



# JENA ECONOMIC RESEARCH PAPERS



# 2012 – 066

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[www.jenecon.de](http://www.jenecon.de)

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact [markus.pasche@uni-jena.de](mailto:markus.pasche@uni-jena.de).

Impressum:

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# Flexible Waste Management under Uncertainty\*

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December 9, 2012

## Abstract

In this paper, we use stochastic dynamic programming to model the choice of a municipality which has to design an optimal waste management program under uncertainty about the price of recyclables in the secondary market. The municipality can, by undertaking an irreversible investment, adopt a flexible program which integrates the existing landfill strategy with recycling, keeping the option to switch back to landfilling, if profitable. We determine the optimal share of waste to be recycled and the optimal timing for the investment in such a flexible program. We find that adopting a flexible program rather than a non-flexible one, the municipality: i) invests in recycling capacity under circumstances where it would not do so otherwise; ii) invests earlier, and iii) benefits from a higher expected net present value.

KEYWORDS: Real Options, Flexibility, Municipal Waste, Recycling

JEL CLASSIFICATION: C61, Q53

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\*The authors gratefully acknowledge Michele Moretto for useful comments. The usual disclaimer applies.

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## 1 Introduction

The design of effective solid waste management strategies is a crucial issue for policy makers not only at the (inter)national level, where guidelines, targets, and strategies are set (US Environmental Protection Agency, 2002; European Commission, 2010), but also at the local level, where waste is actually produced, collected, and treated.

In the last decades, the amount of municipal solid waste produced by industrialized societies has been increasing (Eurostat, 2011; EPA, 2011). This trend, together with growing attention on environmental pollution, human health, and resource recovery, has stimulated a wide debate on the strategies to be implemented to reduce the amount of waste produced and treat the waste collected in an effective and sustainable way (OECD, 2007).<sup>1</sup> In particular, starting from the late 1970s, the U.S. first, and later the EU, introduced a stricter regulation for the construction and operation of landfills<sup>2</sup> in order to promote recycling and incineration /incinerators as alternative disposal methods (EEA, 2009 and Kinnaman, 2006). Incinerators are expensive, however, and their effect on human health is controversial. As a consequence, citizens seem more willing to spend time sorting their waste for recycling than accepting the operation of an incinerator in their neighborhood (Giusti, 2009).<sup>3</sup> Thus, although their profitability is still debated, an increasing number of municipalities have introduced recycling programs (in order) to meet citizens' preferences (see, e.g., Kinnaman, 2006).

In this paper, we consider a municipality designing a new waste management program that integrates the preexistent landfilling with recycling as an alternative waste disposal method.<sup>4</sup> We assume that a price is paid to the municipality for recycled materials and that such a price follows a geometric Brownian motion. We also assume that recycling has higher operative costs than landfilling. The municipality can choose between a non-flexible and a flexible waste management program.

By investing in a non-flexible program (hereafter NFP), the municipality may partially or totally substitute landfilling with recycling. This decision is irreversible and implies that, irrespective of a change in the relative convenience of recycling with respect to landfilling, the purchased recycling capacity must always be fully used.

In contrast, by investing in a flexible program (hereafter FP), the municipality purchases recycling capacity but keeps the option to fully use the preexisting landfilling capacity whenever

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<sup>1</sup>Municipal solid waste can be disposed by essentially adopting four methods: landfilling, incineration, recycling, and composting. See Goddard (1995) for a discussion of these disposal methods.

<sup>2</sup>In the U.S., after the Resource Conservation and Recovery Act of 1976 providing federal guidelines for the operation of landfills, their number (of landfills) has significantly reduced (Kinnaman and Fullerton, 2000b). In the EU, after the Directive 1999/31/EC, which fixed targets for the reduction of biodegradable municipal waste going to landfills, the quantity of waste landfilled has reduced from 68% in 1995 to 33% in 2009 (Eurostat, 2011).

<sup>3</sup>Even though landfilling has strongly reduced, it is still adopted as a residual method together with recycling. As for the EU countries, in 2009 shares of waste landfilled of 14% and 17% were reported by Norway and Luxembourg, respectively. France, Italy, Finland, and the U.K. reported shares in the range of 32% to 50%. Among the EU-12 member states, the highest shares in 2008 were reported by Greece (81%), Portugal (62%), Ireland (62%), and Spain (52%), (Eurostat, 2011).

<sup>4</sup>Note that considering incineration as an alternative disposal method would make no difference in our analysis.

changes in the relative convenience make it profitable. By combining the two disposal methods, the FP guarantees a certain degree of operational flexibility, which may be beneficial under uncertainty about the price for recycled materials. This flexibility, however, comes at a cost. More specifically, we assume that the FP setup requires a sunk investment cost which depends on the chosen recycling capacity, i.e., the chosen degree of flexibility.

The problem faced by the municipality is twofold, and we solve it in two steps. First, the municipality must determine the recycling capacity, taking into account its uncertain profitability and the option of landfilling whenever recycling becomes unprofitable. Second, the municipality must set the investment time threshold, triggering the adoption of the optimally designed FP.

Having designed the optimal FP, we compare the investment in such a program with the investment in an NFP where, as stated above, the option to switch back to landfilling is not available. We find that adopting an FP rather than an NFP gives the municipality two main advantages. First, we show that the municipality may be willing to invest in recycling capacity under circumstances where investment in an NFP would not be undertaken. Second, we show that an investment in an FP may be undertaken earlier than one in an NFP and also provide a higher expected net present value (hereafter NPV).

The intuition behind these results is that the municipality that adopts the FP, by holding the option to switch back to landfilling, may, if needed, adjust the waste disposal operations and so optimally hedge against uncertainty about the profit from recycling. This hedging policy may prove particularly valuable when net revenues from recycling remain low and/or are volatile. In contrast, when net revenues are high and stable, the exercise of the option to switch back to landfilling becomes unlikely and the value of the hedging policy vanishes. Hence, the municipality may, by investing in an FP that guarantees operational flexibility, start recycling when the relative net revenues are too low to justify the investment in an NFP instead. Moreover, this may also occur with a higher payoff in terms of NPV.

Several papers have studied the design of waste management programs in the presence of alternative disposal strategies. In a deterministic frame, some pioneer investigations have been conducted by Huhtala (1997) and Highfill and McAsey (1997, 2001b). Huhtala uses an optimal control model to determine the optimal recycling rate for municipal solid waste. He shows that landfilling is more costly than other disposal alternatives, once the monetary costs of recycling, the social costs of landfilling, and consumers' environmental preferences have been accounted for. Under endogenous waste stream, Highfill and McAsey (1997) study a municipality which must choose between using an (existing and exhaustible) landfill or recycling at higher cost. The authors show that a municipality that recycles will always simultaneously use its landfill. This will last for some time when since landfill use is declining while recycling is increasing. Highfill and McAsey (2001b) extend previous works by including in their analysis a growing income stream. Income is optimally split between consumption and expenditures for waste disposal. Waste disposal must be optimally allocated between recycling, which is considered (as) a backstop technology, and landfilling. The authors show that landfill capacity and initial income have a considerable impact on the optimal recycling

program and recommend considering these factors when designing a waste management program. Recently, Lavee et al. (2009) have analyzed the choice of a municipality that can switch forward and backward between landfilling and recycling but cannot combine them. The choice is determined by taking into account a sunk switching cost and uncertainty about prices for recycled materials. Their main finding is that recycling, due to its uncertain profitability, may not be adopted even when it is less expensive than landfilling. Hence, their analysis advises policy intervention in favor of price stabilization as a tool for enhancing recycling.

Our paper contributes to this literature in two respects. First, under uncertainty about profit from recycling, we study the optimal design of a program where the simultaneous combination of two disposal strategies, i.e., landfilling and recycling, is feasible. Second, we consider how the presence of landfilling as a preexisting and residual method affects i) the degree of operational flexibility in the waste management program and ii) the timing of its adoption.<sup>5</sup>

The remainder of the paper is organized as follows. In Section 2, we present the basic setup of our model. In section 3, we determine the optimal recycling capacity. In section 4, we study investment value and timing. In section 5, we use some numerical examples to illustrate our findings. Section 6 concludes. All proofs are available in the Appendix.

## 2 The Basic Setup

Consider a municipality currently using landfilling as a waste disposal method and contemplating the opportunity of integrating it with recycling. Following Highfill and McAsey (2001), we restrict our analysis to the recycling programs offered by the municipality and do not consider any recycling activity undertaken by individuals on their own initiative. By integrating these two disposal methods, the collected waste may be partially or totally recycled, with the municipality still holding the option of landfilling.<sup>6</sup> Both disposal methods are costly. Denote by  $c^L$  and  $c^R$  the operating costs of landfilling and recycling waste, respectively. We assume that  $c^R - c^L > 0$ .<sup>7</sup> Compared to landfilling, recycling involves additional costs for collection, selection of different types of waste fractions (i.e., plastic, paper, glass), and for their transport to the different recycling plants. Collection costs depend on the requirements of the program, for instance how the recyclables have to be sorted by households (i.e., single-stream or multi-stream), the frequency of the collection of the different sorted waste fractions, and the level of participation in the program. The selection and

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<sup>5</sup>In the real option literature, the value of operational flexibility has been deeply investigated. See, e.g., Kulatilaka (1988, 1993), Triantis and Hodder (1990), and He and Pindyck (1992). In this literature, our paper belongs to a recent family of papers studying investment in flexible systems where the degree of flexibility is optimally chosen. See, e.g., Di Corato and Moretto (2011) on investment in a biogas digester under flexible diet composition and Moretto and Rossini (2012) on partial outsourcing and flexible vertical arrangements.

<sup>6</sup>In our paper, we implicitly consider a non-exhaustible landfill. The reason for this is that we want to focus on the benefit of implementing hedging policies against uncertain recycling profit through a combination of waste disposal technologies. Note that at no loss our frame is sufficiently general to consider an alternative technology such as incineration.

<sup>7</sup>This assumption is in line with Kinnaman (2006, p. 220) reporting that "*On a per-ton basis, recycling is roughly twice as costly as landfill disposal.*" The cost of landfilling may also include the compensation paid to households living near the landfill (Kinnaman, 2006).

processing costs per ton increase with the number of commingled commodities, (EPA, 2012).

Recycled materials are valuable on a secondary market, and the municipality is paid a price  $p_t$  for each unit of recycled waste, where units are expressed in tons.<sup>8</sup>

Let us assume that such a price evolves according to the following geometric Brownian motion:

$$\frac{dp_t}{p_t} = \mu dt + \sigma dz_t, \text{ with } p_0 = p \quad (1)$$

where  $\mu$  is the expected growth rate,  $\sigma$  is the volatility parameter, and  $dz_t$  is the increment of the standard Wiener process satisfying  $E[dz_t] = 0$ ,  $E[dz_t^2] = dt$ .

In the following, we simplify the analysis by considering the optimal disposal of one unit of waste which is potentially recyclable. Such a unit can be thought as including only one specific recyclable material, i.e., glass, paper, plastic, metals, or a mixture of recyclable materials. In the first case,  $p_t$  is the price paid for a specific material. Otherwise,  $p_t$  can be a price vector or, for simplicity, an average price.

## 2.1 A waste management program

Denote by  $W^L$  the waste management program where the collected waste is totally landfilled and by  $W^R$  the program where a portion  $\alpha \in [0, 1]$  of waste is recycled while the rest,  $1 - \alpha$ , is landfilled. When both disposal methods are feasible, the collected waste could be managed in order to minimize the cost of waste disposal,  $c_t$ , that is:

$$\begin{aligned} c_t &= \min \{c^L, c^L(1 - \alpha) + \alpha(c^R - p_t)\} \\ &= c^L + \min \{0, \alpha[c^R - (c^L + p_t)]\} \\ &= c^L - \max \{0, \alpha[(c^L + p_t) - c^R]\} \end{aligned} \quad (2)$$

where  $(c^L + p_t)$  represents the total benefit per unit of recycled waste, i.e., the price paid to the municipality plus the avoided landfilling cost.

The relative convenience of using landfilling or recycling depends on market prices. In this respect, we can have the following two scenarios:

$$c_t = \begin{cases} c^L & \text{for } p_t \leq c^R - c^L \\ (1 - \alpha)c^L + \alpha(c^R - p_t) & \text{for } p_t > c^R - c^L \end{cases} \quad (3)$$

This means that, whenever the current price of recycled material does not cover the increase in the disposal cost, landfilling is the less costly disposal method. In contrast, whenever the current

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<sup>8</sup>The implicit assumption is that the municipality is a price-taker. The arrangements for selling the recycled materials in the secondary markets are different and affected by the national legislative framework. In the U.S., municipalities can sign contracts with private entities providing these services (EPA, 2012). In the EU, there is heterogeneity due to the different approaches adopted by the national Producer Responsibility System (PRO). In Italy, e.g., CONAI, the national PRO, pays municipalities a ‘‘compensation fee’’ for taking back packaging waste from separated waste collections.

price,  $p_t$ , covers the additional disposal cost, the municipality recycles a share  $\alpha \in [0, 1]$  of the collected waste. In the following, we (will) refer to the first and second scenarios as "Landfilling" and "Recycling", respectively.

The analysis can be simplified by noting that the marginal advantage of a waste management program adding the program  $W^R$  to the existing  $W^L$  strictly depends on the benefits the municipality may obtain under each scenario, that is

$$b_t = \begin{cases} 0 & \text{for } p_t \leq d \\ \alpha(p_t - d) & \text{for } p_t > d \end{cases} \quad (4)$$

where  $d = c^R - c^L > 0$  is the additional operating cost to be paid when recycling is preferred to landfilling.

Now, denote by  $W$  an FP, that is, under  $W$ , we allow the municipality to switch between  $W^L$  and  $W^R$ , if this is profitable. Note that, without loss of generality, we assume that, once initiated, the waste management program runs forever.<sup>9</sup> Let  $V(p_t; \alpha)$ ,  $V^L(p_t; \alpha)$ , and  $V^R(p_t; \alpha)$  represent the value of  $W$ ,  $W^L$ , and  $W^R$ , respectively. For  $\alpha \in [0, 1]$ ,  $V(p_t; \alpha)$  is the solution of the following dynamic programming problem (Dixit, 1989, pp. 624-628):

$$\begin{aligned} \Gamma V^L(p_t; \alpha) &= 0 & \text{for } p_t < d \\ \Gamma V^R(p_t; \alpha) &= -\alpha(p_t - d) & \text{for } p_t > d \end{aligned} \quad (5-6)$$

where  $\Gamma = \frac{1}{2}\sigma^2 p_t^2 \frac{\partial^2}{\partial p_t^2} + \mu p_t \frac{\partial}{\partial p_t} - r$  is the differential operator with  $r$  as interest rate.

As shown in appendix A.1, the solution to [5-6] is

$$V(p_t; \alpha) = \begin{cases} V^L(p_t; \alpha) = \alpha O^R p_t^{\theta_1} & \text{for } p_t < d \\ V^R(p_t; \alpha) = \alpha [O^L p_t^{\theta_2} + (\frac{p_t}{r-\mu} - \frac{d}{r})] & \text{for } p_t > d \end{cases} \quad (7)$$

where  $\theta_1 > 1$  and  $\theta_2 < 0$  are the roots of the characteristic equation  $\phi(\theta) = \frac{1}{2}\sigma^2\theta(\theta-1) + \mu\theta - r = 0$  and

$$O^R = \frac{d^{1-\theta_1}}{r} \frac{r - \mu\theta_2}{(r - \mu)(\theta_1 - \theta_2)} > 0, \quad O^L = \frac{d^{1-\theta_2}}{r} \frac{r - \mu\theta_1}{(r - \mu)(\theta_1 - \theta_2)} > 0 \quad (7.1-7.2)$$

In equation (7), we observe that for  $p_t \leq d$  (Landfilling scenario), the value of the FP,  $\alpha O^R p_t^{\theta_1}$ , is simply represented by the option to recycle a portion  $\alpha$  of collected waste as soon as  $W^R$  becomes profitable. Note that the value of this option is increasing in  $p_t$  and decreasing in  $d$ , i.e.,  $c^R - c^L$ . This makes sense, considering that such an option is more valuable if recycled waste is more profitable and less desirable if landfilling becomes relatively more convenient. By contrast,

<sup>9</sup>This is a costless assumption. Recall that the focus of our paper is the comparison between a flexible and non-flexible program. Hence, even assuming a more realistic finite time horizon, our final results would still hold. Finally, note also that, for the sake of simplicity, we abstract from other operative options such as the options to mothball and/or abandon the program once it has been initiated (see, e.g., Dixit and Pindyck, 1994, chaps. 6-7).

when  $p_t > d$  (Recycling scenario), the term  $\alpha O^L p_t^{\theta_2}$  represents the value of the option to switch back to landfilling, which is consistently decreasing in  $p_t$  and increasing in  $d$ . The term  $\alpha(\frac{p_t}{r-\mu} - \frac{d}{r})$  represents instead the net benefit obtained by recycling a portion  $\alpha$  of collected waste.

Finally, denote by  $\widehat{W}$  an NFP where the municipality, once it has/ initiated the program, adopts the disposal regime  $W^R$  forever. Note that in this case the municipality does not hold the option to switch between  $W^L$  and  $W^R$ , i.e.,  $O^R = O^L = 0$ . Hence, the value of the program is simply given by

$$\widehat{V}(p_t; \alpha) = \alpha\left(\frac{p_t}{r-\mu} - \frac{d}{r}\right), \quad (8)$$

which, as above, represents the expected net benefit accruing from the recycling of a portion  $\alpha \in [0, 1]$  of the total collected (recyclable) waste.

### 3 The Optimal Waste Management Program

In this section, we determine the recycling capacity,  $\alpha$ , that a municipality must purchase to ensure an optimal waste management program. As discussed above, when an FP,  $W$ , has been adopted, the municipality holds the options to switch to recycling and back to landfilling. These options are particularly valuable under uncertain profit from recycling since they provide the flexibility needed to conveniently rearrange the waste disposal operations. As can be seen from equations (7.1-7.2), the value of these options depends linearly on the degree of operational flexibility which, in our setup, corresponds to the recycling capacity,  $\alpha$ . However, higher operational flexibility does not come free of cost so that, given the higher investment cost, the municipality may have to give up operational flexibility and invest in a less costly NFP,  $\widehat{W}$ .

Let us denote by  $I(\alpha)$  the sunk investment cost required to add  $W^R$  to the existing  $W^L$ . Assume that it is a function of the recycling capacity  $\alpha$  and that it takes the following convex functional form:

$$I(\alpha) = i_1\alpha + i_2\frac{\alpha^\gamma}{\gamma}, \text{ with } \gamma > 1, i_1 \geq 0 \text{ and } i_2 > 0 \quad (9)$$

where  $i_1$  and  $i_2$  are dimensional investment parameters.

The investment cost in (9) is obtained by summing two components. The first component,  $i_1\alpha$ , captures costs which are linear in the recycling capacity,  $\alpha$ , as, for instance, the cost of informing households about the new collection program, the cost of buying and providing households with specific bins for the waste fraction(s) to be collected separately, the cost of transporting waste, etc. The second component,  $i_2\frac{\alpha^\gamma}{\gamma}$ , accounts for nonlinear costs<sup>10</sup> such as the cost of keeping idle landfilling capacity when recycling or the additional cost of transports to different recycling facilities or to the landfill (see Nagurney and Toyasaki, 2005). Thus, by the second cost component, we mainly want to account for the costs directly related to the implementation of a more complex

<sup>10</sup>In general, we agree on the idea that it should be "less expensive per unit to recycle bottles and newspapers than it is to recycle bottles, newspapers, and refrigerators" (Highfill and McAsey, 2001a, p. 681).



FP allowing for both disposal alternatives.

In order to focus on the role that technological flexibility may have in the adoption of recycling, we assume, in the following, that the first linear cost component,  $i_1\alpha$ , must be paid whenever recycling is adopted while the second component,  $i_2\frac{\alpha^\gamma}{\gamma}$ , is conditional on the municipality having decided to keep the option to switch between landfilling and recycling whenever profitable, i.e., if the municipality invests in an FP. Without loss of generality we may set  $i_1 = 0$  and  $i_2 = i$  and then proceed (in order) to determine the optimal recycling capacity  $\alpha^*$ . Note that this must be done under both “Landfilling” and “Recycling” scenarios, i.e., for  $p_t \leq d$  and for  $p_t > d$ , respectively.

### 3.1 Flexible program: optimal recycling capacity under the "Landfilling" scenario

As discussed above, program  $W^L$  is preferred to  $W^R$  when the price for recycled materials,  $p_t$ , does not cover the increase in disposal costs determined by the introduction of recycling,  $d$ . This implies that by investing in an FP when  $p_t \leq d$  the municipality is only purchasing the option to adopt recycling later as soon as  $p_t > d$ . The optimal recycling capacity,  $\alpha$ , must then maximize the expected net present value,  $NPV^L(p_t, \alpha)$ , of such an option, that is,  $V^L(p_t; \alpha)$ , minus the investment cost,  $I(\alpha)$ . Formally

$$\underline{\alpha}^* = \arg \max NPV^L(p_t, \alpha) \quad \text{s.t. } 0 < \alpha \leq 1 \quad \text{for } p_t \leq d \quad (10)$$

where  $NPV^L(p_t, \alpha) = V^L(p_t; \alpha) - I(\alpha)$ .

Solving the maximization problem yields the following proposition:

**Proposition 1** *The optimal recycling capacity to be adopted in an FP when investing at  $p_t \leq d$  is*

$$\underline{\alpha}^*(p_t) = \begin{cases} \left(\frac{O^R p_t^{\theta_1}}{i}\right)^{\frac{1}{\gamma-1}} & \text{for } 0 < p_t < \underline{p} \\ 1 & \text{for } \underline{p} \leq p_t \leq d \end{cases} \quad (10.1)$$

where  $\underline{p} = \left(\frac{i}{O^R}\right)^{\frac{1}{\theta_1}}$ .

**Proof.** See section A.2 in the appendix. ■

Note that the optimal recycling capacity,  $\underline{\alpha}^*(p_t)$ , is increasing in price  $p_t$ . This makes sense, considering that a higher  $p_t$  implies a higher probability of switching to program  $W^R$  where the municipality starts recycling. Note also that  $\underline{\alpha}^*(p_t)$  is decreasing in the investment cost magnitude,  $i$ , and increasing in the parameter illustrating the convexity of the costs,  $\gamma$ , respectively.<sup>11</sup> This implies that, as expected, a higher recycling capacity is installed as investment costs drop. However, there is a ceiling for recycling capacity. Hence, there exists a price level,  $\underline{p}$ , such that for  $\underline{p} \leq p_t$  it is always worth choosing the highest feasible recycling capacity, i.e.,  $\underline{\alpha}^*(p_t) = 1$ . This means the municipality switches from a waste management program where it landfills the entire amount

<sup>11</sup>Note that  $\frac{dI(\alpha)}{d\alpha} = i\frac{\alpha^\gamma}{\gamma^2}(\gamma \ln \alpha - 1) \leq 0$  in the interval  $\alpha \in [0, 1]$ .

of collected (recyclable) waste to a program where such waste is completely recycled. It is worth noting that the higher the marginal value,  $O^R p_t^{\theta_1}$ , of the option to switch to  $W^R$ , the higher is the desired recycling capacity.

By substituting the optimal recycling capacity,  $\underline{\alpha}^*(p_t)$ , into the net present value function, we obtain:

$$NPV^L(p_t, \alpha^*(p_t)) = \begin{cases} (\frac{O^R p_t^{\theta_1}}{i})^{\frac{\gamma}{\gamma-1}} (1 - \frac{1}{\gamma}) i & \text{for } p_t < \underline{p} \\ O^R p_t^{\theta_1} - \frac{i}{\gamma} & \text{for } \underline{p} \leq p_t < d \end{cases} \quad (10.2)$$

### 3.2 Flexible program: optimal recycling capacity under the "Recycling" scenario

When  $p_t > d$ , recycling is worthwhile and the municipality adopts program  $W^R$  as soon the investment in the flexible program  $W$  has been undertaken. The optimal recycling capacity,  $\alpha$ , is given by the solution of the following problem:

$$\bar{\alpha}^* = \arg \max NPV^R(p_t, \alpha) \quad \text{s.t. } 0 < \alpha \leq 1 \quad \text{for } p_t > d \quad (11)$$

where  $NPV^R(p_t, \alpha) = V^R(p_t; \alpha) - I(\alpha)$ .

**Proposition 2** *The optimal recycling capacity to be adopted in an FP when investing at  $d < p_t < \infty$  is*

a) for  $\Psi > 0$

$$\bar{\alpha}^*(p_t) = \begin{cases} (\frac{O^L p_t^{\theta_2} + \frac{p_t}{r-\mu} - \frac{d}{r}}{i})^{\frac{1}{\gamma-1}} & \text{for } d < p_t < \bar{p} \\ 1 & \text{for } \bar{p} \leq p_t \end{cases} \quad (11.1)$$

b) for  $\Psi \leq 0$

$$\bar{\alpha}^*(p_t) = 1 \quad \text{for } p_t > d \quad (11.2)$$

where  $\Psi = i - O^R d^{\theta_1}$  and  $\bar{p} (> d)$  solves  $Q(\bar{p}) = (O^L \bar{p}^{\theta_2} + \frac{\bar{p}}{r-\mu} - \frac{d}{r}) - i = 0$ .

**Proof.** See section A.3 in the appendix. ■

We observe that  $\bar{\alpha}^*(p_t)$  is increasing in  $p_t$  in the interval  $d < p_t < \bar{p}$ . In this respect, we need to distinguish the presence of two components. First, as  $p_t$  increases, due to larger expected net benefits,  $\frac{p_t}{r-\mu} - \frac{d}{r}$ , the municipality would prefer to invest in high recycling capacity. Second,  $\bar{\alpha}^*(p_t)$  is increasing in the value of the option to switch back to landfilling, i.e.,  $O^L p_t^{\theta_2}$ . This is not surprising, considering that the option to restore  $W^L$  is an extremely valuable hedging policy against fluctuations in the net revenues from recycled waste. However, we also note that such a positive effect is decreasing in  $p_t$ . This is due to the relationship between the value of the option to switch back to landfilling and the distance between  $p_t$  and  $d$  or, differently put, the probability that once the investment has been undertaken,  $p_t$  reaches the region  $p_t < d$  where the program  $W^L$  is less costly than  $W^R$ . Accordingly, the contribution of this second component is decreasing in  $p_t$

since the higher the price of recycled materials, the less likely is the exercise of the option to switch back to  $W^L$ . Studying the impact of investment costs, it is immediate to see that, as in (11.1), the recycling capacity,  $\bar{\alpha}^*(p_t)$ , is decreasing in  $i$  and increasing in  $\gamma$ , respectively. In other words, the lower the investment cost, the higher the recycling capacity installed.

It is worth discussing the role played by the sign of the term  $\Psi = i - O^R d^{\theta_1}$ , which represents the net marginal cost of investing in full capacity. Note that this is, in fact, given by the difference between the marginal investment cost,  $I'(1) = i$ , and the marginal value of the option to switch to  $W^R$ ,  $O^R p_t^{\theta_1}$ , evaluated at the boundary  $p_t = d$ . Thus, if at  $d$  the marginal benefit,  $O^R d^{\theta_1}$ , is higher than the marginal cost,  $i$ , of investing in the last unit of feasible capacity, the municipality invests in the maximum recycling capacity. In contrast, if  $i > O^R d^{\theta_1}$  the municipality may opt for partial recycling capacity. This will occur if prices for recycled material are lower than the level  $\bar{p}$  which, according to (12.1), triggers the choice of a 100% recycling capacity ( $\bar{\alpha}^*(p_t) = 1$ ). Otherwise, again, prices may be sufficiently high to justify  $\bar{\alpha}^*(p_t) = 1$ .

Finally, by plugging the optimal recycling capacity,  $\bar{\alpha}^*(p_t)$ , into  $NPV^R$  we have:

(i) for  $\Psi > 0$ ,

$$NPV^R(p_t, \alpha^*(p_t)) = \begin{cases} \left( \frac{O^L p_t^{\theta_2} + \frac{p_t}{r-\mu} - \frac{d}{r}}{i} \right)^{\frac{\gamma}{\gamma-1}} \left(1 - \frac{1}{\gamma}\right) i & \text{for } d < p_t < \bar{p} \\ O^L p_t^{\theta_2} + \frac{p_t}{r-\mu} - \left(\frac{d}{r} + \frac{i}{\gamma}\right) & \text{for } \bar{p} \leq p_t \end{cases} \quad (11.3)$$

(ii) for  $\Psi \leq 0$ ,

$$NPV^R(p_t, \alpha^*(p_t)) = O^L p_t^{\theta_2} + \frac{p_t}{r-\mu} - \left(\frac{d}{r} + \frac{i}{\gamma}\right) \quad \text{for } d \leq p_t \quad (11.4)$$

### 3.3 Non-flexible program: optimal recycling capacity

Let us now set the optimal recycling capacity,  $\alpha$ , for an NFP  $\widehat{W}$ . Recall that in this case  $I(\alpha) = 0$ . As above, such a capacity is given by

$$\hat{\alpha}^* = \arg \max \widehat{V}(p_t; \alpha) \quad \text{s.t. } 0 < \alpha \leq 1 \quad \text{for } p_t \leq d \quad (12)$$

Note that  $\widehat{V}(p_t; \alpha) > 0$  for  $p_t > \frac{r-\mu}{r}d$ . Thus, by the linearity of  $\widehat{V}(p_t; \alpha)$  in  $\alpha$ , it is straightforward to show that this is the case.<sup>12</sup>

**Proposition 3** *The optimal recycling capacity to be adopted when investing in an NFP is*

$$\hat{\alpha}^*(p_t) = 1 \quad \text{for } p_t > \frac{r-\mu}{r}d \quad (12.1)$$

As expected, the municipality chooses the maximum recycling capacity if the expected net benefits from recycling,  $\frac{p_t}{r-\mu} - \frac{d}{r}$ , are positive and no capacity at all otherwise. The expected

<sup>12</sup>Note that having assumed  $i_1 = 0$  does not affect our results. In fact, even allowing for a more general concave investment function in this case,  $I(\alpha) = i_1 \alpha^\omega$  with  $i_1 > 0$  and  $\omega \leq 1$ , the municipality would still have invested in the highest feasible recycling capacity ( $\hat{\alpha}^*(p_t) = 1$ ).

present value of  $\widehat{W}$  is then given by

$$\widehat{V}(p_t; \widehat{\alpha}^*(p_t)) = \frac{p_t}{r - \mu} - \frac{d}{r}, \text{ for } \frac{r - \mu}{r}d < p_t \quad (12.2)$$

## 4 Investment Value and Adoption Timing

In this section, we study the timing of the investment in an optimal waste management program. To this end, we derive the value of the option to invest and then determine the conditions characterizing an optimal investment time strategy.

First, let us define by  $W^*$  and  $\widehat{W}^*$  the flexible and the non-flexible program where the recycling capacity has been set at its optimal level,  $\alpha^*(p_t)$ . Second, we consider the option to invest in the continuation region  $0 < p_t < \tilde{p}$  where  $\tilde{p}$  is the price threshold triggering investment. The value of such an option is given by

$$F(p_t) = \max_{\tau} E\{e^{-r\tau} NPV^k(p_{\tau})\}, \text{ with } k = \{W^*, \widehat{W}^*\} \quad (13)$$

where  $\tau = \inf\{t \geq 0 \mid p_t = \tilde{p}\}$  is the optimal investment stopping time and

$$NPV^{W^*}(p_t) = \begin{cases} NPV^L(p_t, \alpha^*(p_t)) & \text{for } p_t < d \\ NPV^R(p_t, \bar{\alpha}^*(p_t)) & \text{for } d \leq p_t \end{cases} \quad (13.1)$$

$$NPV^{\widehat{W}^*}(p_t) = \widehat{V}(p_t; \widehat{\alpha}^*(p_t)), \text{ for } \frac{r - \mu}{r}d < p_t \quad (13.2)$$

The problem can be rearranged as follows:<sup>13</sup>

$$F(p_t) = \max_{\tilde{p}} \left[ \left( \frac{p_t}{\tilde{p}} \right)^{\theta_1} NPV^k(\tilde{p}) \right] \quad (14)$$

From the first-order condition of the maximization problem<sup>14</sup> we obtain

$$\tilde{p} = \theta_1 \frac{NPV^k(\tilde{p})}{\frac{\partial NPV^k(\tilde{p})}{\partial \tilde{p}}} \quad (14.1)$$

Finally, the definition of a maximum requires that the following second-order condition should hold at  $\tilde{p}$ :

$$\frac{\tilde{p}}{\theta_1 - 1} \frac{\partial^2 NPV^k(\tilde{p})}{\partial \tilde{p}^2} < \frac{\partial NPV^k(\tilde{p})}{\partial \tilde{p}} \quad (14.2)$$

Let us now study the investment policy under both scenarios, "Recycling" in Section 4.1 and "Landfilling" in Section 4.2.

<sup>13</sup>For the calculation of expected present values, see Dixit and Pindyck (1994, pp. 315-316).

<sup>14</sup>See appendix A.4.

#### 4.1 Investment in a flexible program under the "Recycling" scenario

We consider the option to invest in the subset  $d \leq p_t$  where  $W^R \succ W^L$ . In this case, the recycling capacity would be used as soon as the municipality has invested in  $W^*$ . The desired degree of flexibility,  $\bar{\alpha}^*$ , will be chosen taking into account price volatility and the magnitude of investment costs,  $i$ . In addition, the municipality will choose it, aware that it would still be possible to switch back to  $W^L$ . As discussed above, this consideration should favor investment in a higher recycling capacity.

By using Proposition 2 we can distinguish between two cases in terms of adopted recycling capacity. This will depend on the comparison between the magnitude of the investment cost,  $i$ , and the value of the option to switch to recycling,  $O^R d^{\theta_1}$ . We start by considering case b) where  $\Psi \leq 0$  and the municipality opts for the highest feasible level of flexibility, i.e.,  $\bar{\alpha}^* = 1$ . Substituting (11.4) into (14.1), we obtain the following result:

**Proposition 4** *When  $\Psi \leq 0$  the optimal investment threshold,  $p^*(\geq d)$ , for the adoption of an FP with  $\bar{\alpha}^* = 1$  is given by the solution of the following equation*

$$p^* + \frac{\theta_1 - \theta_2}{\theta_1 - 1} O^L p^{*\theta_2} (r - \mu) - \frac{\theta_1}{\theta_1 - 1} (r - \mu) \frac{i}{\gamma} = \hat{p}^* \quad (15)$$

where  $\hat{p}^* = \frac{\theta_1}{\theta_1 - 1} (r - \mu) \frac{d}{r}$ .

**Proof.** See section A.5 in the appendix. ■

According to Proposition 4, it is worth investing at  $p^* \leq p_t$ . In equation (15),  $\hat{p}^*$  represents the investment threshold for the investment in an NFP, i.e.,  $\hat{p}^* = \arg \max[(\frac{p}{p})^{\theta_1} NPV^{\widehat{W}}(\hat{p})]$ . It is immediate to see that the investment threshold  $p^*$  depends not only on the present value of the disposal cost differential,  $\frac{d}{r}$ , investment cost,  $\frac{i}{\gamma}$ , and on the standard option value considerations<sup>15</sup> but also on the presence of landfilling as a waste management option. This effect is captured by the second term on the RHS of (15). As shown in section A.5,  $\frac{\partial p^*}{\partial O^L} < 0$ . In other words, the more valuable the option to switch back to  $W^L$ , the earlier, in expected terms, the flexible program  $W^*$ , is adopted. Rearranging (15) as follows

$$p^* - \hat{p}^* = \left(\frac{1}{2}\sigma^2\theta_1 + r\right) \frac{i}{\gamma} - \frac{\theta_1 - \theta_2}{\theta_1 - 1} O^L p^{*\theta_2} (r - \mu) \quad (15.1)$$

in Eq. (15.1), the first term of the RHS accounts for the higher investment cost,  $\frac{i}{\gamma}$ , to be paid in order to have a flexible program allowing for both recycling and landfilling. As expected, given that such a program is more costly than a non-flexible one, the municipality should wait longer before investing. Waiting before making an investment takes even longer under uncertainty. Note, in fact, that the first term,  $\frac{1}{2}\sigma^2\theta_1$ , must be added to the user cost per unit of capital,  $r$ , to account

<sup>15</sup>The investment timing should account for the option value arising from new information about the variables affecting the profitability of the investment decision. This consideration implies a higher investment threshold with respect to the one set under the standard NPV approach. See Dixit and Pindyck (1994, chap. 5).

for uncertainty. However, this effect is balanced by the second term in the RHS of (15.1), which represents the option to landfill whenever recycling is less profitable than landfilling. This clearly reduces the uncertainty characterizing the investment. Note that the second effect may also prevail, leading to a faster adoption of the flexible program. This will occur if the following condition is met:

$$O^L p^{*\theta_2} > \left(1 + \frac{\theta_2}{\theta_1 - \theta_2}\right) \frac{i}{\gamma} \quad (15.2)$$

that is, whenever the value of the option to landfill evaluated at  $p^*$  covers a portion of the investment cost  $\frac{i}{\gamma}$  (note that  $1 + \frac{\theta_2}{\theta_1 - \theta_2} < 1$ ).

We now proceed by studying case a) in Proposition 2 where  $\Psi > 0$ . In contrast with the previous case, the municipality may invest in a flexible program  $W^*$  with a lower degree of flexibility, i.e.,  $\bar{\alpha}^* < 1$ , due to the higher investment cost. This should allow a faster adoption of the flexible program by trading off the degree of flexibility with the initial investment cost. Note that not investing in a 100% recycling program ( $\bar{\alpha}^* = 1$ ) makes sense if  $p_t$  is likely to take low values or fall below  $d$ , but this may be regretted when prices for recycled material are high and well above  $d$  since the municipality will not be able to exploit the entire potential of the recycling strategy.

In the appendix, we show that

**Proposition 5** *When  $\Psi > 0$  and provided that*

$$\frac{\bar{p}}{r - \mu} < \Lambda \text{ and } \gamma > \frac{\theta_1}{\theta_1 - 1},$$

*the optimal investment threshold,  $p^{**} (\geq d)$ , for the adoption of an FP with  $\bar{\alpha}^* < 1$  is given by the solution of the following equation*

$$p^{**} + \frac{[\theta_1(\gamma - 1) - \gamma\theta_2]O^L p^{**\theta_2} - \frac{\hat{p}^*}{r - \mu}}{\theta_1(\gamma - 1) - \gamma}(r - \mu) = \hat{p}^* \quad (16)$$

where  $\Lambda = \frac{\theta_2}{\theta_2 - 1} \frac{d}{r} - \frac{\theta_1(\gamma - 1) - \gamma\theta_2}{\theta_2 - 1} \frac{i}{\gamma}$ .

**Proof.** See section A.6 in the appendix. ■

According to Proposition 5, the municipality should invest in the region where  $p^{**} \leq p_t \leq \bar{p}$ . Note that investments in programs with  $\bar{\alpha}^* < 1$  are undertaken when, at  $\bar{p}$ , the expected present value of earnings,  $\frac{\bar{p}}{r - \mu}$ , is not sufficiently high to cover level  $\Lambda$  which, as we show in the appendix, triggers investment in programs with full recycling capacity. By rearranging (16), we obtain

$$p^{**} - \hat{p}^* = \frac{\frac{\hat{p}^*}{r - \mu} - [\theta_1(\gamma - 1) - \gamma\theta_2]O^L p^{**\theta_2}}{\theta_1(\gamma - 1) - \gamma}(r - \mu) \quad (16.1)$$

Again, we stress the role played by the option to switch back to  $W^L$ . In fact,) in this case too<sup>16</sup>  $\frac{\partial p^{**}}{\partial O^L} < 0$ . This implies that also when  $\bar{\alpha}^* < 1$ , the higher the value of the option to restore  $W^L$ ,

<sup>16</sup>See section A.6 in the appendix.

the faster, in expected terms, the flexible program  $W^*$  is introduced.

By comparing the investment in a lower recycling capacity with the investment in a program providing 100% recycling capacity but no option to restore  $W^L$ , we notice that the former may be undertaken earlier if the following condition holds:

$$O^L p^{**\theta_2} > \frac{\frac{\widehat{p}^*}{r-\mu}}{\theta_1(\gamma-1) - \gamma\theta_2} \quad (16.2)$$

that is, when at  $p^{**}$  the value of the option to restore  $W^L$  is higher than a percentage  $\frac{1}{\theta_1(\gamma-1) - \gamma\theta_2} < 1$  of the expected discounted value of revenues accruing from the alternative waste disposal program, namely,  $\frac{\widehat{p}^*}{r-\mu}$ .

We complete our analysis by showing that

**Proposition 6** *When  $\Psi > 0$  and provided that  $\frac{\bar{p}}{r-\mu} \geq \Lambda$ , the optimal investment threshold  $p^{**} (\geq \bar{p})$  for the adoption of an FP with  $\bar{\alpha}^* = 1$  is*

$$p^{**} = p^* \quad (17)$$

**Proof.** See section A.5 in the appendix. ■

The above discussion also applies here. However, note that, since  $\Psi > 0$ , the investment in a management program with full recycling capacity occurs only if at  $\bar{p}$  the expected present value of earnings,  $\frac{\bar{p}}{r-\mu}$ , is higher than level  $\Lambda$ . In fact, if this condition is met, investment occurs at a price level high enough to cover the investment cost.

Finally, by focusing only on the case<sup>17</sup> where  $\bar{\alpha}^*(p^{*(*)}) = 1$  we can also show that a higher expected net present value corresponds to an earlier investment,  $p^{*(*)} < \widehat{p}^*$ . In fact, by comparing at  $p^*$  investment in  $W^*$  with investment in  $\widehat{W}^*$ , it is immediate to show that to conclude our discussion on the relationship between investment

**Proposition 7** *If  $O^L p^{*\theta_2} > \Omega$  then*

$$NPV^{W^*}(p^*) > (p^*/\widehat{p}^*)^{\theta_1} NPV^{\widehat{W}^*}(\widehat{p}^*) \quad (18)$$

where  $\Omega = \frac{d}{r}[1 - (p^*/\widehat{p}^*)^{\theta_1}] - \frac{p^* - \widehat{p}^*(p^*/\widehat{p}^*)^{\theta_1}}{r-\mu} + \frac{i}{\gamma}$ .

In words, this means that a  $W^*$  program guarantees earlier investment and a higher expected net present value when, at the investment time, the value of the option to landfill,  $O^L p^{*\theta_2}$ , covers the sum of investment cost,  $\frac{i}{\gamma}$ , plus the difference between the discounted expected value of the flow of increased disposal costs,  $\frac{d}{r}[1 - (p^*/\widehat{p}^*)^{\theta_1}]$ , and the discounted expected value of the flow of revenues,  $\frac{p^* - \widehat{p}^*(p^*/\widehat{p}^*)^{\theta_1}}{r-\mu}$ , from recycling. Note that both flows start at  $p^*$  and are consistently discounted, taking into account the random time period needed for reaching price level  $\widehat{p}^*$ .

<sup>17</sup>Note that one may easily show that a similar result holds also for the case where  $\bar{\alpha}^*(p^{**}) < 1$ .

## 4.2 Investment in a flexible program under the "Landfilling" scenario

In this section, we examine the investment strategy for the range of values  $p_t < d$  where  $W^L \succ W^R$ . In this range, even if landfilling is still profitable, the municipality may consider the possibility of adopting a flexible waste management program  $W^*$  so that it can switch to  $W^R$  as soon as  $p_t \geq d$ . As we show in appendix A.7, this never occurs. In fact, we prove that

**Proposition 8** *If  $p_t < d$ , the municipality never invests in a flexible waste management program with  $\alpha^* \leq 1$ .*

**Proof.** See section A.7 in the appendix. ■

By plugging (10.2) into (14.2) it is straightforward to show that the second-order condition is violated. This implies that an investment time trigger maximizing (14) does not exist in the range of prices considered. As can be easily seen in (10.2), the option to switch to  $W^R$ , even if valuable, is not worth the investment cost when  $p_t < d$ . Its expected net present value is increasing in  $p_t$  as the probability of a switch increases so that the municipality prefers to postpone the investment. Finally, note also that differently from the option to switch to  $W^L$ , the option to switch to  $W^R$  does not pay any "dividend" when held.

## 4.3 Probability and expected time of adoption

As explained above, due to the presence of uncertainty, the municipality may keep open the option to invest for long time periods. In this respect, it is important, in terms of informing policy makers, to determine, at least in expected terms, the length of such a period. In the following, we will therefore first determine the probability of adoption within a particular time period in the future and then the expected time of adoption.

For this purpose we first remember that investment occurs at stopping time  $\tau = \inf\{t \geq 0 \mid p_t = \tilde{p}\}$ . Note that as prices for recycled materials follow the stochastic process (1),  $\tau$  also becomes a stochastic variable. Hence, denoting by  $T$  the time period in which a first passage through barrier  $\tilde{p}$  may occur, the cumulative probability of investment is given by the following function (see Harrison, 1985, p.14):

$$G(p, T; \tilde{p}) = N \left[ \frac{-\ln(\tilde{p}/p) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right] + e^{2\ln(\tilde{p}/p)(\mu - \frac{1}{2}\sigma^2)/\sigma^2} N \left[ \frac{-\ln(\tilde{p}/p) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right] \quad (19)$$

where  $N[\cdot]$  is the cumulative standard normal distribution.

Following Dixit (1993), the probability of an eventual investment (i.e.,  $T \rightarrow \infty$ ) is

$$G(p, \infty; \tilde{p}) = \begin{cases} 1 & \text{for } \mu \geq \frac{1}{2}\sigma^2 \\ e^{2\ln(\tilde{p}/p)(\mu - \frac{1}{2}\sigma^2)/\sigma^2} & \text{for } \mu < \frac{1}{2}\sigma^2 \end{cases} \quad (20)$$



while the expected investment time is given by

$$E(\tau; p, \tilde{p}) = \begin{cases} \ln(\tilde{p}/p)/(\mu - \frac{1}{2}\sigma^2) & \text{for } \mu > \frac{1}{2}\sigma^2 \\ \infty & \text{for } \mu \leq \frac{1}{2}\sigma^2 \end{cases} \quad (21)$$

Note that for  $\mu \leq \frac{1}{2}\sigma^2$  the expected time is  $\infty$ . This is due to the drift which drives the price away from barrier  $\tilde{p}$ . Since, by (20), the probability of hitting the barrier is lower than one, the expected time tends to  $\infty$  for  $T \rightarrow \infty$ .

Finally, in order to assess the relative profitability of a flexible program with respect to a non-flexible one, we derive from (18) the following measure:

$$\pi(x, \hat{p}^*) = \begin{cases} \frac{NPV^{W^*}(x)}{(\frac{x}{\hat{p}^*})^{\theta_1} NPV^{\widehat{W}^*}(\hat{p}^*)} & \text{for } x < \hat{p}^* \\ \frac{(\frac{\hat{p}^*}{x})^{\theta_1} NPV^{W^*}(x)}{NPV^{\widehat{W}^*}(\hat{p}^*)} & \text{for } x \geq \hat{p}^* \end{cases}, \text{ with } x = \{p^*, p^{**}\}. \quad (22)$$

By (22) we are basically comparing the expected present values of both possible programs evaluated at the same time period. In particular, the comparison occurs at the earliest investment time for the two projects.<sup>18</sup> For instance, if the flexible program is adopted earlier, i.e.,  $x < \hat{p}^*$ , the expected present value of the non-flexible program,  $NPV^{\widehat{W}^*}(\hat{p}^*)$ , is discounted back to  $x$  by using the stochastic discount factor,  $(\frac{x}{\hat{p}^*})^{\theta_1}$ . In contrast, if the adoption of the non-flexible program occurs earlier, i.e.,  $x \geq \hat{p}^*$ , then  $NPV^{W^*}(x)$  is discounted back to  $\hat{p}^*$ . Thus,  $\pi(x, \hat{p}^*)$  indicates, in percentage terms, the value of a flexible program when compared to the value of a non-flexible program. Finally, note that by using the measure  $\pi(x, \hat{p}^*)$  for the comparison, we are taking into account the difference in the probability of adoption and in the expected investment time through the stochastic discount factor  $(\frac{x}{\hat{p}^*})^{\theta_1}$ .

## 5 Numerical Examples

In this section, we use some numerical examples to illustrate the effect of operational flexibility on the investment timing and on the value of a waste disposal program. We start our calculations by setting  $d = 1$ . This is equivalent to normalizing our frame with respect to the additional cost of disposal incurred when recycling.<sup>19</sup> By using (1), it is straightforward to show that the ratio  $q_t = p_t/d$  evolves according to the following Brownian motion:

$$\frac{dq_t}{q_t} = \mu dt + \sigma dz_t, \text{ with } q_0 = q = \frac{p}{d} \quad (23)$$

Note that recycling is consistently profitable for any  $q > 1$  and not profitable otherwise.

In our exercise, we let the other parameters vary as follows:

<sup>18</sup>Clearly, nothing would change if we chose the latest investment time.

<sup>19</sup>Note that we could have normalized the setting from the beginning by assuming that  $q_t = p_t/d_t$  with  $d_t > 0$  evolves according to  $dq_t = \mu q_t dt + \sigma q_t dz_t$ . However, we preferred to keep  $d_t$  since it guarantees a clearer presentation and discussion of our results.

1. **Price trend and volatility** - We let drift,  $\mu$ , and volatility,  $\sigma$ , take values  $\{0, 0.025\}$  and  $\{0.1, 0.2, 0.3\}$ , respectively. Note that a positive drift may capture both an expected increase in the price for recycled material and/or an expected decrease in the gap between recycling and landfilling operating costs. A driftless motion simply implies that, even if fluctuating over time, ratio  $q_t$  on average takes the initial value  $q$ . See Table 2. In each table, we allow for increasing levels of volatility in order to check the impact of uncertainty on the choice of the program and on the investment policy. In particular, we expect that under higher uncertainty i) the municipality prefers a higher degree of flexibility, and ii) investment in a costly flexible program should be delayed. As a result, the municipality must trade off flexibility with investment delay. In light of this trade-off, it is of interest, however, to check how a flexible program performs compared to a non-flexible one.
2. **Investment cost** - We consider the impact of investment cost by letting its magnitude,  $i$ , and convexity,  $\gamma$ , take values  $\{2.5, 5, 10\}$  and  $\{2, 4\}$ , respectively. As discussed above, higher  $i$  and lower  $\gamma$  will likely make a flexible program less desirable since the marginal investment cost of flexibility increases. See Tables 1 and 4.
3. **Interest rate** - We set the interest rate,  $r$ , equal to 5% in our calculations. We then check for the effect of a variation by raising it to 10%. It is immediate to see that when investing in a flexible program, this variation implies, *ceteris paribus*, a higher user cost of capital. Hence, in order to invest, the flexible program must yield higher returns. With a higher interest rate, investment is likely to occur when  $q$  is high enough to cover the higher opportunity cost of capital. See Table 3.

In our calculations, we set 1 as initial level for  $q$ . This means that the additional cost of recycling is covered by revenues from the sale of recycled materials. Note that 1) as discussed above, this does not necessarily trigger investment in any of programs under analysis, and 2) this assumption does not influence the comparison between a flexible and a non-flexible program. In this respect, recall that when comparing the two programs on the basis of probability of investment and expected investment timing, what matters is the temporal distance between the optimal investment thresholds set for each program. We compare the probabilities of eventual investment and investment within 10 years ( $T = 10$ ) with or without flexibility.

In Table 1, we set  $\mu = 0.025$ ,  $r = 0.05$ , and  $\gamma = 2$  and we let vary the investment cost magnitude,  $i$ , and the volatility,  $\sigma$ . We observe that within a flexible program, the municipality would always prefer to purchase the maximum feasible recycling capacity, i.e.,  $\bar{\alpha}^* = 1$ . Furthermore, in the majority of cases the investment in such a program would occur earlier than the investment in a non-flexible program. In particular, we note that this is always the case for the highest level of volatility. This makes sense, considering that under highly uncertain recycling profitability, the municipality may fully exploit the potential of the hedging policy adopted by investing in a flexible program. This consideration is also supported by the performance of the flexible program in terms of expected investment time and value. In fact, we observe that the presence of flexibility may induce

important reductions in the length of time needed for eventual investment. Note that  $\pi(x, \hat{p}^*)$  is always increasing in the level of uncertainty. Interestingly, the flexible program may perform even better in some cases, for instance if  $i = 5$  and  $\sigma = 0.3$ . In this case, the value attached to the flexible program is equivalent to 105% of the value of a non-flexible one. Not surprisingly,  $\pi(x, \hat{p}^*)$  is decreasing in  $i$ , i.e., the higher the investment cost of the flexible program, the lower the net benefit in comparison to the non-flexible one. In general, however, even when  $\pi(x, \hat{p}^*)$  is lower than 1, we note that the score is quite high. This is particularly noteworthy when considering that a non-flexible program is actually costless in terms of investment ( $i = 0$ ). Finally, note that the investment thresholds for both programs are increasing in  $\sigma$ . This is due to the presence of uncertainty and irreversibility which, as is standard in the real option literature, requires a more prudent investment policy. Investment should, in fact, occur at higher price levels. This, in turn, requires waiting longer before investing. This period of inaction may become infinitely long in the presence of a weak expected growth in the net benefit of recycling ( $q$ ). In Table 1, this is actually the case for  $\sigma = 0.3$ . This figure must be analyzed in combination with the probability of investment which, up to Table 1, markedly increases when investing in flexibility.

TABLE 1

The Effect of Investment Cost on Timing and Program Value

		$\sigma = 0.1$		$\sigma = 0.2$		$\sigma = 0.3$	
<b><math>i=2.5</math></b>							
$\bar{p}$	$\bar{\alpha}^*$	-	1	-	1	-	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.18	1.17	1.32	1.58	1.46	2.17
$G(p, t; \hat{p})$		78.37	79.12	68.34	49.38	63.19	34.52
$G(p, \infty; \hat{p})$		100	100	100	100	85	71
$E(\tilde{t}; p, \hat{p})$		8.29	8.03	55.53	92.04	$\infty$	$\infty$
$NPV^{W^*}$	$NPV^{\bar{W}^*}$	26.23	26.97	33.66	43.38	41.77	66.78
$\pi(x, \hat{p}^*)$		0.96		1.01		1.05	
<b><math>i=5</math></b>							
$\bar{p}$	$\bar{\alpha}^*$	-	1	-	1	-	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.28	1.17	1.49	1.58	1.70	2.17
$G(p, t; \hat{p})$		65.47	79.12	55.77	49.38	50.95	34.52
$G(p, \infty; \hat{p})$		100	100	100	100	79	71
$E(\tilde{t}; p, \hat{p})$		12.28	8.03	79.16	92.04	$\infty$	$\infty$
$NPV^{W^*}$	$NPV^{\bar{W}^*}$	28.80	26.97	38.65	43.38	49.49	66.78
$\pi(x, \hat{p}^*)$		0.92		0.98		1.02	
<b><math>i=10</math></b>							
$\bar{p}$	$\bar{\alpha}^*$	-	1	-	1	-	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.45	1.17	1.76	1.58	2.09	2.17
$G(p, t; \hat{p})$		45.34	79.12	39.90	49.38	39.90	34.52
$G(p, \infty; \hat{p})$		100	100	100	100	72	71
$E(\tilde{t}; p, \hat{p})$		18.47	8.03	112.82	92.04	$\infty$	$\infty$
$NPV^{W^*}$	$NPV^{\bar{W}^*}$	32.95	26.97	46.61	43.38	62.02	66.78
$\pi(x, \hat{p}^*)$		0.85		0.92		0.97	

for  $\mu = 0.025$ ,  $r = 0.05$ ,  $T = 10$  and  $\gamma = 2$

In Table 2, we focus on the impact that a change in the expected trend,  $\mu$ , may have. We note that

in the majority of cases, 1) the municipality prefers to invest in the maximum recycling capacity and 2) investment occurs earlier in the presence of flexibility. The expected time of adoption is decreasing in  $\mu$ . In particular,  $E(\tau; p, \tilde{p}) = \infty$  for  $\mu = 0$ . As explained above, this is due to the fact that there is a positive probability of never hitting the investment threshold, i.e.,  $G(p, \infty; \tilde{p}) < 1$ . When considering a flexible program, there are, however, marked gains in terms of probability of adoption as uncertainty rises. The value attached to the disposal program is increasing in  $\mu$ . Higher *NPV* are attached to higher growth in recycling profitability. Note also that for high levels of uncertainty the flexible program performs better when the drift is null (higher  $\pi(x, \hat{p}^*)$ ). This makes sense, considering that if the recycling profitability is expected to grow, the hedging policy available within a flexible program is less valuable. This is due to the lower likelihood of  $q$  falling below 1. In contrast, the activation of the hedging policy is more likely when  $\mu = 0$ . Finally, the positive relationship between  $\pi(x, \hat{p}^*)$  and  $\sigma$  holds also for  $\mu = 0$ .

TABLE 2

The Effect of a Change in the Drift on Timing and Program Value

		$\sigma = 0.1$		$\sigma = 0.2$		$\sigma = 0.3$	
$\mu = 0.025$							
$\tilde{p}$	$\bar{a}^*$	-	1	-	1	-	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.28	1.17	1.49	1.58	1.70	2.17
$G(p, t; \tilde{p})$		65.47	79.12	55.77	49.38	50.95	34.52
$G(p, \infty; \tilde{p})$		100	100	100	100	79	71
$E(\tilde{\tau}; p, \tilde{p})$		12.28	8.03	79.16	92.04	$\infty$	$\infty$
$NPV^{W^*}$	$NPV^{\tilde{W}^*}$	28.80	26.97	38.65	43.38	49.49	66.78
$\pi(x, \hat{p}^*)$		0.92		0.98		1.02	
$\mu = 0$							
$\tilde{p}$	$\bar{a}^*$	1.14	1	-	1	-	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.39	1.37	1.59	1.86	1.80	2.5
$G(p, t; \tilde{p})$		25.08	27.12	35.98	23.33	38.27	19.95
$G(p, \infty; \tilde{p})$		72	73	63	54	56	40
$E(\tilde{\tau}; p, \tilde{p})$		$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
$NPV^{W^*}$	$NPV^{\tilde{W}^*}$	6.57	7.40	12.87	17.27	19.26	30
$\pi(x, \hat{p}^*)$		0.84		1.05		1.11	

for  $i = 5$ ,  $r = 0.05$ ,  $T = 10$  and  $\gamma = 2$

In Table 3, in order to isolate the effect of the interest rate,  $r$ , we set  $\mu = 0$ . We note that by raising  $r$  to 10%, we have two opposite effects on the investment thresholds set for both feasible programs. In fact, while the investment in a flexible program is postponed, the investment in a non-flexible program is anticipated. To explain this difference, again, recall that the investment in a non-flexible program occurs at no cost ( $i = 0$ ). In addition, the current value of future payoffs decreases since they are discounted at a higher rate. This easily explains the rush in investing in a non-flexible program. In contrast, when investing in a flexible program, the effect of discounting on future payoffs is balanced by the effect of a higher user cost for the capital needed to purchase flexibility. In Table 3, the second effect is prevailing in every scenario. This clearly has a huge

impact on the performance of the flexible program when compared to a non-flexible one. Note that  $\pi(x, \hat{p}^*)$  is decreasing in  $r$  and the flexible program performs poorly for  $\sigma = 0.1$ . As above, the performance improves, as uncertainty rises and reaches 90% for  $\sigma = 0.3$ . This confirms again the relevance of flexibility in terms of program value. Similarly, note also that, despite its higher cost, a full recycling capacity is chosen.

TABLE 3

The Effect of a Change in the Interest Rate on Timing and Program Value

		$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$			
$r = 0.1$							
$\bar{p}$	$\bar{\alpha}^*$	1.48	1	1.38	1	1.25	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.51	1.25	1.74	1.56	1.96	1.93
$G(p, t; \hat{p})$		15.55	42.75	28.21	37.80	32.60	33.57
$G(p, \infty; \hat{p})$		66	80	57	64	51	52
$E(\bar{t}; p, \hat{p})$		$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
$NPV^{w^*}$	$NPV^{\bar{w}^*}$	2.86	2.5	5.72	5.58	8.68	9.32
$\pi(x, \hat{p}^*)$		0.44		0.75		0.90	
$r = 0.05$							
$\bar{p}$	$\bar{\alpha}^*$	1.14	1	-	1	-	1
$p^{*(\cdot)}$	$\hat{p}^*$	1.39	1.37	1.59	1.86	1.80	2.5
$G(p, t; \hat{p})$		25.08	27.12	35.98	23.33	38.27	19.95
$G(p, \infty; \hat{p})$		72	73	63	54	56	40
$E(\bar{t}; p, \hat{p})$		$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
$NPV^{w^*}$	$NPV^{\bar{w}^*}$	6.57	7.40	12.87	17.27	19.26	30
$\pi(x, \hat{p}^*)$		0.84		1.05		1.11	

for  $i = 5$ ,  $\mu = 0$ ,  $T = 10$  and  $\gamma = 2$

Finally, we study the effect of investment cost convexity,  $\gamma$ . Recall that in the interval  $\alpha \in (0, 1]$  the investment cost is decreasing in  $\gamma$ . In the three scenarios represented in Table 4, the municipality would always install full recycling capacity within the flexible program. Thus, the effect of  $\gamma$  is highly similar to the effect of a higher  $i$ . As  $\gamma$  decreases, the investment in a flexible program is postponed, and its relative profitability,  $\pi(x, \hat{p}^*)$ , drops. Note that in Table 4 we set  $r = 0.1$  so that, not surprisingly, the investment in a non-flexible program occurs earlier in the majority of cases. It is, however, worth pointing out the strength of a flexible program which improves its performance in terms of  $\pi(x, \hat{p}^*)$  as uncertainty increases.

TABLE 4

The Effect of a Change in the Investment Cost Convexity on Timing and Program Value

		$\sigma = 0.1$		$\sigma = 0.2$		$\sigma = 0.3$	
$\gamma = 1.5$							
$\bar{p}$	$\bar{\alpha}^*$	1.11	1	1.03	1	1	1
$p^{*(c)}$	$\hat{p}^*$	1.52	1.14	1.79	1.42	2.06	1.78
$G(p, t; \hat{p})$		38.05	82.92	38.34	60.24	37.50	47.48
$G(p, \infty; \hat{p})$		100	100	100	100	72	77
$E(\bar{t}; p, \hat{p})$		20.90	6.77	116.50	70.54	$\infty$	$\infty$
$NPV^{w^*}$	$NPV^{\bar{w}^*}$	6.93	5.27	10.89	8.97	15.18	13.71
$\pi(x, \hat{p}^*)$		0.58		0.75		0.86	
$\gamma = 2$							
$\bar{p}$	$\bar{\alpha}^*$	1.11	1	1.03	1	1	1
$p^{*(c)}$	$\hat{p}^*$	1.42	1.14	1.65	1.42	1.88	1.78
$G(p, t; \hat{p})$		48.48	82.92	45.61	60.24	43.71	47.48
$G(p, \infty; \hat{p})$		100	100	100	100	76	77
$E(\bar{t}; p, \hat{p})$		17.47	6.77	100.03	70.54	$\infty$	$\infty$
$NPV^{w^*}$	$NPV^{\bar{w}^*}$	6.44	5.27	9.97	8.97	13.63	13.71
$\pi(x, \hat{p}^*)$		0.66		0.81		0.91	
$\gamma = 2.5$							
$\bar{p}$	$\bar{\alpha}^*$	1.11	1	1.03	1	1	1
$p^{*(c)}$	$\hat{p}^*$	1.36	1.14	1.56	1.42	1.76	1.78
$G(p, t; \hat{p})$		55.72	82.92	50.91	60.24	48.37	47.48
$G(p, \infty; \hat{p})$		100	100	100	100	78	77
$E(\bar{t}; p, \hat{p})$		15.23	6.77	88.89	70.54	$\infty$	$\infty$
$NPV^{w^*}$	$NPV^{\bar{w}^*}$	6.13	5.27	9.28	8.97	12.64	13.71
$\pi(x, \hat{p}^*)$		0.71		0.85		0.94	

for  $i = 5$ ,  $\mu = 0.025$ ,  $T = 10$  and  $r = 0.1$

## 6 Conclusion

The design of a waste management program is a crucial choice for a municipality. This must be made by taking into account, on the one hand, the constraints fixed on the disposal methods available and, on the other hand, the economic profitability of the adopted program. Despite the increasing popularity of recycling programs, their actual economic profitability remains weak. This is mainly due to the level of prices paid for recycled materials, which, for some specific materials, do not even cover the cost of recycling. In addition, investment in recycling capacity is also discouraged by the volatility characterizing the dynamic of prices in secondary markets for raw materials.

In our paper, we propose to hedge against uncertainty about profit from recycling by operating a flexible waste management program allowing for the use of landfilling whenever recycling is not profitable. The landfilling disposal strategy, which is generally less costly than recycling, could then act as a buffer in periods when prices for recycled materials are too low. Clearly, the operation of a more complex waste management program, allowing for the option to restore landfilling whenever needed, may impose additional investment costs on the municipality. However, we have shown that

the value of operational flexibility may cover the additional cost and have a positive impact on the decision to invest in recycling capacity. This is mainly explained by the presence of flexibility, which may substantially reduce uncertainty about recycling profitability. This positive effect may lead to two interesting results. First, investment in recycling within a flexible program may occur earlier than within a non-flexible program. Second, it may result in a higher expected net present value.

## A Appendix

### A.1 The value of the flexible program

The general solution to the differential equations (5) and (6) takes the form:<sup>20</sup>

$$\begin{aligned} V^L(p; \alpha) &= O_1^R p^{\theta_1} && \text{for } p_t < d \\ V^R(p; \alpha) &= O_2^L p^{\beta_2} + \alpha \left( \frac{p}{r-\mu} - \frac{d}{r} \right) && \text{for } p_t > d \end{aligned} \quad (\text{A.1.1-A.1.2})$$

where  $\theta_1 > 1$  and  $\theta_2 < 0$  are the roots of the characteristic equation  $\phi(\theta) = \frac{1}{2}\sigma^2\theta(\theta - 1) + \mu\theta - r = 0$ . In (5) and (6) the first term indicates the value of the option to switch to the alternative operational arrangement. In (6) the second term represents the value of benefits (cost reduction) accruing operating forever under regime  $W^R$ . Clearly, since the availability of strategic options always increases the value of a project, the constants  $O_1^{LR}$  and  $O_2^L$  must be non-negative.<sup>21</sup>

At  $p_t = d$ , the standard pair of conditions for an optimal waste disposal policy must hold. That is, value-matching

$$V^L(d, \alpha) = V^R(d, \alpha) \quad (\text{A.1.3})$$

and the smooth-pasting

$$V_p^L(d, \alpha) = V_p^R(d, \alpha). \quad (\text{A.1.4})$$

Solving the program [A.1.3-A.1.4] yields

$$\begin{aligned} O_1^R &= \alpha \frac{d^{1-\theta_1}}{r} \frac{r - \mu\theta_2}{(r - \mu)(\theta_1 - \theta_2)} = \alpha O^R \\ O_2^L &= \alpha \frac{d^{1-\theta_2}}{r} \frac{r - \mu\theta_1}{(r - \mu)(\theta_1 - \theta_2)} = \alpha O^L \end{aligned}$$

### A.2 Proof of Proposition 1

Suppose that  $p_t < d$ . The optimal share of recycling is given by

$$\underline{\alpha}^*(p_t) = \arg \max \left\{ \alpha O^R p_t^{\theta_1} - i \frac{\alpha^\gamma}{\gamma} \right\}, \text{ s.t. } 0 < \alpha \leq 1.$$

From the first-order condition for (A.2.1) we obtain

$$\underline{\alpha}^*(p_t) = \left( \frac{O^R p_t^{\theta_1}}{i} \right)^{\frac{1}{\gamma-1}} \quad (\text{A.2.2})$$

<sup>20</sup>Note that under  $W^L$  the general solution to (5) should take the form  $V^L(p_t, \alpha) = O_1^R p_t^{\theta_1} + O_2^R p_t^{\theta_2}$ . However, since the value of the option to switch to  $W^R$  vanishes as  $p_t \rightarrow 0$  then we set  $O_2^R = 0$ . Similarly, under  $W^R$  the general solution to (6) should be  $V^R(p_t, \alpha) = \alpha \frac{p_t - d}{r} + O_1^L p_t^{\theta_1} + O_2^L p_t^{\theta_2}$ . However, the option to switch to  $W^L$  is valueless as  $p_t \rightarrow \infty$  and then we set  $O_1^L = 0$ .

<sup>21</sup>See Dixit and Pindyck (1994, chps. 6 and 7) for a thorough discussion.



As can be easily shown, the second-order condition is always satisfied. Taking into account the upper and lower bound for  $\underline{\alpha}^*$  we have:

$$\underline{\alpha}^*(p_t) = \begin{cases} (\frac{O^R p_t^{\theta_1}}{i})^{\frac{1}{\gamma-1}} & \text{for } p_t < \underline{p} \\ 1 & \text{for } \underline{p} \leq p_t < d \end{cases} \quad (\text{A.2.3})$$

where  $\underline{p} = (\frac{i}{O^R})^{\frac{1}{\theta_1}}$ .

### A.3 Proof of proposition 2

Suppose  $p_t \geq d$ . The optimal share of recycling is given by

$$\bar{\alpha}^*(p_t) = \arg \max \{ \alpha O^L p_t^{\theta_2} + \alpha (\frac{p_t}{r-\mu} - \frac{d}{r}) - i \frac{\alpha^\gamma}{\gamma} \}, \text{ s.t. } 0 < \alpha \leq 1.$$

The first-order condition for (A.3.1) yields

$$\bar{\alpha}^*(p_t) = (\frac{O^L p_t^{\theta_2} + \frac{p_t}{r-\mu} - \frac{d}{r}}{i})^{\frac{1}{\gamma-1}} \quad (\text{A.3.2})$$

It is easy to check that the second-order condition holds always. To be feasible  $\bar{\alpha}^*(p_t)$  must belong to the interval  $(0, 1]$ . Let's then show under which conditions  $\bar{\alpha}^*(p_t)$  belongs to such interval. First, let's introduce the convex function  $U(p_t) = O^L p_t^{\theta_2} + \frac{p_t}{r-\mu} - \frac{d}{r}$  and  $\bar{p}$  as solution of the equation  $U(\bar{p}) = i$ . Note that since  $U(d) = \frac{d(r-\mu\theta_2)}{r(r-\mu)(\theta_1-\theta_2)} > 0$  and  $U'(d) = \frac{\theta_1(r-\mu\theta_2)}{r(r-\mu)(\theta_1-\theta_2)} > 0$  then  $\bar{\alpha}(p_t) > 0$  and  $\bar{\alpha}'(p_t) > 0$  for  $p_t \geq d$ . In order to define the interval where  $\bar{\alpha}(p_t) < 1$  we need to impose that  $U(p_t) < i$ . Finally, we simply need to check under which conditions  $U(d) < i$ . That is,

$$\frac{d(r-\mu\theta_2)}{r(r-\mu)(\theta_1-\theta_2)} = O^R d^{\theta_1} < i \quad (\text{A.3.3})$$

Note that if (A.3.3) holds then it exists a  $\bar{p} (> d)$  such that  $\bar{\alpha}^*(p_t) < 1$  for  $d < p_t < \bar{p}$  and  $\bar{\alpha}^*(p_t) = 1$  for  $\bar{p} \leq p_t$ . Otherwise,  $\bar{\alpha}^*(p_t) = 1$  for  $d \leq p_t$ .

### A.4 Optimal investment timing

Let's maximize the objective function in (14) with respect to  $\tilde{p}$ . The first-order condition is:

$$(\frac{p_t}{\tilde{p}})^{\theta_1} [\frac{\partial NPV^k(\tilde{p})}{\partial \tilde{p}} - \frac{\theta_1}{\tilde{p}} NPV^k(\tilde{p})] = 0 \quad (\text{A.4.1})$$

It is immediate to derive (14.1) from (A.4.1).

A necessary and sufficient condition for the definition of a maximum at  $\tilde{p}$  is given by the following

second-order condition:

$$\begin{aligned}
 & -\frac{\theta_1}{\tilde{p}}\left(\frac{p_t}{\tilde{p}}\right)^{\theta_1}\left[\frac{\partial NPV^k(\tilde{p})}{\partial \tilde{p}} - \frac{\theta_1}{\tilde{p}}NPV^k(\tilde{p})\right] + \\
 & +\left(\frac{p_t}{\tilde{p}}\right)^{\theta_1}\left[\frac{\partial^2 NPV^k(\tilde{p})}{\partial \tilde{p}^2} - \frac{\theta_1}{\tilde{p}}\left(\frac{NPV^k(\tilde{p})}{\partial \tilde{p}} - \frac{NPV^k(\tilde{p})}{\tilde{p}}\right)\right] < 0
 \end{aligned} \tag{A.4.2}$$

Plugging (A.4.1) into (A.4.2) and rearranging, we obtain condition (14.2):

$$\tilde{p}\frac{\partial^2 NPV^k(\tilde{p})}{\partial \tilde{p}^2} < (\theta_1 - 1)\frac{\partial NPV^k(\tilde{p})}{\partial \tilde{p}} \tag{A.4.3}$$

## A.5 Proof of propositions 4 and 6

It is easy to show that condition (14.2) holds for a maximum at  $p^*$ . Substituting  $NPV^R(p^*, 1)$  into (14.2) we obtain

$$\begin{aligned}
 O^L\theta_2(\theta_2 - 1)p^{*\theta_2-1} & < (\theta_1 - 1)(O^L\theta_2p^{*\theta_2-1} + \frac{1}{r - \mu}) \\
 -\frac{d^{1-\theta_2}}{r}\frac{r - \mu\theta_1}{r - \mu}\theta_2p^{*\theta_2} & < \frac{p^*}{r - \mu}(\theta_1 - 1) \\
 -\frac{r - \mu\theta_1}{r(\theta_1 - 1)}\theta_2 & < \left(\frac{p^*}{d}\right)^{1-\theta_2} \\
 1 & < \left(\frac{p^*}{d}\right)^{1-\theta_2}
 \end{aligned} \tag{A.5.1}$$

Note that  $\frac{\partial p^*}{\partial O^L} < 0$ . In fact, taking the derivative with respect to  $O^L$  on both sides of (15) we obtain

$$\begin{aligned}
 \frac{\partial p^*}{\partial O^L} & = -\frac{\theta_1 - \theta_2}{\theta_1 - 1}p^{*\theta_2}\left(1 + \frac{\theta_2 O^L}{p^*}\frac{\partial p^*}{\partial O^L}\right)(r - \mu) \\
 & = \frac{-\frac{\theta_1 - \theta_2}{\theta_1 - 1}p^{*\theta_2}}{1 + \frac{r - \mu\theta_1}{\theta_1 - 1}\frac{\theta_2}{r}\left(\frac{p^*}{d}\right)^{\theta_2-1}}(r - \mu) \\
 & = -\frac{\frac{\theta_1 - \theta_2}{\theta_1 - 1}p^{*\theta_2}}{1 - \left(\frac{p^*}{d}\right)^{\theta_2-1}}(r - \mu) < 0
 \end{aligned} \tag{A.5.2}$$

### A.5.1 Existence and uniqueness of the threshold $p^*$

Let's first define the function  $\Phi(p_t) = \frac{\theta_1 - \theta_2}{\theta_1 - 1}O^L p_t^{\theta_2}(r - \mu) + p_t - \frac{\theta_1}{\theta_1 - 1}\left(\frac{d}{r} + \frac{i}{\gamma}\right)(r - \mu)$ . Note that  $\Phi(p_t)$  is convex and  $\Phi(p^*) = 0$ . Thus, to prove that an unique  $p^* > d$  exists for  $\Psi \leq 0$ , it suffices to show that  $\Phi(d) < 0$ . That is:

$$\begin{aligned}
 \Phi(d) & = -\left[\frac{\theta_1}{\theta_1 - 1}\left(\frac{d}{r} + \frac{i}{\gamma}\right)(r - \mu) - d - \frac{r - \mu\theta_1}{\theta_1 - 1}\frac{d}{r}\right] \\
 & = -\frac{\theta_1}{\theta_1 - 1}\frac{i}{\gamma}(r - \mu) < 0
 \end{aligned} \tag{A.5.3}$$

Similarly, in order to prove the existence of an unique investment threshold,  $p^{**} = p^* \geq \bar{p}$ , for  $\Psi > 0$  we need to show that  $\Phi(\bar{p}) \leq 0$ . Using results in section A.3 we obtain  $O^L \bar{p}^{\theta_2} = i - (\frac{\bar{p}}{r-\mu} - \frac{d}{r})$ . Substituting and rearranging, we have

$$\begin{aligned}\Phi(\bar{p}) &= \frac{\theta_1 - \theta_2}{\theta_1 - 1} O^L \bar{p}^{\theta_2} (r - \mu) + \bar{p} - \frac{\theta_1}{\theta_1 - 1} \left( \frac{d}{r} + \frac{i}{\gamma} \right) (r - \mu) \\ &= \left[ -O^L \bar{p}^{\theta_2} (\theta_2 - 1) - \frac{d}{r} - i \left( 1 + \theta_1 \frac{1 - \gamma}{\gamma} \right) \right] \frac{r - \mu}{\theta_1 - 1} \leq 0\end{aligned}$$

It follows that a necessary and sufficient condition for  $p^* \geq \bar{p}$  is given by the following inequality:

$$\begin{aligned}O^L \bar{p}^{\theta_2} &\leq \frac{\frac{d}{r} + i \left( 1 + \theta_1 \frac{1 - \gamma}{\gamma} \right)}{1 - \theta_2} \\ \frac{\bar{p}}{r - \mu} &\geq \Lambda = \frac{\theta_2}{\theta_2 - 1} \frac{d}{r} - \frac{\theta_1 (\gamma - 1) - \gamma \theta_2}{\theta_2 - 1} \frac{i}{\gamma}.\end{aligned}\tag{A.5.4}$$

## A.6 Proof of proposition 5

Substituting  $NPV^R(p^{**}, \bar{\alpha}^*(p^{**}))$  (where  $0 < \bar{\alpha}^*(p^{**}) < 1$ ) into (14.1), we obtain:

$$\begin{aligned}p^{**} &= \theta_1 \frac{O^L p^{**\theta_2} + \frac{p^{**}}{r-\mu} - \frac{d}{r}}{O^L \theta_2 p^{**\theta_2-1} + \frac{1}{r-\mu}} \left( 1 - \frac{1}{\gamma} \right) \\ &= \left[ \frac{\theta_1 \left( 1 - \frac{1}{\gamma} \right)}{\theta_1 \left( 1 - \frac{1}{\gamma} \right) - 1} \frac{d}{r} - O^L p^{**\theta_2} \frac{\theta_1 \left( 1 - \frac{1}{\gamma} \right) - \theta_2}{\theta_1 \left( 1 - \frac{1}{\gamma} \right) - 1} \right] (r - \mu)\end{aligned}\tag{A.6.1}$$

from which it follows (16).

Let's now check for condition (14.2). Plugging  $NPV^R(p^{**}, \bar{\alpha}^*(p^{**}))$  into (14.2) yields

$$\begin{aligned}&\left( \frac{O^L p^{**\theta_2} + \frac{p^{**}}{r-\mu} - \frac{d}{r}}{i} \right)^{\frac{\gamma}{\gamma-1}-2} \left[ \frac{p^{**}}{\gamma-1} \frac{(O^L \theta_2 p^{**\theta_2-1} + \frac{1}{r-\mu})^2}{i} + \frac{O^L p^{**\theta_2} + \frac{p^{**}}{r-\mu} - \frac{d}{r}}{i} O^L \theta_2 (\theta_2 - 1) \frac{p^{**\theta_2}}{p^{**}} \right] \\ &< \frac{\theta_1 - 1}{p^{**}} \left( \frac{O^L p^{**\theta_2} + \frac{p^{**}}{r-\mu} - \frac{d}{r}}{i} \right)^{\frac{\gamma}{\gamma-1}-1} \left( O^L \theta_2 p^{**\theta_2} + \frac{p^{**}}{r-\mu} \right)\end{aligned}$$

Using (A.6.1) and rearranging we obtain

$$\begin{aligned}
 \frac{(O^L \theta_2 p^{**\theta_2} + \frac{p^{**}}{r-\mu})^2}{i(\gamma-1)} &< \frac{O^L p^{**\theta_2} + \frac{p^{**}}{r-\mu} - \frac{d}{r}}{i} [O^L \theta_2 p^{**\theta_2} (\theta_1 - \theta_2) + (\theta_1 - 1) \frac{p^{**}}{r-\mu}] \\
 \frac{(O^L \theta_2 p^{**\theta_2} + \frac{p^{**}}{r-\mu})^2}{i(\gamma-1)} &< \frac{O^L \theta_2 p^{**\theta_2} + \frac{p^{**}}{r-\mu}}{\theta_1(1-\frac{1}{\gamma})i} [O^L \theta_2 p^{**\theta_2} (\theta_1 - \theta_2) + (\theta_1 - 1) \frac{p^{**}}{r-\mu}] \\
 (O^L \theta_2 p^{**\theta_2} + \frac{p^{**}}{r-\mu}) \frac{\theta_1}{\gamma} &< O^L \theta_2 p^{**\theta_2} (\theta_1 - \theta_2) + (\theta_1 - 1) \frac{p^{**}}{r-\mu} \\
 \frac{p^{**}}{r-\mu} [\theta_1(1-\gamma) + \gamma] &< -O^L \theta_2 p^{**\theta_2} [\theta_1(1-\gamma) + \theta_2 \gamma] \\
 (\frac{p^{**}}{d})^{1-\theta_2} [\theta_1(1-\gamma) + \gamma] &< \frac{\theta_1 - 1}{\theta_1 - \theta_2} [\theta_1(1-\gamma) + \theta_2 \gamma] \tag{A.6.2}
 \end{aligned}$$

Note that  $\frac{\theta_1-1}{\theta_1-\theta_2}[\theta_1(1-\gamma) + \theta_2 \gamma] < 0$ . This implies that if  $1 < \gamma \leq \frac{\theta_1}{\theta_1-1}$ , the inequality in (A.6.2) does not hold. Then let's restrict the analysis to the set where  $\gamma > \frac{\theta_1}{\theta_1-1}$ . Dividing on both sides by  $[\theta_1(1-\gamma) + \gamma]$  we obtain

$$(\frac{p^{**}}{d})^{1-\theta_2} > \frac{\theta_1 - 1}{\theta_1 - \theta_2} \frac{\theta_1(1-\gamma) + \theta_2 \gamma}{\theta_1(1-\gamma) + \gamma} > 1 \tag{A.6.3}$$

Finally, using (A.6.3) it is easy to show that  $\frac{\partial p^{**}}{\partial O^L} < 0$ :

$$\begin{aligned}
 \frac{\partial p^{**}}{\partial O^L} &= -\frac{\theta_1(\gamma-1) - \gamma\theta_2}{\theta_1(\gamma-1) - \gamma} p^{**\theta_2} (1 + \frac{\theta_2 O^L}{p^{**}} \frac{\partial p^{**}}{\partial O^L})(r-\mu) \\
 &= \frac{-\frac{\theta_1(\gamma-1) - \gamma\theta_2}{\theta_1(\gamma-1) - \gamma} p^{**\theta_2}}{1 - \frac{\theta_1-1}{\theta_1-\theta_2} \frac{\theta_1(1-\gamma) + \theta_2 \gamma}{\theta_1(1-\gamma) + \gamma} (\frac{p^{**}}{d})^{\theta_2-1}} (r-\mu) < 0 \tag{A.6.4}
 \end{aligned}$$

### A.6.1 Existence and uniqueness of the threshold $p^{**}$

Let's provide the conditions that guarantee existence and uniqueness of the optimal threshold when  $d \leq p^{**} < \bar{p}$ . We introduce the function  $\Theta(p_t) = \frac{\theta_1(\gamma-1) - \gamma\theta_2}{\theta_1(\gamma-1) - \gamma} O^L p_t^{\theta_2} (r-\mu) + p_t - \frac{\theta_1(\gamma-1)}{\theta_1(\gamma-1) - \gamma} \frac{d}{r} (r-\mu)$ . Note that  $\Theta(p_t)$  is convex and  $\Theta(p^{**}) = 0$ . Hence, if  $\Theta(d) \leq 0$  then a unique  $p^{**} \geq d$  exists. This can be easily verified:

$$\begin{aligned}
 \Theta(d) &= \frac{d}{r} \frac{r-\mu\theta_1}{\theta_1-\theta_2} \frac{\theta_1(1-\frac{1}{\gamma}) - \theta_2}{\theta_1(1-\frac{1}{\gamma}) - 1} + d - \frac{\theta_1(1-\frac{1}{\gamma})}{\theta_1(1-\frac{1}{\gamma}) - 1} \frac{d}{r} (r-\mu) \\
 &= -\frac{d}{r} \frac{r-\mu\theta_1(1-\frac{1}{\gamma}) - \frac{r-\mu\theta_1}{\theta_1-\theta_2} [\theta_1(1-\frac{1}{\gamma}) - \theta_2]}{\theta_1(1-\frac{1}{\gamma}) - 1} \\
 &= -\frac{d}{r} \frac{\theta_1 \frac{r-\mu\theta_2}{\gamma}}{(\theta_1-\theta_2)[\theta_1(1-\frac{1}{\gamma}) - 1]} < 0 \tag{A.6.5}
 \end{aligned}$$

We complete the analysis by studying the condition  $p^{**} < \bar{p}$ . Note that, using the properties of  $\Theta(p_t)$ , this is equivalent to having  $\Theta(\bar{p}) > 0$ . After substituting, we obtain

$$\begin{aligned}
 \Theta(\bar{p}) &= \frac{\theta_1(\gamma - 1) - \gamma\theta_2}{\theta_1(\gamma - 1) - \gamma} O^L \bar{p}^{\theta_2} (r - \mu) + \bar{p} - \frac{\theta_1(\gamma - 1)}{\theta_1(\gamma - 1) - \gamma} \frac{d}{r} (r - \mu) \\
 &= \gamma \frac{1 - \theta_2}{\theta_1(\gamma - 1) - \gamma} O^L \bar{p}^{\theta_2} (r - \mu) + i(r - \mu) - \frac{\gamma}{\theta_1(\gamma - 1) - \gamma} \frac{d}{r} (r - \mu) \\
 &= \gamma \frac{r - \mu}{\theta_1(\gamma - 1) - \gamma} [(1 - \theta_2) O^L \bar{p}^{\theta_2} + \frac{i}{\gamma} (\theta_1(\gamma - 1) - \gamma) - \frac{d}{r}] > 0
 \end{aligned} \tag{A.6.6}$$

which is positive if and only if

$$\begin{aligned}
 O^L \bar{p}^{\theta_2} &> \frac{\frac{d}{r} + i(1 + \theta_1 \frac{1-\gamma}{\gamma})}{1 - \theta_2} \\
 \frac{\bar{p}}{r - \mu} &< \Lambda = \frac{\theta_2}{\theta_2 - 1} \frac{d}{r} - \frac{\theta_1(\gamma - 1) - \gamma\theta_2}{\theta_2 - 1} \frac{i}{\gamma}.
 \end{aligned} \tag{A.6.7}$$

Last, note that if condition (A.6.7) does not hold then  $p^{**} = p^*$ .

## A.7 Proof of proposition 8

Denote by  $p^+$  the investment threshold and consider the case  $p^+ < \underline{p}$ . Plugging (10.2) into (14.2) yields

$$\begin{aligned}
 p^+ \left( \frac{\gamma}{\gamma - 1} \theta_1 - 1 \right) \left( \frac{O^R}{i} p^{+\theta_1} \right)^{\frac{\gamma}{\gamma - 1}} \frac{i\theta_1}{p^{+2}} &< (\theta_1 - 1) \left( \frac{O^R}{i} p^{+\theta_1} \right)^{\frac{\gamma}{\gamma - 1}} \frac{i\theta_1}{p^+} \\
 \left( \frac{O^R}{i} p^{+\theta_1} \right)^{\frac{\gamma}{\gamma - 1}} i\theta_1 \left( \frac{\gamma}{\gamma - 1} \theta_1 - 1 \right) &< \left( \frac{O^R}{i} p^{+\theta_1} \right)^{\frac{\gamma}{\gamma - 1}} i\theta_1 (\theta_1 - 1) \\
 \frac{\gamma}{\gamma - 1} \theta_1 - 1 &< \theta_1 - 1 \\
 \frac{\theta_1}{\gamma - 1} &< 0,
 \end{aligned} \tag{A.7.1}$$

which never holds since  $\gamma > 1$  by assumption.

Finally, it is immediate to verify that (14.2) does not hold also for  $\underline{p} \leq p^+ < d$ . In fact, following the same steps we obtain

$$\begin{aligned}
 p^+ (\theta_1 - 1) O^R \theta_1 p^{+\theta_1 - 2} &< (\theta_1 - 1) O^R \theta_1 p^{+\theta_1 - 1} \\
 0 &< 0
 \end{aligned} \tag{A.7.2}$$

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