions of these games. We believe that this is not worthwhile: As we know that repeatedly playing a stage game is always a Nash equilibrium in the repeated game, our results show that any number of hawks can be established in the (repeated version of) the T-period Fixed Action game. For Fixed Link games we did not come to this conclusion. Hence, the repetition of T-period Fixed Link games might lead to additional equilibria. However, as the T-period Fixed Link games have the same equilibria as the Fixed Link game, and these do not suffice to implement additional equilibria in the repeated version, we do not believe such equilibria to occur by repeating the T-period Fixed Action game again.

4 Experiment

To evaluate our theoretical prediction from Section 2, we designed an experimental test. This section summarizes the corresponding experimental design, hypotheses, and experimental results.

4.1 Design

To evaluate the hypotheses described, we conducted laboratory experiments at the Karlsruhe Institute of Technology. Each experimental session lasted approximately 1.5 hours. For all sessions we recruited a total of 162 participants using ORSEE (Greiner, 2004) from a pool of students in Karlsruhe. At the beginning of each experimental session, we randomly assigned the participants to groups of six. We handed out written instructions to each participant, describing the experimental setup. After all participants had read the instructions, they played one treatment implemented using zTree (Fischbacher, 2007) at a computer terminal. Finally, we paid the participants in private, depending on their success in the treatment.

The baseline treatment (Treatment Basic) is equivalent to the Network Hawk-Dove game and consisted of 50 periods. In every period, participants could specify other participants they wanted to establish links to and the action they wanted to play. At the end of each period, the experimental software calculated participants payoff (with the payoff function being identical to our running example in Table 2). For each link a participant had established, he had to pay $k = 50$ points. All linked participants then played the Hawk-Dove game. That is to say, if a participant played dove and his neighbor hawk, he received 40 points, while his neighbor received 80 points. If both participants played hawk, they received 20 points each. They received 80 points when coordinating toward dove action. After payoff calculation, the computer terminal showed the actions, all links, and the individual performance to each participant.

We conducted two modifications of Treatment Basic: (1) Treatment Fixed Link and (2) Treatment Fixed Action. Treatment Fixed Link was identical to 10 times playing the T-period Fixed Link game with $T = 5$. In Treatment Fixed Action, the T-period Fixed Action game with $T = 5$ was played 10 times. That is to say, we limited the strategy sets of the players in periods $2, ..., 5, 7, ..., 10$ and allowed them to choose both, actions and links, in periods $1, 6, ...$

At the end of the experiment, all participants received a show-up fee of 5.00 €. For $1,000$ points earned during the experiment, a participant received 1.00 €. On average, each participant earned 11.39 €.

4.2 Hypotheses

To begin with, we summarize the results of the preceding sections and apply the parameters of our experiments to these results (Table 3). We have shown (Theorem 2) that the payoff increases with the number of doves in the network. As we motivated in the introduction, network experiments with (anti-)coordination games have one central property: Theory predicts inefficient outcomes (e.g., Jackson and Watts, 2002, Bramoulle et al., 2004). However, in experiments efficient strategy profiles occur frequently (Corten and Buskens, 2010,Corbae and Duffy, 2008, Berninghaus et al., 2012). This result is not surprising as experimental tests of several other games show a tendency toward efficient outcomes (Engelmann and Strobl, 2004).

We have further shown that when analyzing the Network Hawk-Dove game as a finitely repeated game, such efficient outcomes are also theoretically plausible in the repeated Network Hawk-Dove game and the T-period Fixed Action game. However, they do not occur in T-period Fixed Link games. To confirm these theoretical predictions, we designed an experiment in which the overall payoff is maximal if the number of hawks n_H is 0 or 1. This efficient strategy profile can be established both in the repeated Network Hawk-Dove game and the T-period Fixed Action game but does not constitute an equilibrium in the T-period Fixed Link game. Hence, this setup allows us to check whether our theoretical predictions actually occur in an experimental setup.

Table 3: Number of hawks in Nash equilibrium

	Network Hawk-Dove	Fixed Action	Fixed Link
General Prediction ($a > b > k > c > d$)			
One-shot	$n_H^* \geq \frac{a-b}{a-b+c-d}(n-1)$	$n_H^* \in \{0, ..., n\}$	$n_H^* \in \{0, ..., n\}$[7]
Repeated	$n_H^* \in \{0, ..., n\}$	$n_H^* \in \{0, ..., n\}$[8]	$n_H^* \geq \frac{a-b}{a-b+c-d}(n-1)$
Our example ($a = 80 > b = 60 > k = 50 > c = 40 > d = 20$; $n = 6$)			
One-shot	$n_H^* \geq 2.5$	$n_H^* \in \{0, ..., 6\}$	$n_H^* \in \{0, ..., 6\}$
Repeated	$n_H^* \in \{0, ..., 6\}$	$n_H^* \in \{0, ..., 6\}$	$n_H^* \geq 2.5$

Based on our theoretical predictions, we expect the following properties in Treatment Basic:

Hypothesis 1. *In Treatment Basic, we expect ...*

a) a trend toward efficient strategy profiles, ...

b) more hawks in period 50 *than in periods* 1 *to* 49*, and ...*

c) links to doves only.

Equilibrium predictions for the Fixed Action game are identical to these predictions except for the trend toward more hawks in period 5. However, due to the reduced strategy set and a higher simplicity of the game, we expect there will be fewer deviations from the equilibrium strategy than in the Network Hawk-Dove game.

Hypothesis 2. *In Treatment Fixed Action, we expect ...*

a) the number of hawks and links to be similar to the number of hawks in periods 1 *to* 49 *of Treatment Basic and ...*

b) fewer deviations from the equilibria than in Treatment Basic.

[7]For the neighborhoods $n_H^i > \frac{a-b}{a-b+c-d}n^i$ has to hold.

[8](If $T(a-b+c) < (T-1)k - d$ holds, $n_H \geq \frac{T(a-b)-(T-1)k}{T(a-b+c)-(T-1)k-d}(n-1)$ has to be satisfied.

Hypothesis 2 (b) results from our expectation that the Fixed Action game simplifies the decision situation. Subjects who only adjust their links have a much simpler task than subjects who have to choose both, links and actions, as the latter do not have to coordinate on the desired number of hawks and doves but merely adjust their links to the given strategy profile.

For the Fixed Link game predictions are identical to the predictions of the repeated Network Hawk-Dove game in period 5. However, similar to the Fixed Action game, we expect that participants benefit from the higher simplicity of the game and that we see fewer deviations from equilibria.

Hypothesis 3. *In Treatment Fixed Link, we expect ...*

a) the number of hawks to be higher than the number of hawks in periods 1..49 *of Treatment Basic and in all periods of Treatment Fixed Action and ...*

b) fewer deviations from the equilibria than in Treatment Basic.

Similar to Hypothesis 2 (b), Hypothesis 3 (b) accounts for the lower complexity of the T-period Fixed Link game.

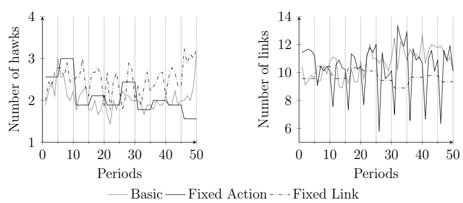
4.3 Results

A first glance at the data (see Fig. 1) shows that the number of hawks remains almost stable throughout the game in Treatments Fixed Link and Fixed Action, with the number of hawks being higher in Treatment Fixed. As expected, the graphs in Figure 1 suggest an increase in the number of hawks in the last period of Treatment Basic, whereas the development of the number of hawks resembles that in Treatment Fixed Action in all preceding periods.

When we compare the number of links, again Treatment Basic and Treatment Fixed Action resemble each other - except for a steep decrease in the number of links in Treatment Basic in periods $6, 11, ...$, i.e., in all periods in which the one-shot Network Hawk-Dove game is played. As expected, the number of links in Treatment Fixed Links is below the number of links in both other treatments.

To sum up, a first glance at the data is in line with our expectations except for Treatment Fixed Action. In this treatment, players tend to establish fewer links than in all other periods when choosing actions and links simultaneously.

Figure 1: Development of behavior during the experiment



One might criticize that a comparison between the treatments is difficult, because in Treatment Basic subjects can choose both, links and actions, in every period, while in Treatments Fixed Action and Fixed Link they can only do so in a subset of periods. Hence, we focus our analyses on all periods of the game and discuss differences between periods $1, 6, ..$ and the other periods of Treatments Fixed Action and Fixed Link whenever they occur.

4.4 Treatment Basic

In Treatment Basic, the number of hawks in the population, 2, is on average above that of all groups (see Table 4). While the number of hawks is around 2 during the first periods, it sharply increases to 3 on average in period 50. A binomial test confirms this increase (p=0.04), which is in line with Hypothesis 1 (b).

Table 4: Number of hawk players per group

Grp.	Basic			Fixed	Fixed Link		
	$t = 1..49$	$t = 50$	All	Action	$t = 1, 6, ...$	other t	All
1	2.04	3.00	2.06	2.20	2.30	2.90	2.78
2	2.24	4.00	2.28	2.20	0.90	1.70	1.54
3	2.45	3.00	2.46	2.00	2.60	2.73	2.70
4	2.76	4.00	2.78	2.60	3.60	3.18	3.26
5	2.39	3.00	2.40	2.00	2.50	2.73	2.68
6	1.24	2.00	1.26	2.00	3.70	2.48	2.72
7	1.88	2.00	1.88	2.20	2.40	2.08	2.14
8	1.49	4.00	1.54	1.80	2.70	2.45	2.50
9	1.76	3.00	1.78	2.00	1.70	2.43	2.28
Avg.	2.03	3.11	2.05	2.11	2.49	2.52	2.51

The number of hawks during the first 49 periods is higher than the number of hawks in efficient Nash equilibria, i.e., 0 or 1, and no group in our experiments reached an average of 1 or less. However, in all groups except one the average number of hawks is below 2.5, the minimum number of hawks in the one-shot equilibrium of the Network Hawk-Dove game. Hence, we observe a tendency toward efficient Nash equilibria as predicted by Hypothesis 1 (a).

Table 5: Avg. number of links per group

Grp.	Basic			Fixed Action			Fixed
	$t = 1..49$	$t = 50$	All	$t = 1, 6, ...$	other t	All	Link
1	12.04	12.00	12.04	10.63	10.60	10.62	10.30
2	11.14	13.00	11.18	11.80	12.40	11.92	9.40
3	10.02	11.00	10.04	9.95	10.30	10.02	9.60
4	7.43	3.00	7.34	8.58	10.00	8.86	9.80
5	10.16	12.00	10.20	10.43	11.30	10.60	10.40
6	13.00	6.00	12.86	9.30	9.60	9.36	9.30
7	10.49	12.00	10.52	10.55	11.50	10.74	11.60
8	11.57	10.00	11.54	11.20	11.10	11.18	7.70
9	10.35	12.00	10.38	9.65	11.00	9.92	9.30
Avg.	10.69	10.11	10.68	10.23	10.87	10.36	9.71

We attribute the low number of hawks to difficulties in establishing links. In a population of 2 hawks, we expect 14 links in equilibrium, i.e., 8 links from hawks to doves (4 per hawk player to each dove) and 6 links between doves (to link all 4 doves). However the average number of links is between 10 and 11 per group (see Table 5). In addition, the links should decrease in period 50. Here all 3 hawks should link to all 3 doves, and the doves should link to each other, resulting in 12 links. The number of links increases in 5 of 9 groups, not confirming the expected decrease (Wilcoxon, p=0.953, Z=-0.059).

To further analyze the link quality, we calculate different types of links relative to the possible number of links (see Table 6). For example, 45% observed (H,H) links in group 1 means that 45% of possible links between two hawks were established. Recall that links to hawks should not be established while all links to doves exist in equilibrium. Hence, an average of 35% of all links between two hawks is established, and 11% of all links from doves to hawks exist on average. This result clearly rejects Hypothesis 1 (c). In addition, 40% of all links between doves are not established. Only links from hawks to doves persist to a high extent (about 80% are established, while only 4% are missing).

Given the deviations in the established links from the equilibrium prediction, it is not surprising that the number of hawks in the population is

Table 6: Detailed analysis of links in Treatment Basic

Grp.	(H,H)-links		(D,D)-links			(D,H)- or (H,D)-links			
	Obs.	Bil.	Obs.	Bil.	Mis.	Obs.	Bil.	Mis.	(D,H)
1	45%	9%	76%	12%	36%	90%	5%	14%	12%
2	30%	4%	70%	6%	36%	86%	2%	16%	9%
3	13%	1%	46%	3%	57%	91%	1%	10%	4%
4	39%	3%	51%	2%	51%	52%	4%	52%	19%
5	35%	3%	63%	5%	42%	80%	2%	22%	7%
6	44%	4%	84%	15%	31%	96%	13%	16%	18%
7	22%	2%	80%	6%	26%	67%	2%	35%	8%
8	41%	0%	74%	2%	29%	86%	4%	18%	11%
9	49%	4%	56%	5%	49%	86%	5%	20%	12%
Avg.	35%	3%	67%	6%	40%	82%	4%	23%	11%

Abbreviations: Obs. = Observed, Bil. = Bilateral, Mis. = Missing

2 instead of 0 or 1 in the efficient equilibrium. In addition, this allows for efficiency increases in Treatments Fixed Action and Fixed Link, in which the subjects can focus on establishing the best links or choosing the right actions only.

4.5 Treatment Fixed Action

We now compare the results of Treatment Basic with those of Treatment Fixed Action with a focus on the differences in the established links. Our analysis concentrates on periods 1 to 49 of Treatment Basic in the light of the results of our theoretical prediction and behavioral results, clearly showing different behavior in the last period. The number of hawks in the population is identical in both Treatment Basic and Treatment Fixed Action (Mann Whitney U, p=0.931, U=39.0), confirming Hypothesis 2 (a).

The number of links does not differ between Treatments Basic and Fixed Action (Mann Whitney U, p=0.489, U=32.0). Nevertheless, the lower number of links in periods $1, 6, ...$ (see Fig. 1) is significant (Mann Whitney U, p=0.002,U=7.0). In our analysis of Treatment Fixed Action, we therefore focus on periods $2, ..., 5, 7, ...$ and only mention periods $1, 6, ...$ explicitly if behavior during these periods differs.

Table 7: Detailed analysis of links in Treatment Fixed Action

Grp.	(H,H)-links		(D,D)-links			(D,H)- or (H,D)-links			
	Obs.	Bil.	Obs.	Bil.	Mis.	Obs.	Bil.	Mis.	(D,H)
1	6%	0%	75%	8%	34%	89%	3%	14%	4%
2	13%	1%	88%	12%	23%	100%	9%	8%	11%
3	6%	1%	69%	14%	45%	99%	2%	4%	4%
4	17%	2%	62%	8%	46%	90%	1%	11%	1%
5	2%	0%	72%	10%	38%	90%	1%	11%	1%
6	6%	0%	69%	13%	44%	89%	1%	13%	3%
7	7%	0%	74%	6%	32%	97%	1%	4%	1%
8	0%	0%	67%	9%	42%	100%	0%	0%	0%
9	0%	0%	62%	8%	46%	93%	2%	10%	2%
Avg.	6%	1%	71%	10%	39%	94%	2%	8%	3%

Abbreviations: Obs. = Observed, Bil. = Bilateral, Mis. = Missing

In line with our expectation (see Hypothesis 2), (b) the number of links between hawks (Mann Whitney U, p=0.000, U=1.5) and the number of links from hawks to doves (Mann Whitney U, p=0.014, U=12.0) is lower in Treatment Fixed Action than in Treatment Basic (see Tables 6 and 7). Bilateral, i.e., inefficient, links between two hawks (Mann Whitney U, p=0.006, U=10.0) or between hawks (Mann Whitney U: p=0.094, U=21.0) and doves as well as missing links from hawks to doves (Mann Whitney U: p=0.000, U=4.0) are also less frequent in Treatment Fixed Action. Even links from doves to hawks do not occur as often (Mann Whitney U: p=0.001, U=5.0). This clearly confirms Hypothesis 2 (b). However, these results do not hold if periods $1, 6, ...$ are also subject of the analysis. Here all significant effects except for the number of links between two hawks and links from doves to hawks vanish.

The fraction of players playing all optimal links is about 30% from period 5 on (except for periods $6, 11, ...$) in Treatment Fixed Action. In Treatment Basic, it takes longer until these levels are reached, in line with the results on links involving hawks. The situation changes when we analyze links between two doves (see Fig. 2). In Treatment Fixed Action, the fraction of missing links between doves increases over time toward levels close to 100%. That is to say, in later periods doves are almost never linked. Throughout the game, the fraction of links between doves (and the number of missing links between doves) does not vary (Mann Whitney U, p=0.796, U=37.0; p=1.000, U=40.0). We attribute this to the fact that most links between doves are established after every Network Hawk-Dove game played but are removed afterwards, which, in turn, we attribute to a lack of reciprocity. Subjects try to alternate in paying for the all links between doves. However, they fail to do so, which is confirmed by the fact that in Treatment Fixed Action bilateral links between doves are more frequent (Mann Whitney U, p=0.040, U=17.0), yielding fewer links and contradicting Hypothesis 2.

The results of Treatment Fixed Action clearly confirm Hypothesis 2 - except for one aspect: Subjects fail in reciprocally establishing links between two doves. This result is especially surprising because, in contrast to Treatment Basic, subjects in Treatment Fixed Action can focus on the links only.

4.6 Treatment Fixed Link

The number of hawks in Treatment Fixed Link is about 2.5 and therefore significantly higher than in the first 49 periods of Treatment Basic (Mann Whitney U, p=0.050, U=18.0) and also significantly higher than in Treatment Fixed Action (Mann Whitney U, p=0.024, U=15.0), confirming Hypothesis 3 (a). However, it is still lower than in period 50 of Treatment Basic (Mann Whitney U, p=0.077, U=20.0).

We investigate this by checking whether the played action (hawk or dove) is the best response to the strategy profile of all other players (see Fig. 3). Throughout the game, for almost the same fraction, i.e., about 50% of the players who play dove, dove is in fact the best strategy. This percentage is quite low. It indicates that half of the dove players could increase their payoff by switching to hawk. As for Treatment Basic, this is not surprising: Here, in an efficient Nash equilibrium, several players resort to dove so as not to be punished in the long run, although a switch to hawk would increase their payoff in the short run. However, this does not hold for Treatment Fixed Link. Here subjects play a one-shot Nash equilibrium. A deviation does not have a negative consequence in the long run. The result is more intriguing when we focus on the number of hawks playing the optimal action, given the strategy profile of all others (see Fig. 3, right chart). Throughout the game, the fraction of hawks who should play hawk to maximize their payoff is higher in Treatment Basic than in Treatment Fixed Link.

In consequence, the fraction of players maximizing their payoff (see Ta-

Table 8: Fraction of best actions given other links and actions per group

Grp.	Basic						Fixed Action		Fixed Link	
	$t = 1..49$		$t = 50$		All					
	D	H	D	H	D	H	D	H	D	H
1	62%	90%	100%	67%	63%	90%	60%	92%	79%	44%
2	71%	95%	100%	0%	71%	93%	55%	91%	21%	84%
3	91%	96%	100%	100%	91%	96%	63%	85%	79%	60%
4	69%	62%	100%	25%	69%	62%	76%	78%	83%	61%
5	69%	86%	100%	67%	70%	86%	59%	92%	83%	75%
6	29%	96%	25%	50%	29%	95%	58%	88%	80%	74%
7	33%	94%	75%	100%	34%	94%	62%	93%	57%	77%
8	27%	96%	100%	0%	29%	94%	64%	95%	49%	70%
9	61%	92%	100%	100%	62%	92%	66%	98%	56%	71%
Avg.	57%	90%	89%	57%	58%	89%	63%	90%	65%	68%

ble 8) by playing hawk is higher (Mann Whitney U, p=0.003, U=8.0) in periods 1 to 49 of Treatment Basic than in Treatment Fixed Link, while this does not hold for doves (Mann Whitney U, p=0.489, U=32.0). However, this changes in period 50 of Treatment Basic. Here for more doves playing dove is the best alternative in Treatment Basic compared to Treatment Fixed Link (Mann Whitney U, p=0.014, U=13.0), while no such effect exists for doves (Mann Whitney U, p=0.605, U=34.0). Hence, this result contradicts Hypothesis 3 (b).

To sum up, we observe an increase in the number of hawks in Treatment Fixed Link compared to the first periods of Treatments Basic and Fixed Action. However, this increase is not as high as that in the last period of Treatment Basic. We attribute this to the problems of subjects in finding the best response to the strategy profiles they face. Similar to Treatment Fixed Action, the increased simplicity does not help to better identify and play the equilibria.

4.7 Summary

Treatments Basic and Fixed Action yield almost the same number of hawks per population, namely about 2. First, given the theoretical predictions of the Network Hawk-Dove Game, these are more hawks than we expect in an efficient Nash equilibrium. We attribute this deviation to the players' problems when choosing their optimal links. Second, in Treatment Fixed Link subjects resort to hawk less often (about 2.5 per network) than expected by the Nash prediction. This is the consequence of not playing the best response to the strategies of others.

Table 9: Payoff per capita and per period

Grp.	Basic			Fixed	Fixed
	$t = 1..49$	$t = 50$	All	Action	Link
1	661.63	560.00	659.60	623.40	357.80
2	679.59	390.00	673.80	640.00	492.40
3	654.08	610.00	653.20	559.80	396.80
4	377.14	130.00	372.20	497.00	387.60
5	651.22	680.00	602.00	640.40	533.60
6	651.22	420.00	646.60	526.40	459.80
7	645.31	840.00	649.20	675.80	472.00
8	735.71	60.00	722.20	669.80	429.40
9	599.39	600.00	599.40	608.00	415.80
Avg.	622.72	476.67	619.80	604.51	438.36

We finally examine the payoffs in all three treatments (see Table 9). While payoffs in Treatments Basic and Fixed Action do not differ (Mann Whitney U, p=0.387, U=30.0), they are significantly higher than in Treatment Fixed Links (Basic: Mann Whitney U, p=0.003, U=8.0; Fixed Action: Mann Whitney U, p=0.000, U=2.0), clearly showing that the increase in simplicity does not improve payoffs.

5 Discussion

Like Berninghaus et al. (2012), we investigate Network Hawk-Dove games. However, their work differs from ours in two aspects: (1) They play the corresponding experiments in continuous time. (2) Their theoretic analysis considers one-shot versions of the game only. In this section, we discuss the advantages and disadvantages of both approaches.

Berninghaus at al. (2012) motivate the usage of continuous time experiments with lower coordination failures. Such failures in Network Hawk-Dove games should be expected since the best choice of action and links depends on the choices of all other players in the group. As we are interested in equilibrium play, it seems an intriguing idea to find a way to shorten the time needed to reach these equilibria. However, our experiment shows that coordination failures are not reduced: Subjects find equivalent strategies both if they play in continuous time, as in (Berninghaus, 2012), and in discrete time, as in this study. Efficiency in our study is around 60%, while it is around 76% in the study with continuous time. This decrease in efficiency results from slower adaptations to the strategy profiles of others rather than from different behavioral patterns, confirming that discrete time experiments lead to the same strategy profiles as continuous time experiments and offer no additional insights.

However, a disadvantage of continuous time experiments is their lack of control: While we can limit the strategy sets in certain periods, this is not possible in continuous time experiments. The goal of Berninghaus et al. (2012) has been to investigate the strategy adaptation process, namely to find out whether subjects first choose actions and adjust their links accordingly or the other way around. They show that in continuous time experiments, subjects tend to choose actions and fix links afterwards. Our design with discrete time allows for additional insights. Apart from the fact that the interplay of actions and links is confirmed, we observe that subjects who adjust their actions to a given network never reach the levels of efficiency reachable if actions are chosen first. We can even measure the loss in efficiency imposed by this limitation. Moreover, we even see the impact on the adaptation process for the different setups. Hence, our experiments suggest that discrete time experiments should be preferred over experiments in continuous time.

Berninghaus et al. (2012) use a theoretic analysis of the one-shot Network Hawk-Dove game to predict behavior. They do this using the Nash concept, which is not in line with their own theoretical predictions and a new concept derived from their experimental results. We believe that our approach, i.e., using the Nash concept and extending the analysis to a multi-period model, is preferable to the introduction of a new concept as this leaves a central question open: When to apply which concept?

6 Conclusion

In this paper, we have extended existing game-theoretic analyses of the one-shot Network Hawk-Dove game (Bramoulle et al., 2004, Berninghaus and Vogt, 2006) to the finitely repeated version of the game. We have shown that, although efficient strategy profiles and Nash equilibria of the one-shot game differ, efficient strategy profiles are sub game perfect Nash equilibria in the finitely repeated version of the game. This helps to better understand the behavior observed in anti-coordination games with endogenous network formation. Here the subjects typically tend to play efficient strategy profiles, although game-theoretic models of the one-shot games predict outcomes with low payoffs only. Our results are especially worthwhile because a small design change, i.e., limiting the strategy set to action selection in some periods, has resulted in our game-theoretic model not predicting efficiency and the subject not reaching efficiency in the lab.

Our results suggest that in real world (online) social networks, humans should choose their actions wisely and adjust their contacts accordingly. In your own company, do encourage employees to decide early whether to become a manager or an employee and encourage them to interact. In the online world, let community members choose their strategies early and let them build links easily.

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A Experimental Instructions

In this appendix, we list our experimental instructions. The first subsection consists of a translation of the German instructions used during the experiments. Both latter subsections are translations of the paragraphs modified for Treatments Fixed Action and Fixed Link.

A.1 Treatment Basic

Welcome to this experiment and thank you very much for your participation. You will receive 5.00 € for showing up in time. During the experiment, you will have the opportunity to earn additional money. Please stay calm and switch of your mobile phone. Please read these instructions carefully which are identical for all participants. Communication among the participants is not allowed. If you do not follow these rules, we impose a fine of 5.00 € on you. If you have any questions, please raise your hand. The experimenter will then come to you and answer your question in private. The endowment of 5.00 € for showing up in time as well as any other amount of money, you earn during the experiment, will be paid to you in cash at the end of the experiment. We will pay you privately to ensure that no other participant becomes aware of the amount of your payment. Your payment depends on your own decisions as well as the decisions of other participants. The payoff in the experiment is measured in points. The points you earn during the experiment will be converted into Euro at the end of the experiment and paid to you. You find the conversion rate at the end of this document. You and all other participants enter their decisions independently of other participants in individual computer terminals.

Course of the experiment

At the beginning of the experiment, you are randomly assigned to five other participants, forming groups of six. Each member of your group will be randomly assigned to one of six positions. The participants in one group will not necessarily sit side by side. The composition of the group and the positioning remain unchanged. In the following, "participants stands for the participants in your group. The participants of other groups are not considered in the remainder of these instructions.

This experiment consists of 50 periods. At the beginning of each period, you have to make two decisions:

- You decide on the strategy you will play during the next period.
- You decide on the connections you establish to participants you want to play with.

While making your decision, you will see the following display on your computer screen (see Figure 4). Your position and the period you play in are visualized in the upper area of the screen. On the left side of the display you decide on your strategy. On the right side you choose the partners to whom you want to establish connections. By marking several different other participants you can establish connections to more than one participant.

Your payoff

At the end of each period, your payoff is calculated based on the decisions of all participants. The result of this calculation, your decisions and the decisions of all other participants are shown on the computer screen.

Table 10: Payoff

		Your fellow player	
		Strategy A	Strategy B
You	Strategy A	You: 20, Fellow Player: 20	You: 80, Fellow Player: 40
	Strategy B	You: 40, Fellow Player: 80	You: 60, Fellow Player: 60

The calculation of your payoff is as follows. For each connection you established to another participant you face costs at height of 50 per connection. In addition you receive a positive payoff for each participant you are connected to. Therefore, it does not matter whether the other participant, you or both established the connection. The payoff depends on your strategy and the strategy of the connected participant (see Table 10). If both of you chose Strategy A, you receive a payoff of 20 points. If you chose Strategy A and the other participant chose Strategy B, you receive 80 points. If the other participant chose Strategy A and you Strategy B, you receive 40 points. If both of you chose Strategy B, you receive 60 points. You receive a payoff per period calculated as the payoff from all games with other participants minus the costs for the connections established by you.

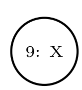
Example: Imagine you are Participant 1. You have established a connection to Participant 2. Participant 3 has established a connection to you. You play Strategy A, Participant 2 plays Strategy A and Participant 3 plays Strategy B. Your Payoff is calculated as follows: The connection to Participant 2 costs 50 points, the connection of Participant 3 to you is free of charge for you. You receive 20 points from the game with Participant 2 and 80 points from the game with Participant 3. Summed up, you receive 50 points $(80 + 20 - 50)$.

Information at the end of each period

In the end of each period you see your earnings during the last period and the current structure of the network. In the upper area of the corresponding display you see your position and the period you currently play in. On the left half you see the costs for connections established by yourself, the payoff from every game with another participant you are connected to and your earnings during the last period, namely the sum of your connection costs and the payoffs per game. On the right half of the display you see the structure of the network. The visualization follows these conventions:

- Every participant, even yourself, is visualized by a circle. Within the circle stands an identification number of the participant and the strategy, namely A or B, the corresponding participant is playing.

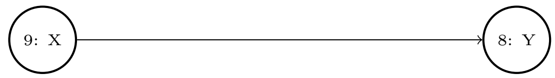
  Example: The following visualization represents Participant 9 playing Strategy X.



- Connections between participants are visualized with arrows. Each arrow starts at the participant who established the connection and points to the participant to whom the connection was established.

  Example: The following visualization represents a connection between

Participant 9 and Participant 8. Participant 9 has established the connection.



- If both participants established a connection to each other, the arrow has heads at both ands. If no connection exists, no arrow connects the circles representing the participants directly.

End of the experiment

The experiment ends after 50 periods. Throughout all periods the groups and the assignments to positions do not change. Please remain seated during all periods and only get up when asked.

In the end of the experiment your points are converted to money. For 1.000 points you earned during the experiment, you receive 1.00 €. Your earnings are rounded to the next amount dividable by 5 Cent.

A.2 Treatment Fixed Action

The instructions of Treatment Fixed Action were identical to the instructions of Treatment Basic, except for two paragraphs in the section ”Course of the experiment.

The end of the second paragraph (“At the beginning of each period [...] want to play with.”) was replaced by “At the beginning of each period, you decide on the connections you establish to participants you want to play with. At the beginning of the first period, and then after every 5 periods (namely, Period 6, Period 11,) you also decide on the strategy you will play during the next period.”

The third paragraph (“While making your decision [...] to more than one participant.”) was extended by “During the periods, in which you cannot decide which strategy you play, the left part of the display is empty. During these periods you play the strategy of the preceding period.”

A.3 Treatment Fixed Link

In the instructions of Treatment Fixed Link, we modified the same two paragraphs of Treatment Basic as in Treatment Fixed Action.

In particular, the second paragraph ("This experiment consists of [...] want to play with.") now read "At the beginning of each period, you decide on the strategy you will play during the next period. At the beginning of the first period, and then after every 5 periods (namely, Period 6, Period 11,) you also decide on the connections you establish to participants you want to play with."

After the third paragraph ("While making your decision [...] to more than one participant.") we added "During the periods, in which you cannot decide to whom you want to establish connections to the right part of the display is empty. During these periods the network structure remains identical to the structure of the preceding round for you and all other participants."

Figure 2: Fraction of players having optimal links given played actions

Figure 3: Fraction of players using optimal action given played actions

Figure 4: Decision Screen