



# JENA ECONOMIC RESEARCH PAPERS



# 2010 – 035

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[www.jenecon.de](http://www.jenecon.de)

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact [markus.pasche@uni-jena.de](mailto:markus.pasche@uni-jena.de).

Impressum:

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# Imperfect Recall and Time Inconsistencies: An experimental test of the absentminded driver “paradox”\*

M. Vittoria Levati, Matthias Uhl, Ro'i Zultan

## Abstract

Absentmindedness is a special case of imperfect recall that according to Piccione and Rubinstein (1997b) may lead to time inconsistencies, since the optimal strategy of an absentminded player, as calculated before entering the game tree, is not consistent with maximization of expected payoffs given consistent beliefs once called upon to act. Aumann, Hart and Perry (1997a) question their analysis and argue that optimization at the action stage must be carried out with respect to the strategy at the current node only, hence resolving the possible dynamic inconsistency. The present paper explores this issue from a behavioral point of view by creating an experimental game involving absentmindedness. Participants' strategies are elicited separately in a planning stage and in an action stage in different games sharing the same game form. The behavior observed in the experiment exhibits time inconsistencies, as suggested by Piccione and Rubinstein's analysis. We introduce a cognitive principle, which best explains the data.

*Keywords:* imperfect recall, absentmindedness,  
dynamic inconsistency, experiment

*JEL Classification:* C72, C91, D03, D81, D83

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# 1 Introduction

Dynamic consistency is a compelling fundamental tenet of rational behavior: once a decision maker makes a plan, she should carry it out as long as there is no relevant change in the decision environment. Notwithstanding its normative appeal, the principle of dynamic consistency has been systematically invalidated by empirical evidence, therefore calling for a revision of the normative theories, as in the case of decision-making under risk (e.g. Kahneman and Tversky, 1979) or ambiguity (e.g. Gilboa and Schmeidler, 1989; Epstein and Schmeidler, 2003). Conversely, Piccione and Rubinstein (1997b, henceforth PR) have drawn attention to a particular case of dynamic inconsistency that arises exactly from standard rational decision theory. PR considered a specific type of imperfect recall, which they termed “absentmindedness”, where a single history passes through two decision nodes in an agent’s information set. They showed that considerations that would bear no impact on the analysis of decision problems with perfect recall become crucial to the analysis when absentmindedness (and, to a lesser extent, other forms of imperfect recall) is involved.

Most importantly, one has to ascertain whether a decision maker, upon reaching an information set, expects to make the same decisions as she does now in other occurrences of the information set, or whether she takes her decisions at those occurrences to be fixed.<sup>1</sup> PR show that when the former is assumed, time inconsistencies are likely to emerge under absentmindedness. Conversely, the opposite assumption guarantees consistency. PR state that they “find both [assumptions] to have some intuitive appeal and neither to be universally valid.” (p. 20).

Absentmindedness and the paradoxical results associated with it are most usefully illustrated by the problem of the “absentminded driver”, a simplified version of which is presented in Figure 1’s game tree.<sup>2</sup>

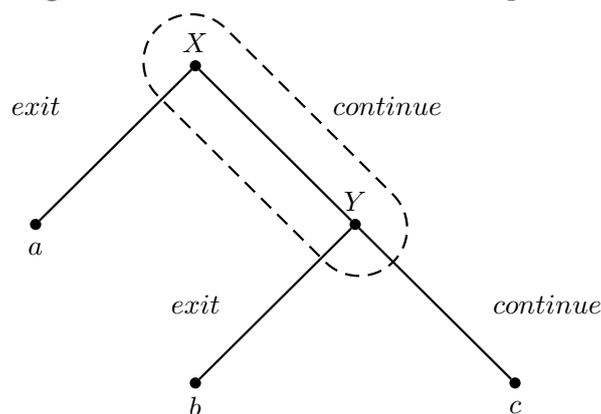
The absentminded driver starts her journey at node  $X$  where she can either “exit” (for a payoff of  $a$ ) or “continue” to  $Y$  where she faces the same choice. If at  $Y$  she exits, she gets a payoff of  $b$ ; if she continues beyond  $Y$ , she earns  $c$ . The driver suffers from absentmindedness in the sense that she is unable to distinguish between nodes  $X$  and  $Y$ , both of them being in the same information set. PR demonstrated that if the highest payoff is at the second exit (i.e.,  $b > a; c$ ), an agent’s plan before she starts her journey (the *planning stage*) is inconsistent with her beliefs once she reaches a decision node (the *action stage*) as long as she assigns some positive probability to being at  $Y$ .

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<sup>1</sup>This assumption underlies the ‘modified multiselves approach’, in which decisions in different decision nodes within the information set are taken to be carried out by independent agents who share the decision maker’s payoffs.

<sup>2</sup>We speak of game (rather than decision) tree to stay in the framework of game theory.

Figure 1: The absentminded driver problem



In other words, the decision maker is tempted to change her initial plan when the time comes to make a decision.<sup>3</sup> This observation was termed by PR “the absentminded driver paradox”.

In the same issue of *Games and Economic Behavior*, Aumann et al. (1997a, henceforth AHP) replied to PR. Whereas PR presented both ways of reasoning about the problem as potentially valid, possibly depending on other features of the situation, AHP took a more restrictive normative stand. They argued that optimization at the information set *must* be carried out with respect to the strategy at the current decision node while considering the rest of the play as fixed, as in PR’s modified multiselves approach. Consistent with PR’s analysis, AHP showed that, in this case, the planning-optimal decision is also action-optimal, thus resolving the paradox.

Notwithstanding having been brought out more than a decade ago, the absentminded driver paradox and the theoretical controversy surrounding it have not yet been settled. Game theory has proven unable to provide a general normative prescription because different game-theoretical approaches lead to conflicting results. Indeed, while AHP argue for the planning-optimal strategy as the unique normative prescription, Binmore (1996) considers any non-committing plan irrelevant to the analysis.

In this paper we aim at expanding the study of absentmindedness beyond the philosophical normative debate.<sup>4</sup> Absentmindedness is prevalent in real-world situations. For

<sup>3</sup>Note that the highest payoff  $b$  can never be reached through pure or mixed strategies. The following analyses therefore always consider *behavioral strategies*, allowing the decision maker to randomize over her actions independently at  $X$  and at  $Y$ .

<sup>4</sup>Theoretical discussions of the paradoxes arising under absentmindedness can be found in Battigalli (1997), Gilboa (1997), Grove and Halpern (1997), Halpern (1997), Aumann et al. (1997b), and Lipman (1997), which are summarized and countered in Piccione and Rubinstein (1997a), Binmore (1996), and Board (2003). So far, the experimental studies of absentmindedness have not attempted a direct test of the absentminded driver paradox. Huck and Müller (2002) tested a related game which is comparable only to the action stage, while Deck and Sarangi (2009) devised a procedure for inducing absentmindedness in the laboratory without testing the paradox.

example, we often meet someone for the second time, not remembering their name or even whether we've asked for it when we first met. A further relevant example involves two of the authors' mothers, who take prescriptions to moderate blood pressure on a daily basis. They are often faced with the decision about whether to take a pill or not as they cannot remember if they took the pill that day. This problem is exacerbated if many similar decisions are to be made over time; for example, if there are several medications, each to be administered according to a different schedule.<sup>5</sup> The prevalence of situations involving absentmindedness calls for a better understanding of how the theoretical debate reflects on actual behavior. Such an understanding would benefit the design of institutions dealing with situations in which absentmindedness may lead to suboptimal behavior from a planner's perspective.

Consequently, and in contrast to previous studies, we explore absentmindedness from a descriptive point of view: we design an experimental game that we model as a game with absent-mindedness, which shares the essential features of the game depicted in Figure 1.<sup>6</sup> We then compare the actual behavior in the game with the (qualitative) theoretical predictions.<sup>7</sup>

To achieve this, we let participants 'become' absentminded by loading them with many decisions. Deck and Sarangi (2009) have shown how absentmindedness in the lab can be achieved by means of cognitive overload resulting from information abundance.<sup>8</sup> Building on their procedure, we implement an environment in which many driving "maps" are presented to the participants, with each map being uniquely identified not only by the game tree (defined by the payoffs  $a$ ,  $b$ , and  $c$ ), but also by a distinct color.<sup>9</sup> This makes it next to impossible to recall whether a map has been previously encountered.<sup>10</sup> Thus,

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<sup>5</sup>PR pointed out that organizations are also susceptible to absentmindedness, as their decisions are made by different individuals. For example, two employees in a firm are called upon at different times to place a certain procurement order. If one of them places the order, then the other will be informed, but in lack of information an employee does not know whether the other has refrained from placing the order or has not had the opportunity to decide yet.

<sup>6</sup>For consistency with the previous literature, we use the game of Figure 1 in the theoretical exposition. The variant we employ in our experiment will be introduced and analyzed in Section 4.2.2, after the experimental design has been introduced. It will be shown that our experimental game does not qualitatively change the theoretical predictions.

<sup>7</sup>We acknowledge that absentmindedness is a distinctive theoretical concept that does not map onto empirical situations in any straightforward way. Yet, it is not different from any other theoretical concept that abstracts from the way in which it is manifested in the real world. The challenge we are facing is to imbue the experimental participants with the preferences and information of the theoretical agent.

<sup>8</sup>When people must process large amounts of information within a short time span, the limited capacity of their short-term memory causes cognitive overload (see, e.g., Kareev and Warglien, 2003). Short-term memory capacity refers to the number of items that an individual can retain at one time and is classically estimated to be  $7 \pm 2$  (Miller, 1956; Shiffrin, 1976; Kareev, 2000); but see Cowan (2001), for a lower estimate.

<sup>9</sup>As argued by, e.g., Kahneman's (1973) model of divided attention, focusing on multiple stimuli should deteriorate memory performance.

<sup>10</sup>Our data confirm that participants are unable to recall whether they have already seen the current

this paradigm places our participants in the same position as an absentminded agent, effectively allowing us to explore whether they exhibit inconsistencies between decisions made at a planning stage and at an acting stage. For ease of terminology, we shall refer in the following to the analysis leading to time inconsistencies as PR's approach, and to the alternative analysis as AHP's approach. The reader should keep in mind that PR do not reject the latter, whereas AHP reject the former.

To these two approaches we add a third one rooted in behavioral cognitive considerations. The theoretical analyses of PR and AHP assume that a decision maker is able to perform sophisticated calculations over the distribution of beliefs about the states of the world. In line with previous research in cognitive psychology, we argue instead that an absentminded individual (not knowing where she is during the journey) may consider a specific contingency, such as being at node  $Y$ , commit to it, and act as if the considered contingency was the true state of the world. Thus, when encountered with the problem of the absentminded driver, one might tend to exit more than dictated by her planning strategy because of the occurrence of "being at  $Y$ " in her mentally constructed state of the world, rather than because of sophisticated optimization over beliefs.

Our experiment is constructed in a way which enables us to juxtapose the predictions of the three approaches. Each participant goes through both a planning stage and an action stage. In the planning stage, the participants are shown each driving map once, and for each map they choose one behavioral strategy to be independently implemented at both decision nodes. In the action stage, the (potentially) absentminded participants are shown each map twice and every time they choose one behavioral strategy. To distinguish between PR optimization and our cognitive explanation, we elicit the state of the world that a participant considers at each specific period. We achieve this by asking participants in the action stage to guess the node to which their current decision applies. The guess is taken as a proxy for the contingency that the participant considers, i.e., the one that is more salient in the participant's mind. Note that this procedure may influence participants to choose their strategies to be consistent with the stated beliefs, resulting in an artifact time inconsistency due to the experimental procedure. Therefore we run additional treatments with new participants, using the same procedure but without belief elicitation.

Absentmindedness in the action stage is manipulated in two different ways, which we term "induced absentmindedness" (henceforth IND treatment) and "imposed absentmindedness" (henceforth IMP treatment). In the IND treatment, the two decision nodes are presented in the natural order, i.e., the first time a participant observes a map, her decision applies to the first node, and the second time she sees the same map, her decision

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map.

applies to the second node. Keeping the nodes to the natural order has the drawback that first decision nodes  $X$  are more likely to appear earlier in the stage. A participant, although absentminded, may realize this and accordingly decide to “continue” with high probability during the early part of the stage, changing this strategy midway through the stage. The actual decision nodes and the participant’s strategies would then be correlated simply because both are independently contingent on the number of maps displayed so far. This participant would therefore emulate the game behavior of a non-absentminded individual, as well as show a correspondence between actual nodes and guesses.

To control for this experimental artifact, we introduce the IMP treatment, where a random device selects with equal probability either (1) the realistic scenario, where the decisions made the first and the second time a map is presented apply to, respectively, the first and the second node, or (2) the artificial scenario, where the first decision applies to the second node and the second decision applies to the first node. Consequently, when presented with a map, a participant cannot know whether she is at node  $X$  or at node  $Y$ . This treatment may, however, alter the optimal strategy. If participants were able to coordinate their decisions such that they choose “continue” at one node and “exit” at the other, this guarantees them a payoff of  $\frac{a+b}{2}$ , which is higher than the payoff obtainable from a behavioral strategy (this argument is elaborated in Section 6(e) of AHP).<sup>11</sup> To circumvent this issue, we use in the IMP treatment the same procedure as in the IND treatment. This procedure precludes the coordination of decisions because the cognitive overload makes it practically impossible to identify different instances of the same map, and therefore to employ different strategies at the two different nodes.<sup>12</sup>

Our results generally indicate significant time inconsistencies across treatments and decision problems, in line with PR. The belief data supports our cognitive interpretation of the effect.

The remainder of the paper is organized as follows. The next section presents the formal arguments put forth by PR and AHP. In Section 3 we develop our behavioral approach. Section 4 details the experimental design and experimental implementations of imperfect recall. Section 5 discusses our experimental results, and Section 6 has some concluding remarks.

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<sup>11</sup>This holds for all of our experimental game trees. Only when payoff  $c$  is very large compared to  $a$ , payoff maximization still prescribes behavioral strategies, but trees of this kind are not present in our experiment.

<sup>12</sup>Coordination of decisions would lead to an overall probability to “continue” closer to 0.5 in the action stage compared to the planning stage. This is ruled out by our data.

## 2 Theoretical background

Consider the absentminded decision problem shown in Figure 1. Denote the probability of “continue”, which defines the behavioral strategy, by  $p$ . At the planning stage, the decision problem is to maximize  $(1 - p)a + p(1 - p)b + p^2c$  over  $p$ . Straightforward computations show that the optimal behavioral strategy is

$$(1) \quad p^* = \frac{b - a}{2(b - c)}.$$

Take now into account the action stage. Once the driver is on the road and arrives at an intersection,<sup>13</sup> she does not know whether this is the first or second intersection. Let  $\alpha$  be the probability the driver assigns to being at  $X$ , and let  $q$  be the strategy at the other decision node. Indicate by  $H(p, q, \alpha)$  the expected payoff given  $p$ ,  $q$ , and  $\alpha$ . PR take the probability of “continue” at the current and at the other intersection as being the same and therefore maximize  $H(p, p, \alpha) = \alpha[(1 - p)a + p(1 - p)b + p^2c] + (1 - \alpha)[(1 - p)b + pc]$  over  $p$ , holding  $\alpha$  fixed. Solving this problem yields:

$$\bar{p} = \frac{\alpha(2b - a - c) + c - b}{2\alpha(b - c)},$$

which is strictly smaller than  $p^*$  for any  $\alpha < 1$ . Thus, in PR’s argumentation, unless the driver believes, unreasonably, to be at the first node  $X$  with probability 1, her optimal strategy at the action stage is inconsistent with her optimal plan.

AHP claim that PR’s analysis is “flawed” (p. 102) in its formulation. They argue for the normative requirement that “when at one intersection, he (the driver) can determine the action *only there*, and *not* at the other intersection – where he isn’t” and “whatever reasoning obtains at one (intersection) must obtain also at the other” (p. 104), as in PR’s alternative, multiselves approach. Accordingly, the planning-optimal strategy  $p^*$  is also action-optimal if it maximizes payoff at the action stage assuming that  $p^*$  is played at the other intersection. In AHP’s analysis, the expected payoff at the action stage is

$$(2) \quad H(p, q, \alpha) = \alpha[(1 - p)a + p(1 - q)b + pqc] + (1 - \alpha)[(1 - p)b + pc],$$

where  $\alpha$  is not held fixed, but is determined by  $q$ ; in particular, it is the consistent belief  $\alpha = \frac{1}{1+q}$ .<sup>14</sup>

<sup>13</sup>Throughout the paper we use the terms ‘intersection’ and ‘node’ interchangeably.

<sup>14</sup>Rational beliefs under absentmindedness are also a source of controversy. PR develop two non-converging notions of consistent beliefs (see also Aumann et al., 1997b). Starting with Elga (2000), this issue has since spawned a vast literature in philosophy under the title of the Sleeping Beauty problem (at the time of writing, an online bibliography edited by Pust, 2011, lists over 50 papers in philosophy

Suppose that the strategy at the other intersection is the one prescribed by the optimal plan  $p^*$  as defined in (1), i.e.,  $q = p^* = (b - a)/(2(b - c))$ . Then the probability that the current intersection is  $X$  is  $\alpha = 2(b - c)/(3b - 2c - a)$ . Substituting  $q$  for  $(b - a)/(2(b - c))$  and  $\alpha$  for  $2(b - c)/(3b - 2c - a)$  into (2), the expected payoff becomes

$$(3) \quad H\left(p, \frac{b - a}{2(b - c)}, \frac{2(b - c)}{3b - 2c - a}\right) = \frac{b(a + b) - 2ac}{3b - a - 2c},$$

which does not depend on  $p$ . Hence  $p = p^*$  maximizes (3) and therefore  $p^*$  is both planning- and action-optimal. AHP thus prove that in the absentminded driver problem studied here the planning-optimal strategy is the *unique* action-optimal strategy.

### 3 Behavioral hypothesis

Normatively, upon reaching the information set in the game of Figure 1, the absentminded decision maker should form a consistent belief about the actual decision node she is in, and assess the probability of obtaining each possible payoff given this belief. This process is akin to making probability judgments based on uncertain categories (e.g., Murphy and Ross, 1994). We illustrate the similarity using the following example from Ross and Murphy (1996):

[S]uppose you are walking through the woods and see an animal to the side of the trail. You think it is a dog and want to make a prediction about how it will react to your presence. Should you continue along the trail or not?  
(p. 736)

In order to make this decision, you need to estimate the probability that the unknown animal will be hostile. This requires you to pay attention to the behavior not only of a dog but also of another animal (say a raccoon). Specifically, you should consider the probability of hostility in both dogs and raccoons, weighted by the probability that the animal is a dog or a raccoon (Anderson, 1991). For example, if you believe (i) that the animal is a dog with probability 0.80 and a raccoon with probability 0.20, and (ii) that dogs are hostile with probability 0.10 and raccoons with probability 0.75, then the probability of encountering hostility is  $(0.80 \cdot 0.10) + (0.20 \cdot 0.75) = 0.23$  (Ross and Murphy, 1996).

Nevertheless, Murphy and Ross (1994) suggested that people focus on a single category when making predictions. In the example above, people are expected to consider the animal to be a dog as the most likely contingency, and therefore they estimate the

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journals).

probability of encountering hostility as 0.10. In other words, their estimation will not be affected by the probability of hostility in raccoons. This conjecture was supported by a series of experiments (Murphy and Ross, 1994; Malt et al., 1995; Ross and Murphy, 1996). In line with the single-category view, Lagnado and Shanks (2003) proposed the *commitment heuristic*, according to which, when people consider a category, they “*mentally commit to its truth*” (p. 162).<sup>15</sup>

Similarly, we postulate that the absentminded participants in our experiment apply a commitment heuristic, by which they sometimes consider one contingency (i.e., one of the two exits) and commit to its truth. Consideration of the possibility of being at the second exit and commitment to this possibility will create a tendency to exit more than prescribed by the planning-optimal strategy. Hence, this Commitment Effect (henceforth CE) gives rise to time-inconsistencies similar to those suggested by PR. In the following we formalize and analyze CE and its implications in Figure 1’s problem.

Denote by  $\sigma_i$  the strategy that the decision maker would like to choose at decision node  $i$  ( $i \in \{X, Y\}$ ) if she knew that  $i$  would be the actual state of the world. With probability  $\alpha$  the absentminded driver considers node  $X$ , which induces a tendency to move according to  $\sigma_X$ , and with complementary probability,  $1 - \alpha$ , she considers  $Y$ , which induces a tendency to move according to  $\sigma_Y$ .<sup>16</sup> This tendency should be interpreted as a shift from the planning-optimal strategy  $p^*$  towards  $\sigma_i$ , where the weight given to  $p^*$  is independent of  $i$ . Accordingly, the expected observed mean strategy,  $\hat{p}$ , should lie between  $p^*$  and the expected  $\sigma_i$ :

$$\min(p^*, \alpha\sigma_X + (1 - \alpha)\sigma_Y) < \hat{p} < \max(p^*, \alpha\sigma_X + (1 - \alpha)\sigma_Y).$$

In the following we will show that

$$(4) \quad \alpha\sigma_X + (1 - \alpha)\sigma_Y < p^*,$$

so that  $\hat{p} < p^*$ , and CE indeed leads to time inconsistencies in the same direction as predicted by PR.

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<sup>15</sup>This is a generalization of the original heuristic of Lagnado and Shanks (2003), whose investigation was in the more specific context of hierarchical structures. Another related approach is the “As-If” model, which hypothesizes a threshold of (un)certainly, after which people act “as if” they are entirely certain of the true state of the world (Gettys and Willke, 1969; Howell et al., 1971; Gettys et al., 1973).

<sup>16</sup>For simplicity, we assume that the probability of considering, and committing to  $X$  is given by  $\alpha$ . The predictions remain qualitatively unchanged if this assumption is somewhat relaxed, insofar as one commits to  $X$  ( $Y$ ) with a probability that is weakly increasing (decreasing) in  $\alpha$ , and symmetry holds in the sense that if  $\alpha = 0.5$ , then each node is considered with equal probability.

First, note that the driver always wishes to exit at  $Y$ , implying  $\sigma_Y = 0$ . Therefore inequality (4) reduces to

$$(5) \quad \alpha\sigma_X < p^*.$$

When  $a > c$ , it is not obvious what one should do at  $X$  because the action-optimal strategy hinges upon the choice at  $Y$ . In this case, the strategy at  $X$  can be construed to be the same as in the planning stage, i.e.  $\sigma_X = p^*$ . Condition (5) thus implies that the driver would not like to follow her planning-optimal strategy as long as  $\alpha < 1$ , therefore exhibiting time inconsistency.

On the other hand, if  $c > a$ , wishing to continue at the first intersection is a dominant strategy, i.e.  $\sigma_X = 1$ . In such a case, time inconsistency arises due to the assumption that  $X$  is more likely to be considered as  $\alpha$  increases. In our experiment the objective  $\alpha$  is 0.5 (each map is indeed presented to the participants twice). Since  $c > a$  implies  $p^* > 0.5$ , inequality (5) holds.<sup>17</sup>

The case with  $c = a$  is somewhat ambiguous because  $\sigma_X = 1$  is only weakly dominant. Although it seems still reasonable to take  $\sigma_X = 1$ , this would imply  $\hat{p} = p^* = 0.5$  and hence no time inconsistency. Therefore CE has no clear predictions.

## 4 Experimental design

To disentangle the predictions of the three different approaches presented in the previous sections, the experiment consists of two phases: the planning stage where participants provide their plan *before* starting the “journey”, and the action stage in which participants indicate their choices *during* the journey. According to AHP, no difference in behavior between the stages should be detected. According to PR, participants should always exit more in the action stage. Finally, according to CE, participants should systematically exit more in the action stage only when they guess to be at the second node.

### 4.1 Phase 1: Planning stage

The first phase involves sequential decisions. In this phase, participants were shown 14 game trees of the type depicted in Figure 1 in 4 different colors (yellow, green, blue, purple) so that they faced a total of 56 maps.<sup>18</sup> For every map, each participant had to specify her behavioral strategy.

<sup>17</sup>This result remains unchanged if we replace the experimental  $\alpha = 0.5$  with the consistent  $\alpha = \frac{1}{1+\sigma_X}$ .

<sup>18</sup>Recall that a “map” is uniquely identified by both the game tree (and thus the payoffs) and the color.

To implement the randomizing mechanism associated with behavioral strategy play, we used a technique similar to that provided by Huck and Müller (2002). Participants were asked to imagine an urn with 100 balls. They could determine the composition of the urn, i.e., how many balls would stand for “exit” and how many for “continue”. Once the composition of the urn was decided, the computer randomly drew one ball from the urn (afterwards replaced). If the ball showed “exit”, then the participant had to take the first exit and earned  $a$ . If the ball showed “continue”, then the participant continued to the second intersection and the computer randomly drew a second ball. If this was an “exit”-ball, then the participant had to take the second exit and earned  $b$ ; otherwise, the participant stayed to the end and earned  $c$ .

For each of the 56 maps, participants could indicate their desired mixture of “exit” and “continue” balls by entering a number in both an “exit”-box and a “continue”-box, where the two numbers had to add up to 100. We made clear that the strategy chosen by a participant referred to both intersections. Although this procedure allowed participants to implement randomized strategies, participants could, if they wanted to, enter 100 in one box and 0 in the other, therefore playing a pure strategy.

To familiarize participants with the task – including both the game form and the randomization procedure – we gave them 15 minutes of practice to experiment with different payoffs and strategies. Participants filled in payoffs in a blank tree and indicated a behavioral strategy, after which they received the resulting probability distributions over outcomes as well as the expected payoff. Next, particular payoff realizations could be obtained by pressing a “travel”-button repeatedly.

## 4.2 Phase 2: Action stage

The second phase corresponds to the action stage. Given that each map has two decision nodes, in this phase participants encountered each map exactly twice, and every time they had to specify a strategy. The same map was never shown consecutively. In addition to the 56 maps seen during the planning stage, participants were presented with 16 (4 trees  $\times$  4 colors) filler maps, i.e., maps inserted at the beginning and throughout the stage only as scribbles and, as such, excluded from the data analysis.<sup>19</sup> Since each of the 72 maps was shown twice, the action stage consisted of 144 game decisions/periods.

Strategies were elicited employing the same randomization technique as in the first phase.<sup>20</sup> Participants were required to decide twice on each map regardless of their deci-

<sup>19</sup>The rationale behind the filler maps is discussed in Section 4.2.1 below.

<sup>20</sup>Although people, in reality, can choose only one of the two actions, we allow participants to randomize in the action stage for being consistent not only with the planning stage, but also with the theoretical models.

sion at the first decision node; i.e., having chosen to exit with certainty at the first intersection ( $p = 0$ ) did not exclude making a choice for the second intersection. Although this procedure somewhat alters the original game, it does not affect the qualitative predictions of the different theories (see Section 4.2.2 below).

Depending on how absentmindedness was brought about and on whether beliefs were elicited or not we discriminate among four treatments.

#### 4.2.1 Inducing and imposing absentmindedness

In the IND treatment, the payoff for each map was computed in the natural way; i.e., the first time a participant saw a map, her decision applied to the first decision node, and the second time she encountered the same map, her decision applied to the second node. As discussed above, the disadvantage of this treatment is the possible artifact correlation between actual nodes and strategies caused by the timing of the experiment. Indeed, in the early part of the stage, when not many maps have already been encountered, the likelihood of first decision nodes and the beliefs to be at  $X$  are both rather high. In this sense, the experiment's timing could serve as a cue for the current node.

To avoid this shortcoming, we performed the IMP treatment, in which the payoff for each map was determined by randomly matching strategies with intersections. Specifically, with 50% probability, the decision made when a map was first encountered applied to the first intersection and the decision made when the same map was displayed for the second time applied to the second intersection; with complementary probability, the connection between decisions and intersections was reversed: the first decision applied to the second intersection and the second decision to the first intersection. The participants were aware of the random device that linked their strategies to the nodes, but not of the order that was actually implemented.

When participants start the action stage, their memory is not yet overloaded and thus they might remember the maps that were displayed.<sup>21</sup> Therefore only filler maps were used in the first 10 periods.

#### 4.2.2 Theoretical analysis of the experimental game

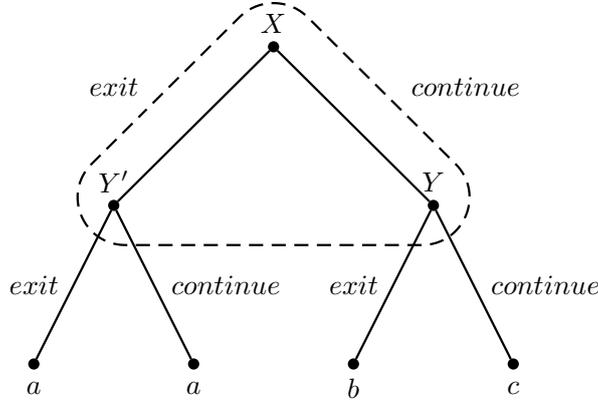
Like the original problem presented in Figure 1, our experimental version of the problem entails absentmindedness: each time that a participant is called to choose a strategy, her information set includes both the first and second intersections. However, our procedure dictates that each participant encounters both decision nodes regardless of her strategy, although the decision made at the second node affects her payoff only if the choice at the

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<sup>21</sup>At the very beginning of the IND treatment, participants know with certainty that their decisions are for the first intersection.

first node was to continue. This alteration slightly changes the game to the one depicted in Figure 2.<sup>22</sup>

Figure 2: The experimental absentminded driver problem



To see that the experimental game does not qualitatively alter PR’s analysis, denote by  $\alpha_X$ ,  $\alpha_Y$ , and  $\alpha_{Y'}$  the probabilities that the driver assigns to being at  $X$ ,  $Y$ , and  $Y'$ , respectively. Since the driver passes through  $X$  and through either  $Y$  or  $Y'$  exactly once, any consistent belief must be such that  $\alpha_X = \alpha_Y + \alpha_{Y'} = 0.5$ . First, note that the planning-optimal strategy remains as in Equation (1). Following PR’s analysis of the action-optimal strategy, maximize the expected payoff  $H(p, p, \{\alpha_X, \alpha_Y, \alpha_{Y'}\}) = 0.5[(1 - p)a + p(1 - p)b + p^2c] + \alpha_Y[(1 - p)b + pc] + \alpha_{Y'}(a)$  over  $p$  to obtain

$$\bar{p} = \frac{b - a}{2(b - c)} - \alpha_Y,$$

which is strictly smaller than  $p^*$  for any  $\alpha_Y > 0$ . This proves that the theoretical argument provided in Section 2 remains valid in the experimental version of the absentminded driver problem. Furthermore, the argumentation of Section 3 follows through, as the decision made at  $Y'$  has no consequences.

### 4.2.3 Belief elicitation

Given the importance of beliefs to disentangling CE from PR’s argument, we elicited point beliefs about the intersection at which a participant thinks to be.<sup>23</sup> Moreover, in the IND treatment (where absentmindedness is achieved by means of cognitive overload), the elicited beliefs serve as a memory test, aimed at verifying how well participants can recall and, thus, whether or not they are absentminded.

<sup>22</sup>For simplicity of presentation, we abstract from whether the randomization of the behavioral strategy is carried out internally, as in the original game, or by an external mechanism, as in the experimental

Table 1: Elicitation of beliefs about the current decision node

Your choice	Option	If your guess is <i>correct</i> you WIN	If your guess is <i>wrong</i> you LOSE
○	<i>A</i>	1	1
○	<i>B</i>	3	5
○	<i>C</i>	5	15

The elicitation procedure was as follows. In each period, participants were asked to guess whether they were at the first or second intersection and to place a bet on their guess being correct. Specifically, participants were asked to choose one of the three options depicted in Table 1. Each option is associated with a gain and a loss depending on the guess being, respectively, correct and not. The possibility of gains should incentivize participants to remember maps, even though the concomitant possibility of losses should urge those who suffer from imperfect recall to select option *A*.<sup>24</sup> The joint choice of belief and bet was performed concurrently with the choice of game strategy.

To control for a possible impact of belief elicitation on participants' choices, we conducted both the IMP and IND treatments also without eliciting beliefs. Therefore, our experiment comprises four treatments: IMP with and without belief elicitation (henceforth IMP-With and IMP-Without), IND with and without belief elicitation (henceforth IND-With and IND-Without).

### 4.3 Experimental game trees

The game trees used in the experiment are shown in Table 2. Trees 1–14 were presented to the participants in both phases. Trees 15–18 were presented in phase 2 only: they are filler trees, which we exclude from all analyses.

In addition to ten *paradox trees*, we included two *optimal exit* and two *optimal stay trees*, where the existence of a dominant strategy excludes time inconsistencies due to absentmindedness. The optimal stay trees have the largest payoff at the end (i.e.,  $c > a, b$ ). The optimal exit trees have the largest payoff at the first exit (i.e.,  $a > b, c$ ). The unique optimal pure strategy in these trees is “continue” at both nodes and “exit” at both nodes, respectively. Hence, recall plays no role. Behavior in these trees will provide

game.

<sup>23</sup>In the IMP treatment beliefs are tantamount to guessing the outcome of a fair coin toss.

<sup>24</sup>The numbers in Table 1 are chosen so that the expected payoff from option *A* exceeds the expected payoff from the other two options whenever the probability assigned to being correct is lower than  $2/3$ . Only if the probability of being correct is greater than  $5/6$ , a risk neutral decision maker should opt for *C*.

Table 2: Experimental game trees

Tree number	$a$	$b$	$c$	$p^*$	$\alpha = 1/(1 + p^*)$	Tree type
1	20	50	30	0.75	0.57	Paradox
2	10	80	30	0.70	0.59	Paradox
3	0	40	10	0.67	0.60	Paradox
4	10	50	20	0.67	0.60	Paradox
5	30	90	40	0.60	0.63	Paradox
6	30	70	30	0.50	0.67	Paradox
7	20	80	10	0.43	0.70	Paradox
8	30	60	20	0.38	0.73	Paradox
9	30	70	10	0.33	0.75	Paradox
10	30	50	10	0.25	0.80	Paradox
11	50	10	30			Optimal exit
12	60	40	20			Optimal exit
13	20	10	50			Optimal stay
14	10	30	60			Optimal stay
15	0	50	10			Filler
16	10	80	40			Filler
17	30	90	50			Filler
18	10	60	20			Filler

a check of participants' understanding of the task.

#### 4.4 Procedures

The computerized experiment was conducted in the controlled environment of the laboratory of the Max Planck Institute of Economics (Jena, Germany) in April and December 2009. It was programmed in z-Tree (Fischbacher, 2007). The participants were undergraduate students from the Friedrich-Schiller University of Jena. They were recruited using the ORSEE software (Greiner, 2004). Upon entering the laboratory, the participants were randomly assigned to visually isolated computer terminals. The instructions distributed at the beginning informed the participants that the experiment consisted of two phases, and explained the rules of the first phase only. Written instructions on the second phase were distributed at the end of the first one (a translation of the German instructions for both phases is reproduced in the Appendix). Before starting the experiment, participants had to answer a control questionnaire testing their comprehension of the rules.

We ran two sessions per treatment. Thirty-six students participated in each treatment (so that a total of 144 participants were involved in the experiment). In all treatments, participants did not know the number of maps (and thus periods) beforehand. Moreover, they received no information on the random draws determining their period-payoff and

Table 3: Continue choices for the optimal exit and optimal stay trees

Tree	Proportion of optimal choices		Mean Strategy <sup>a</sup>	
	Plan (1)	Action (2)	Plan (3)	Action (4)
11	92.01%	56.16%	2.07 (0.51)	22.56 (2.13)
12	93.58%	77.60%	1.99 (0.55)	6.86 (1.04)
13	92.36%	84.03%	96.78 (0.90)	94.77 (0.91)
14	92.88%	87.59%	97.50 (0.67)	96.14 (0.75)

<sup>a</sup> Standard errors (based on 144 observations) in parentheses.

the earnings from their guesses until the end of the experiment.

Each session lasted about two hours. We did not impose any time limit on choices, although we strongly encouraged participants to make each decision rather fast. Money in the experiment was denoted in ECU (Experimental Currency Unit), where 10 ECU = 7 euro cents. Participants were informed that the sum of all payoffs accumulated during the several periods of the two phases would determine their final monetary payoff.<sup>25</sup> The average earnings per participant were €35.40 (including a €2.50 show-up fee).

## 5 Experimental results

### 5.1 Planning stage

We use the data from phase 1 (planning stage) to check whether participants understood the task they were facing and behaved according to the incentives. Since phase 1 is the same for all four treatments, we pool the data across treatments so that our analysis relies on 144 individuals.<sup>26</sup>

For the optimal exit and optimal stay trees, we expect participants to behave optimally in close to 100% of the decisions. Table 3 shows that, in the planning stage, the proportion of optimal choices for trees 11 to 14 is above 90% and the mean strategy is to take the optimal action with over 0.95 probability (see columns (1) and (3), respectively). The proportion of optimal choices is lower in the action stage, but the mean strategies are close to optimal.

Realistically, for the paradox trees, we do not expect participants to be able to compute the exact optimal strategy  $p^*$ . However, if participants are sensitive to the payoffs

<sup>25</sup>By paying a small monetary amount over a large number of periods we try to induce risk neutrality.

<sup>26</sup>For each participant and each tree, we take the average over the four colors.

they can obtain from each tree, we do expect that their choices would be strongly correlated with the optimal strategy across trees. Averaging over the 144 participants for each of the 10 paradox trees, we find that choices in these trees are indeed significantly correlated with  $p^*$  ( $\rho = 0.906$ ,  $p < 0.001$ ).

## 5.2 Planning vs. action

The mean strategies by paradox tree, phase and treatment are summarized in Tables 4 and 5. A graphical representation of the same data is provided in Figure 3, where the trees are ordered on the horizontal axis by  $p^*$ .

The mean strategies are lower in the action stage than in the planning stage for all 10 paradox trees and in all four treatments (compare column (1) with (2) and column (7) with (8) in Tables 4 and 5). This difference is statistically significant in all but 3 cases, according to Wilcoxon signed rank tests with continuity correction relying on 36 independent observations (see columns (3) and (9) in the tables). Overall, the probability assigned to continue in the action stage is, on average, 85.1% and 69.0% of that assigned in the planning stage, in treatments IMP and IND, respectively.

Table 6 reports the results of two generalized linear random-effects models (based on Poisson distributions) regressing continue choices on *Phase* (which takes values 0 for the planning stage and 1 for the action stage) while controlling for  $p^*$ , belief elicitation (*BE*, which equals 1 for the treatment with belief elicitation and 0 otherwise), and the interaction of *BE* with  $p^*$  and *Phase*.

The estimated coefficient on *Phase* is negative and highly significant in both treatments, thereby confirming that participants tend to continue less in the action stage in both manipulations of absentmindedness. In the IMP treatment, continue choices do not depend on belief elicitation (the coefficient on *BE* is not significant) and are positively correlated with  $p^*$  regardless of whether beliefs are elicited or not. Moreover, the negative effect of *Phase* on continue choices is less pronounced when  $BE = 1$  (i.e., with belief elicitation). Turning to the IND treatment, continue choices are weakly significantly higher with belief elicitation whereas the other effects are as in IMP. We conclude that time inconsistencies exist in the data, in line with PR's argument and in contradiction to AHP's normative analysis.

The effect of belief elicitation on participants' behavior is also explored via a series of two-sided Wilcoxon rank sum tests comparing the 36 independent continue choices in IMP-With (IND-With) and IMP-Without (IND-Without). The  $p$ -values (reported in Table 7) confirm that for all but three trees belief elicitation does not affect behavior.

Table 4: Continue choices for the paradox trees in the IMP treatments

Tree	$p^*$	IMP-WITH						IMP-WITHOUT		
		Plan <sup>a</sup> (1)	Action <sup>a</sup> (2)	$Plan\ vs.\ Action$ $p\text{-value}^b$ (3)	Action $\beta = X^a$ (4)	Action $\beta = Y^a$ (5)	$\beta = X$ vs. $\beta = Y$ $p\text{-value}^c$ (6)	Plan <sup>a</sup> (7)	Action <sup>a</sup> (8)	$Plan\ vs.\ Action$ $p\text{-value}^b$ (9)
1	75.000	61.458 (2.138)	56.361 (2.504)	0.036	60.588 (2.732)	46.019 (3.643)	0.001	63.882 (2.175)	58.427 (2.226)	0.085
2	70.000	63.181 (2.859)	61.844 (2.541)	0.278	66.745 (2.979)	48.903 (4.249)	0.003	67.840 (2.267)	60.601 (2.605)	0.023
3	66.667	74.076 (2.941)	69.354 (2.464)	0.006	74.762 (2.449)	59.426 (3.926)	0.001	80.007 (2.078)	76.177 (2.796)	0.221
4	66.667	60.722 (1.986)	59.132 (2.832)	0.391	62.781 (2.872)	49.559 (3.931)	0.001	63.840 (2.263)	58.028 (2.393)	0.066
5	60.000	56.667 (2.066)	51.302 (3.076)	0.047	56.205 (3.446)	39.809 (3.863)	0.001	61.028 (2.540)	50.962 (2.630)	0.002
6	50.000	50.653 (1.871)	41.545 (2.682)	0.001	46.325 (2.978)	34.473 (3.626)	0.000	53.250 (1.722)	43.059 (2.489)	0.001
7	42.857	43.174 (2.277)	34.556 (2.903)	0.003	37.828 (3.057)	31.256 (4.163)	0.016	48.035 (2.936)	35.385 (3.334)	0.000
8	37.500	42.389 (2.323)	34.194 (2.770)	0.000	36.844 (2.880)	30.222 (3.968)	0.005	43.778 (2.834)	34.604 (2.720)	0.000
9	33.333	39.215 (2.692)	30.299 (2.932)	0.000	31.071 (3.021)	29.275 (3.615)	0.024	42.917 (2.835)	30.375 (2.956)	0.000
10	25.000	36.993 (2.644)	30.490 (2.611)	0.020	32.046 (2.995)	30.546 (3.548)	0.067	40.875 (2.402)	30.028 (2.621)	0.000

<sup>a</sup> Standard errors (based on 36 independent observations) in parentheses.

<sup>b</sup> Two-sided Wilcoxon signed rank tests with continuity correction relying on 36 independent observations (averages over the 4 colors for each participant).

<sup>c</sup> Two-sided Wilcoxon signed rank tests with continuity correction. Number of independent observations for each map:  $N_1 = 25$ ;  $N_2 = N_3 = 28$ ;  $N_m = 27$  for  $m = 4, \dots, 9$ ;  $N_{10} = 28$ .

Table 5: Continue choices for the paradox trees in the IND treatments

Tree	$p^*$	IND-WITH						IND-WITHOUT		
		Plan <sup>a</sup> (1)	Action <sup>a</sup> (2)	Plan vs. Action $p$ -value <sup>b</sup> (3)	Action $\beta = X^a$ (4)	Action $\beta = Y^a$ (5)	$\beta = X$ vs. $\beta = Y$ $p$ -value <sup>c</sup> (6)	Plan <sup>a</sup> (7)	Action <sup>a</sup> (8)	Plan vs. Action $p$ -value <sup>b</sup> (9)
1	75.000	62.951 (2.849)	42.278 (3.624)	0.000	73.504 (3.328)	36.391 (3.908)	0.000	63.743 (1.601)	49.701 (2.589)	0.000
2	70.000	67.799 (3.121)	41.806 (3.714)	0.000	70.333 (4.016)	29.515 (3.558)	0.000	68.167 (2.149)	50.660 (2.679)	0.000
3	66.667	77.965 (3.363)	44.410 (4.242)	0.000	84.336 (3.701)	31.732 (4.505)	0.000	76.646 (2.004)	53.448 (3.531)	0.000
4	66.667	65.993 (3.218)	40.257 (4.132)	0.000	64.878 (4.789)	32.393 (4.071)	0.000	61.875 (1.542)	48.274 (2.955)	0.000
5	60.000	63.278 (3.142)	39.899 (3.843)	0.000	70.310 (2.949)	25.583 (3.779)	0.000	60.507 (2.049)	43.549 (3.027)	0.000
6	50.000	52.903 (2.332)	35.139 (2.936)	0.000	69.301 (3.732)	19.345 (2.722)	0.000	53.750 (1.780)	35.833 (2.582)	0.000
7	42.857	44.146 (3.169)	29.948 (3.000)	0.001	54.356 (4.419)	17.719 (2.976)	0.000	48.889 (1.819)	33.198 (2.272)	0.000
8	37.500	46.222 (3.180)	30.101 (3.002)	0.000	52.313 (3.958)	21.610 (2.461)	0.000	45.174 (2.223)	35.122 (2.387)	0.000
9	33.333	41.604 (3.083)	29.385 (3.048)	0.000	51.665 (4.224)	19.611 (2.678)	0.000	46.215 (2.351)	32.528 (2.779)	0.000
10	25.000	40.382 (3.037)	27.556 (2.490)	0.000	45.529 (4.302)	20.632 (2.731)	0.000	45.139 (2.272)	30.750 (2.661)	0.000

<sup>a</sup> Standard errors (based on 36 independent observations) in parentheses.

<sup>b</sup> Two-sided Wilcoxon signed rank tests with continuity correction relying on 36 independent observations (averages over the 4 colors for each participant).

<sup>c</sup> Two-sided Wilcoxon signed rank tests with continuity correction. Number of independent observations for each map:  $N_1 = 31$ ;  $N_2 = 26$ ;  $N_3 = N_4 = 32$ ;  $N_5 = 31$ ;  $N_6 = 29$ ;  $N_7 = 27$ ;  $N_8 = N_9 = 30$ ;  $N_{10} = 33$ .

Figure 3: Mean continue choices in the four treatments

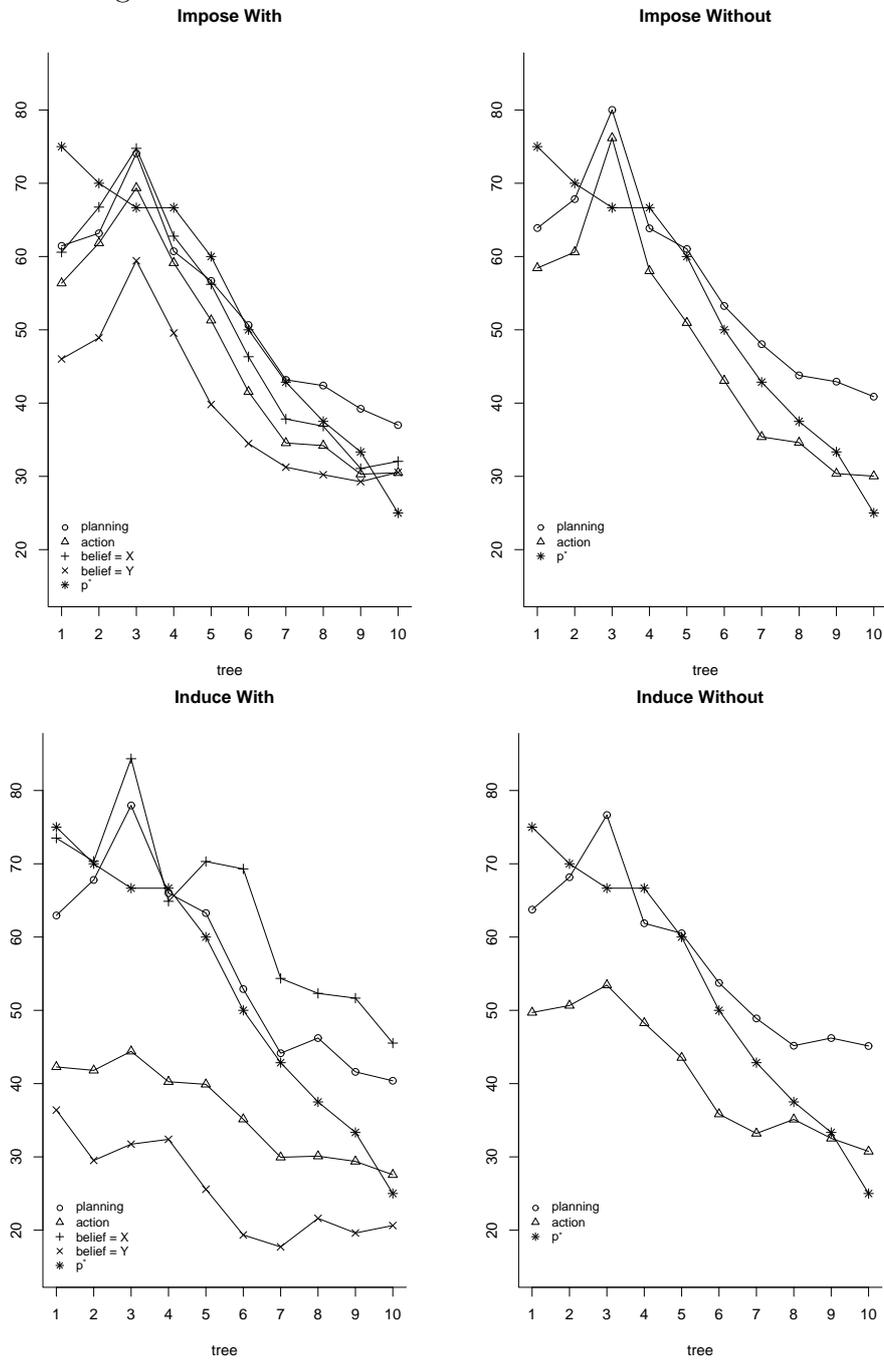


Table 6: Time inconsistencies: generalized linear mixed-effects regression on continue choices

	<b>IMPOSE</b>			<b>INDUCE</b>		
	Coeff	Std.Error	<i>p</i> -value	Coeff	Std.Error	<i>p</i> -value
Intercept	26.022	2.195	0.0000	49.234	2.701	0.0000
<i>Phase</i>	-10.313	0.651	0.0000	-17.094	0.882	0.0000
<i>BE</i>	-0.868	3.085	0.7794	6.433	3.737	0.0896
<i>p</i> *	0.793	0.017	0.0000	0.488	0.023	0.0000
<i>BE</i> × <i>Phase</i>	1.521	0.906	0.0931	-2.824	1.206	0.0192
<i>BE</i> × <i>p</i> *	-0.0658	0.024	0.0067	-0.108	0.030	0.0004

Table 7: Comparing continue choices with and without belief elicitation in the action stage

Tree	<b>IMPOSE</b>		<b>INDUCE</b>	
	WITH vs. WITHOUT	<i>p</i> -value	WITH vs. WITHOUT	<i>p</i> -value
1		0.681		0.028
2		0.021		0.021
3		0.101		0.101
4		0.636		0.069
5		0.955		0.316
6		0.713		0.761
7		0.933		0.248
8		0.901		0.195
9		0.924		0.305
10		0.831		0.528

### 5.3 Bets and beliefs

We first look at the bets made on the beliefs to ascertain the extent to which the participants *feel* absentminded. Taking the betting choices in treatment IMP-With as a baseline, we find that, overall, participants were unlikely to choose the risky bets  $B$  and  $C$ . In fact, participants are even more likely to choose the safe option  $A$  in IND-With compared to IMP-With (84.2% vs. 79.4%). Choices of the highly risky option  $C$  are quite rare in both treatments, although more frequent in IND-With (4.9% vs. 2.0%), where option  $B$  is selected less frequently (10.8% vs. 18.5%). We conclude that, moving from the IMP-With to the IND-With treatment, the shift to the cautious option  $A$  is larger in magnitude than the shift to the risky option  $C$ . This indicates that participants are not systematically more confident about their bets in treatment IND-With than in treatment IMP-With, in which they are absentminded by definition.

The mean continue choices by stated beliefs (labelled  $\beta$ ) are presented in columns (4) and (5) of Tables 4 and 5. The mean strategy chosen when participants guess to be at node  $Y$ <sup>27</sup> is significantly lower than that chosen when participants guess to be at node  $X$  for all 10 trees and for both IMP-With treatment and IND-With treatment, in line with the CE prediction.

To test the correlation between stated beliefs and game strategies while controlling for other variables, we use the generalized linear random-effect models reported in Table 8. The model regresses continue choices on *Belief*, controlling for  $p^*$ , *Actual node*, and *Period*. Continue choices are negatively and significantly correlated with the stated beliefs. However, in the IND-With treatment this result can be due to participants not being absentminded, i.e., knowing which decision node they are at. Indeed, in Model 1 of the IND-With panel, strategies are also correlated with the actual decision node, indicating that participants' beliefs are more accurate than expected by chance. Yet, this can happen not only if participants can recall their history, but also if they use the period as a cue. To control for this possibility, we perform Model 2 (which includes *Period* among the covariates). We find that the effect of the actual node disappears when controlling for *Period*, while the effect of *Belief* remains unchanged. In line with the unequivocal correspondence between beliefs and decisions observed in the IMP-With treatment, we conclude that, in the IND-With treatment, the correspondence is, at least in part, not due to an artifact. This observation supports the CE hypothesis.

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<sup>27</sup>In the experiment the maps were presented as Figure 1's original game tree. Hence we denote the second exit by  $Y$ , which encompasses both  $Y$  and  $Y'$  in Figure 2.

Table 8: Beliefs-strategies contingencies

	IMP-WITH				IND-WITH			
	Model 1		Model 2		Model 1		Model 2	
	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Intercept	23.227 (2.803)	0.000	23.667 (2.822)	0.000	83.226 (2.974)	0.000	82.079 (2.999)	0.000
<i>Belief</i>	-11.338 (0.745)	0.000	-11.280 (0.746)	0.000	-37.341 (0.936)	0.000	-37.079 (0.939)	0.000
<i>p</i> *	0.755 (0.027)	0.000	0.759 (0.027)	0.000	0.378 (0.028)	0.000	0.388 (0.029)	0.000
<i>Actual node</i>	-0.352 (0.554)	0.525	-0.342 (0.554)	0.537	-2.243 (0.679)	0.001	-0.350 (0.921)	0.704
<i>Period</i>			-0.012 (0.009)	0.171			-0.048 (0.016)	0.002

Note: Numbers in parentheses are estimated standard errors.

## 6 Conclusions

The vast majority of theoretical and experimental research effort to understand rational decision making has so far been confined to situations of perfect recall. Nonetheless, imperfect recall is likely to play a significant role in many real-world decision problems. Firms or countries, for example, are often modeled as single players, although different elements in their strategies have to be decided by different agents, sometimes lacking information about the decisions of other parts of the aggregate player (cf. PR, Binmore, 1996). Furthermore, even a single person is likely to suffer from imperfect recall as storing and accessing huge amounts of information is practically impossible.

Some issues arising from imperfect recall are well illustrated by the paradox of the absentminded driver.<sup>28</sup> This paper joins the theoretical efforts devoted to the paradox, and complements the theoretical discussions by providing a positive analysis of the problem. Specifically, we report on an experiment designed to compare behavior in a planning stage and an action stage of a decision problem featuring absentmindedness. In the minimal setting, as implemented in our IMP-WITHOUT treatment, the decision task is almost identical in the two stages, with the only difference being that in the action stage participants provide two strategies, whereas in the planning stage they provide a single strategy to be implemented twice. Despite the fact that payoff is maximized by the same strategy in both cases, we find that this difference is enough to lead to a systematic variance in behavior, as suggested by PR. Namely, participants tend to exit more in the action stage than in the planning stage. This result is supported by the findings in the IND treatment, in which absentmindedness is implemented in a more natural way.

Examining the elicited beliefs, we find a significant correlation between stated beliefs and game strategy: participants assign, on average, a lower probability to continue when they guess to be at the second (rather than the first) node. This finding is not consistent with PR's analysis, which never makes a normative distinction between different times in which an information set is reached. It is, however, predicted by the commitment heuristic. According to our interpretation, when faced with the uncertainty inherent in the decision problem under absentmindedness, participants employ a simple heuristic, by which they consider a single deterministic state of the world and mentally commit to it. As the belief elicitation taps into the mentally-constructed state of the world, this process leads to a contingency between decisions and stated beliefs,

We should note that this contingency between beliefs and strategies is merely correlational. Therefore we cannot completely rule out alternative explanations. One such explanation is that trembles in the strategy lead to systematically different beliefs driven

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<sup>28</sup>Other directions of research focus on the analysis of players with bounded complexity (see, e.g., Abreu and Rubinstein, 1988; Lehrer, 1988; Rubinstein, 1986).

by a preference for consistency. E.g., participants who indicate a high probability to continue refrain from guessing that they are at the second exit in order to avoid the apparent discrepancy between the two choices. However, this explanation does not rationalize why planning and action decisions should differ. Hence, it cannot account for the observed difference between plans and actions, which is apparent regardless of belief elicitation. Our interpretation, on the other hand, is based on a single process which fully predicts and organizes the data.

Our results are also related to the theoretical analysis developed by Binmore (1996), who modeled the absentminded driver paradox as a repeated decision problem, somewhat akin to our action stage. It is noteworthy that the behavior we observe in the action stage conforms to the normative prescription of Binmore (1996), although he assumes that the decision maker can remember all her past decisions and thus derive a frequency-based belief over decision nodes. Conversely, our participants did not receive feedback between maps and encountered each information set twice.

To sum, this paper presents a first step in understanding the behavior of people in situations involving absentmindedness. PR offered two ways to analyze games with imperfect recall, which differ with regard to a player's beliefs about her strategy when the same information set occurs again. If we assume that the player expects to follow the same reasoning as she does in the current occurrence of the information set, time inconsistencies may arise. AHP reject this assumption, and therefore argue that optimal strategies are always time consistent. Conversely, we provide conclusive descriptive evidence for the existence of the time inconsistencies, and elucidate the process driving these inconsistencies. Our behavioral data support PR's assertion that both approaches carry some validity, and may be useful in understanding and predicting behavior in some situations. Thus, our contribution is on two levels. First, our conclusions should be considered in behavioral applications, e.g., when designing institutions for people who may suffer from absentmindedness. Second, we contribute to the theoretical debate, by providing an example of how the analysis provided by PR illuminates actual behavior, in contrast with AHP's position. In this paper we focus on absentmindedness caused by the cognitive limitations of human decision makers. We leave it to future research to determine whether other related phenomena, such as absentmindedness of organizations, differ in this respect.

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## Appendix: Experimental instructions

NOT FOR PUBLICATION

This appendix reports the instructions (originally in German) we used for the IND-With treatment. The instructions for the other treatments were adapted accordingly and are available upon request.

### INSTRUCTIONS

Welcome and thanks for participating in this experiment. Please remain quiet and switch off your mobile phone. Stow away any reading or writing materials: your table should contain only these instructions. Do not speak to the other participants. Communication between participants will lead to the automatic end of the session with no payment to anyone. Whenever you have a question, please raise your hand and one of the experimenters will come to your place.

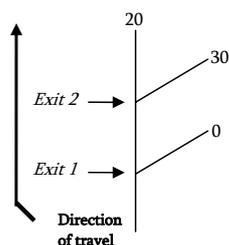
You will receive 2.50 euros for having shown up on time. The experiment allows you to earn additional money. Since your earnings during the experiment will depend on your decisions, and may depend on chance, the better you understand the instructions, the more money you will be able to earn.

In this experiment you will not interact with any other participant. The decisions of the other participants will not affect your earnings and, similarly, your decisions will not affect the earnings of the other participants.

During the experiment, we shall not speak of euros but rather of ECU (Experimental Currency Unit). ECU are converted to euros at the following exchange rate: 10 ECU = 7 euro cents.

### DETAILED INFORMATION ON THE EXPERIMENT

Imagine yourself driving up the highway as you see in the following picture.



When you approach Exit 1, you have to decide whether you want to continue on the highway or you want to take that exit. If you decide to take Exit 1, then you terminate your journey. Otherwise, you continue to Exit 2, where again you must decide whether to continue on the highway or to exit. The numbers on the highway tell you the amount of money in ECU that you would earn based on where you choose to go. In the example above, you would earn: 0 ECU if you take Exit 1; 30 ECU if you continue at Exit 1 and take Exit 2; 20 ECU if you continue at

both exits. In this example, your maximum earnings would be achieved by continuing at Exit 1 and then taking Exit 2.

In the experiment, you will be shown several highways differing in the amount of ECU they yield. Each highway will be presented to you in 4 different colors (yellow, green, blue, red). In the following, we shall refer to a highway in a specific color as a map. Highways with the same earnings but different colors correspond to different maps. You can think of the different colors as different days in which you are driving on the same highway. Suppose that you are shown the highway depicted in the above example first in yellow and then in blue. Your travel on the yellow highway takes place on one day and your travel on the blue highway takes place on another day. Therefore, your earnings from the two travels are independent of each other.

The experiment consists of two phases. The instructions for the first phase follow on this page. The instructions for the second phase will be distributed to you at the end of the first phase. This is done to avoid confusion between the two phases. Your payoff from any of the two phases is determined only by what you do in that phase and is not affected by what you do in the other phase.

At the end of today's session, i.e., after the second phase, the amount of ECU you have earned in each period of the two phases will be added up. The resulting sum will be converted to euros and paid to you in cash and privately (i.e., without the other participants knowing your earnings) together with the show-up fee of 2.50 euros.

## **PHASE 1**

There will be a series of periods in this phase. In each period, you will be shown a map (i.e., a highway in a specific color). Like in the example above, it is as though you are starting at the bottom of the map and driving up the highway, and your payoff will depend on where you go. For each map, you must make a single decision that applies to both exits.

For each map, you decide as follows. Imagine an urn with 100 balls. You can determine how many of these balls stand for "continuing" and how many stand for "exiting". Once you have decided the composition of the urn, the computerized program will randomly draw a ball from the urn (and put it back afterward). If the randomly drawn ball shows "exit", then you take the first exit and earn the corresponding amount of ECU. If the ball shows "continue", then you continue to Exit 2 and the program will randomly draw a second ball from the urn. Depending on whether the ball shows "continue" or "exit", you get the corresponding earnings.

To determine the composition of the urn, you must enter a number in each of the two boxes that you will see on the screen below the map. One box is labeled "exit" and the other is labeled "continue". The number you enter in the exit-box determines the number of exit-balls in the urn. Likewise, the number you enter in the continue-box determines the number of continue-balls in the urn. The sum of the two numbers you enter must be 100.

EXAMPLE 1. If you enter 80 in the continue-box and 20 in the exit-box, then the urn will contain exactly 80 continue-balls and 20 exit-balls. This means that when the program randomly

draws the first ball from the urn, you will have 80% chance of continuing on the highway and 20% chance of taking Exit 1. If the first randomly drawn ball shows “continue”, then at the second random draw (corresponding to Exit 2) you will again have 80% chance of continuing on the highway and 20% chance of taking Exit 2. Therefore you will have  $80\% \times 80\% = 64\%$  chance of continuing beyond Exit 2 and  $80\% \times 20\% = 16\%$  chance of taking Exit 2.

EXAMPLE 2. If you enter 40 in the continue-box and 60 in the exit-box, then the urn will contain exactly 40 continue-balls and 60 exit-balls. This means that when the program randomly draws the first ball from the urn, you will have 40% chance of continuing on the highway and 60% chance of taking Exit 1. If the first randomly drawn ball shows “continue”, then at the second random draw (corresponding to Exit 2) you will again have 40% chance of continuing on the highway and 60% chance of taking Exit 2. Therefore you will have  $40\% \times 40\% = 16\%$  chance of continuing beyond Exit 2 and  $40\% \times 60\% = 24\%$  chance of taking Exit 2.

EXAMPLE 3. If you enter 0 in the continue-box and 100 in the exit-box, then the urn will contain only exit-balls. This means that you will take Exit 1 with certainty.

As it is evident from examples 1 and 2 above, you can make a decision that does not mean exiting or continuing with certainty. However, if you wish to make such a decision, you can do so by entering 100 in one box and 0 in the other (like in example 3 above).

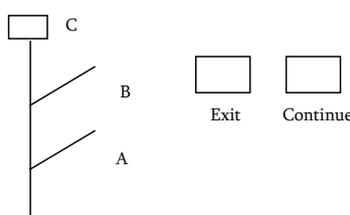
During the two phases of the experiment you will have to make many decisions. If you make each decision in 20 seconds, the two phases will last more than one hour. Thus, in order for the experiment to take not too long, we strongly encourage you to decide rather fast.

You will NOT receive any information about the random draws, and thus your earnings, until the end of today’s session.

**Practice stage**

Before the experiment starts you will have 15 minutes of practice to get familiar with your task. This stage is conducted only to help you learn how the experiment works, and does not count towards your payoff. During this time you can choose different highways to practice on, and see the consequences of different choices for each highway.

On your screen you will see a highway with three empty payoff-boxes (A, B, and C in the picture below), one exit-box, and one continue-box.



In order to choose a highway to experiment with, you have to enter a number between 0 and 200 in each of the three payoff-boxes A, B, and C. The number you enter in the payoff-box labeled A stands for the ECU you would earn if you take Exit 1. Similarly, the numbers you

enter in the payoff-boxes labeled B and C stand for the ECU you would earn if you take Exit 2 or continue at both exits, respectively.

The exit-box and the continue-box allow you to determine the composition of the urn from which the program will make the random draw(s) deciding where you go and thus how much you earn. The numbers you enter in the continue-box and the exit-box must add up to 100.

Once you have entered a number in each of the 5 boxes, if you press the button marked “I want to test this highway and this composition of the urn” you will see on the screen:

- the chances you have to take Exit 1, to take Exit 2, or to continue beyond Exit 2 based on the numbers you have entered in the continue-box and in the exit-box;
- the expected payoff in ECU given your choices.

You can change all or some of the numbers you have entered as many times as you want, and then press the button to know the consequences (as explained above) of your choices.

After you have chosen a highway and a composition of the urn for that highway, you can experience “travelling” along the highway and observing whether you end up in A, B, or C. For that you can press the button marked “Travel”. Each time you press this button, the program will make the random draw(s) based on your decision, and show the result. In order to make many travels and see what happens for each of them, you can press the “Travel”-button as many time as you like.

During the 15 minutes of practice, you can repeat all the steps above as many times as you wish. Notice that you can enter a number in the three payoff-boxes only in the practice stage. Thereafter, the payoffs that you can earn are given.

Before the practice stage starts, you will have to answer some control questions to verify your understanding of the rules of the experiment.

*Please remain seated quietly until the experiment starts. If you have any questions please raise your hand now.*

## **PHASE 2**

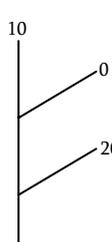
In this phase, you will be asked to imagine yourself travelling along the highway. During your travel you encounter the exits one after the other.

During this phase you will again see a series of maps. But, differently from phase 1, you will see each map exactly twice. Since a map is identified by both a highway with some earnings and a color and since each highway is shown to you in 4 different colors, you will see the same highway  $2 \times 4 = 8$  times, but only twice in the same color.

The first time you see a map, your decision applies to Exit 1; the second time you see the map, your decision applies to Exit 2. The maps are displayed in a preselected order and you will never see the same map in two consecutive periods. Hence the map you will see in period 1 will not be used in period 2, but it may appear again in period 3, or period 4, or any other period during this second phase.

Like in phase 1, for each map you must decide how many of the 100 balls contained in an urn should stand for “continuing” and how many for “exiting”. Below each map you will again see a continue-box and an exit-box, in each of which you must enter a number. The sum of the two numbers you enter must always be 100.

What is different is that in phase 2, the program will randomly draw only one ball from the urn. The drawn ball will determine your decision for the current exit of the shown map. If, for instance, you are shown a map for the first time and enter 20 in the exit-box and 80 in the continue-box, you will have 20% chance of taking Exit 1 and 80% chance of continuing. On the other hand, if you are shown a map for the second time and enter 50 in the exit-box and 50 in the continue box, you will have 50% chance of taking Exit 2 and 50% chance of continuing. Of course, where you end up depends on the two decisions you make for a particular map as well as on the random draw. The table below shows 3 possible cases.



	First time you see a map (Exit 1 decision)	Second time you see a map (Exit 2 decision)	Earnings
<b>Case 1</b>	10 in exit-box and 90 in continue-box; a continue-ball is randomly drawn	50 in exit-box and 50 in continue-box; a continue-ball is randomly drawn	10
<b>Case 2</b>	10 in exit-box and 90 in continue-box; an exit-ball is randomly drawn	50 in exit-box and 50 in continue-box; a continue-ball is randomly drawn	20
<b>Case 3</b>	10 in exit-box and 90 in continue-box; a continue-ball is randomly drawn	50 in exit-box and 50 in continue-box; an exit-ball is randomly drawn	0

Notice that even though you enter 100 in the exit-box (therefore deciding to exit with certainty) the first time you see a map, you will still be shown the map a second time. In this case it does not matter what decisions you make the second time.

You do not know how many periods there are in this phase. During the phase you will not be informed of how many periods are left, nor of whether you are at the first or second exit of the current map.

For each map, before making your decision about the number of exit-balls and continue-balls that should be contained in the urn, the computer will ask you to guess whether you think of being at Exit 1 or at Exit 2; that is, whether you think to see the current map for the first time or for the second time. You will have to place a bet on your guess being correct by choosing one of the three options shown below:

Your choice	Option	If your guess is <i>correct</i> you WIN	If your guess is <i>wrong</i> you LOSE
0	A	1	1
0	B	3	5
0	C	5	15

Note that option B and C offer higher payoffs if you are correct, but also carry higher losses if you are wrong. Therefore you are advised to choose option C only if you are very sure that you are correct, option B if you think that you are probably correct, and option A if you are very unsure that you are correct.

*Please remain seated quietly until the experiment starts. If you have any questions please raise your hand now.*