Product Quality in Scientific Competition

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Abstract

The paper presents a linear model of product quality in scientific competition. The only outputs of research are published papers; the only inputs are labor and papers by other researchers, which are cited when used. Researchers compete for status, measured as their rank in a citations count. If quality is hereditary in the production process, competition and self-fulfilling expectations can establish a quality scale.

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1 Introduction

Science is an industry in which norms of product quality play an important role. In contrast to quality norms for consumer goods, quality norms in science are not imposed from outside the industry. Rather, they emerge endogenously within scientific competition.

Of course, researchers believe in ideals and norms even before they enter science. Curiosity is often considered an important motivation for choosing a career in science, and it is hard to see how somebody could be curious without being interested in the truth about matters. Hence, the love of truth might be a norm that researchers carry over into science.

However, neither curiosity nor a love of truth are sufficient for judging the quality of a research paper. It takes long training to appreciate the products of science. The quality norms established in science are not easily explained to outsiders. In fact, they are also a matter for debate among insiders. Both facts indicate that quality norms are endogenous to science.

Product quality in science is traditionally analyzed by philosophers under the heading of methodology. While early contributions assumed methodological norms to be reducible to logical analysis, Popper (1959) demonstrated that, actually, methodological norms must be social conventions (see also Jarvie 2001).

This suggests that at least some aspects of scientific methodology are susceptible to an economic approach. However, the economic analysis of science (e.g., Stephan 1996) has so far ignored product quality. The same is true of other approaches to the scientific analysis of science (see Merton 1973, Hull 1988, Simonton 1999), which, while contributing significantly to our understanding of scientific competition, do not systematically considers norms of product quality.

The present paper uses a very basic model of scientific competition in order to explain the emergence of a quality scale. Following Merton (1973) and Hull (1988), researchers' behavior is explained by status seeking. In contrast to recent work in the philosophy of science, which uses a constitutional-economics perspective (Zamora Bonilla 2002, Luetge 2004), there is no centralized decision making. Or in other words: it is not assumed that the norms established in scientific competition are *ex ante* efficient from the researchers' point of view.

The paper is concerned with basic research; incentives in applied areas or technology are presumably quite different. The model is based on a much simplified account of the research process. The tangible outputs of research are published papers containing ideas and results, that is, theories, models, observation reports, and so on. The only inputs in addition to labor are ideas and results of other researchers contained in published papers. Thus, as a shortcut, the production process can be described as the production of papers by means of papers and, of course, labor.

It is assumed that researchers compete for status, which rises when other researchers make use of their original ideas or results. Given the usual citation rules, which require the citation of papers if ideas and results from these papers are used, status can be measured in a citations count. Of course, citations rules themselves have to explained. The formal model of product quality takes citation rules as given. However, the model is supplemented by an informal explanation of the incentive compatibility of citation rules.

In scientific competition, product quality is important for the selection of inputs. Researchers use the work of others if they think that it is good enough to build upon it (Hull 1988). Hence, if a paper gets a negative evaluation, its ideas and results will not be used by other researchers, which means that the author fails in the quest for status. Thus, norms of product quality are relevant for the selection of input, and, therefore, citation behavior.

The model presented in this paper tries to capture this basic point. It identifies a necessary condition for the establishment of quality norms. The quality of the input must affect the quality of the output, that is, quality must be hereditary. Without this link between input and output, there is no reason to be selective in the choice of inputs.

The simplest model where the basic idea can be demonstrated assumes that a quality norm classifies papers into two jointly exhaustive categories, X and Y. Using (and citing) X-papers raises the chances of producing an X-paper, while using (and citing) Y-papers raises the chances of producing a Y-paper. If researchers expect X-papers to earn more citations than Y-papers, they prefer X-papers as an input for their own research.

Although many equilibria are possible depending on the exact specifications of the model, the following three simple and plausible equilibria should exist under many conditions: a neutral equilibrium where researchers ignore the categories when deciding whether to cite a paper; an X-preference equilibrium, where everybody cites only X-papers; and a Y-preference equilibrium, where everybody cites only Y-papers. These equilibria can be established by self-fulfilling expectations: researchers behave in that way because they expect others to behave in the same way.

In models with more than two categories, self-fulfilling expectations can establish a quality scale. In general, several such scales are possible. This leaves some room for preconceptions imported into science, which can establish focal points. However, not just any kind of expectation turns out to be self-fulfilling.

Two classes of models in the literature show some relation to the basic model of scientific competition. In markets, producers often have to choose between the production of high-quality or low-quality goods. If quality can only be observed after the purchase, production of high-quality goods raises a problem of cooperation between producers and their customers: producers must keep their promise of delivering high quality, and their customers must trust them and pay the price for high-quality products. The problem can be solved if producers can rationally invest in a reputation for delivering high quality (Rasmussen 1994a: 131-134). Necessary conditions for such a solution are given by the folk theorem for infinitely repeated games (see, e.g., Fudenberg and Tirole 1991: 150-165).

Another competitive process related to scientific competition is the output of judges in systems of common law, where a judge may earn status by creating precedent (Miceli and Coşgel 1994, Rasmusen 1994b). Following precedent, or *stare decisis* in lawyers' terminology, involves citation, while not following precedent might earn citations if a new precedent is thereby created, but runs the risk of reversal. Rasmusen (1994b) considers a repeated game where *stare decisis* is a matter of cooperation among judges. Again, the folk theorem states necessary conditions for such cooperation to be possible. Miceli and Coşgel (1994), on the other hand, invoke preferences of judges, who face a trade-off between following precedent or their private inclinations.

In contrast, the present model of scientific competition requires no cooperation between researchers. The relevant problem is a coordination problem; the folk theorem is not invoked.

The paper proceeds as follows. Section 2 motivates the basic assumptions of the paper. Section 3 states the model of scientific competition. Section 4 discusses and extends the results of section 3.

2 Basic Assumptions and Motivation

2.1 Product Quality

There are two kinds of quality norms in science. Procedural norms recommend a certain procedure. Evaluation norms evaluate the products of science in the light of certain regulative ideas (H. Albert 1978: 31). Regulative ideas in science are realism—or, in plain words, truth—, mathematical correctness, simplicity, explanatory power, political relevance, empirical success, fruitfulness, and so on.

In the case of a procedural norm, a researcher can achieve high quality by just following the recommendation. In economics, the rules for the design of economic experiments are a good example, for instance, Vernon Smith's rule to pay subjects according to performance instead of compensating them for mere participation (see, e.g., Camerer 2003: 34-42). Researchers can decide whether to comply with such a rule or not.

In the case of evaluation norms, there is no procedure that ensures high quality. Researchers can try to produce work that has high quality in the light of certain regulative ideas; for instance, they can try to invent a simple model whose consequences are consistent with given observations. Yet, despite the author's efforts, the model may remain complicated; it may have overlooked consequences that are inconsistent with observations; and the consequences the author seemingly deduced from the model may actually not follow because of mathematical errors.

Section 3's model is concerned with evaluation norms. Explaining evaluation norms is the more basic task because a procedural norm is usually adopted on the basis of evaluation norms. Pay for performance, for instance, is recommended because it gives control over the subjects' motivations. Improved experimental control, in turn, allows for more severe testing, which is an improvement if one aims at finding true theories. Given some payment scheme in an experiment, it can therefore be asked whether the scheme actually establishes control, which may be doubtful if payment is too low. Moreover, payment for performance makes it difficult to test certain theories, for instance, theories about intrinsic motivation. Thus, the pay-for-performance rule is not an unambiguous and unconditional recommendation; it needs interpretation in the light of regulative ideas.

Some regulative ideas suggest a quality scale, while others suggest a dichotomy. Theories range from very simple to highly complicated, while mathematical deductions are either correct or not. However, already by considering two different regulative ideas at once, the category scheme becomes a matrix, and there is no longer an obvious scale. Section 3's model just assumes $k \leq 2$ different categories and shows how a quality scale can emerge as an equilibrium. Which of the categories is to be interpreted as high quality is determined endogenously. Since, however, the categorization is based on evaluation norms, there is no recipe to produce high quality with certainty; researchers can only influence their probability of producing high quality.

The quality of research may gradually emerge during the research stage or in the process of writing. On the other hand, researchers may learn about the true quality of their ideas or results only long after publication since classification can be difficult depending on the categories in question. In order to avoid problems of sequential decision making, the model assumes that researcher learn about the quality of their research only upon publication. Then, quality is established once and for all and can be ascertained by any reader, which is certainly a simplification, although cases where the evaluation of an idea or result changes drastically over time seem to be relatively rare.

2.2 Incentives of Researchers

Following Merton (1973) and Hull (1988), it is assumed that researchers compete for status; other motives like financial gain or curiosity are not taken into account. Status is determined by the use of ideas, while citation is only an indication of the use of ideas. A basic model, then, could ignore the question of citation rules and concentrate on the use of ideas alone, assuming that researchers are able to monitor the use of ideas by some means or other. However, the connection between status and citation is so important in practice that I prefer to include citations even in a basic model, although in a much simplified fashion.

Section 3's model is based on the assumption that researchers perfectly adhere to the priority rule of citing the first work where an idea they use was published; hence, use and citation coincide. Section 4 explains in an informal way why citation rules are incentive compatible, thus indicating extensions of the model where the citation decision is modeled explicitly.

It is assumed that the status of researchers is equivalent to their rank in a citations count. This is a simplification on two counts. First, it is possible that the importance of the citing papers is relevant for status, which suggests the use of a cumulative impact factor instead of a citations count. Second, once an idea is common currency, it is often used without formal citation, as in the case of Nash equilibrium.[1]

In the model, the only compensation for research is status. Implicitly, it is assumed that researchers receive an income—for instance, a compensation for teaching (Dasgupta and David 1994)—that is not affected by their successes or failures in research. Thus, if research has some value outside research, researchers supply a public good (Nelson 1959, Arrow 1962).

The present paper is not concerned with the welfare economics of science. In the model, the level of scientific activity is exogenous. Subsidies, as discussed by Nelson (1959) and Arrow (1962), may be necessary for, and effective in, raising the level of scientific activity, although even without subsidies, competition can lead to excessive private investment.[2] Given that much of science is subsidized today, the problems connected with subsidies

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[1] On the practice of eponymy, that is, using the name of a researcher to refer to an idea, a result, or even a field of research, see Merton (1973) and Stigler (1999: ch. 14).

[2] See Stephan (1996: 1206-1207) for the relevant literature. See also Congleton (1989), who additionally considers competition between different status games.

are important. However, before turning to the influences of subsidies, a basic model of scientific competition without subsidies should be analyzed as a point of reference.

The model assumes that the output of research is published in the form of scientific papers. No referee process exists; every paper is published. Thus, if quality norms are established, the only reason is that researchers accept them in their own interest; they are not imposed by editors or referees. The argument for abstracting from the peer review process is the same as in the case of subsidies: a basic model of quality norms in scientific competition is needed as a reference point.[3]

The paper does not consider productivity differences between researchers, although it is well-known that these are in fact quite large (see Simonton 1997). However, these differences are probably not relevant for the emergence of quality norms. Given that productivity differences are ignored anyway, we might as well assume that every researcher produces just one paper. Cooperation between researchers in the production of papers is ignored.

The inputs into research are labor, that is, time spent on research, and previously published papers, which stand for the ideas or results they contain. Labor and papers may be substitutes or complements, that is, making use of the previously published ideas and results may save labor or may cost labor. It is assumed that, *ceteris paribus*, researchers minimize their labor input. This assumption contradicts the impression that researchers enjoy research so much that they take more time for it once their time budget increases (e.g., when they get rid of some official function). However, even researchers for whom research is a superior good economize on the time spent on one paper in order to gain time for writing another one. Thus, what seems to be a special assumption in the present simplified model emerges as a conclusion from a more general and more realistic model: researchers economize on the time spent on writing each single paper.

Given that the priority rule is perfectly adhered to (or that the use of ideas is perfectly monitored in some other way), it does not pay to plagiarize because this earns no citations and no status. Perfect observance of the priority rule, however, requires more than a willingness to observe the rule. It presupposes knowledge of the relevant literature, which may be imperfect because there is too much of it. Under such circumstances, the priority norm may be widely acknowledged but still violated inadvertently because reading a significant part of the literature is too costly.

If knowledge of the literature is imperfect, copying a paper (the most

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[3] For possible inefficiencies introduced by the peer review process and a solution proposal see Frey (2003).

drastic case of plagiarism) is a potentially successful strategy. Those who adopt it may actually provide a service to the scientific community since, instead of increasing the number of different papers, they give previously overlooked papers a further run. Since the priority rule is still observed, if imperfectly, by most researchers, those who copy papers save labor but take the risk of not earning a citation because somebody reads both their paper and the original. In equilibrium, then, the two pure strategies of copying papers and of publishing original research may have the same expected payoff.

While the analysis of such opportunistic strategies is interesting,[4] there seems to be no connection with the issue of quality norms. Therefore, the present paper assumes that all researchers read and know all of the relevant literature. Papers that are no longer read are outdated or unavailable and can no longer be copied with a chance of earning citations.

In the model, a continuum of researchers publish a continuous flow of papers. In such a setting, competition between researchers is perfect in the sense that it does not pay to ignore a previously published idea just because using it means that a competitor earns a citation. When researchers decide whether to use and cite papers of competitors, they consider only the effect on their own citations count.

2.3 Quality and Citation Probability

It is assumed that only the quality of a paper determines the citation probabilities for the paper. Hence, the citation process involves both randomness and selection.

For each researcher writing a new paper, a sample of papers from the available literature turns out to be relevant. From the perspective of the researcher, there are good reasons why these papers are relevant. From the perspectives of previous researchers, however, the sample is a random sample. This reflects the idea that each researcher considers a new problem, or takes a new perspective on some old problem, making it unpredictable for previous researchers whether their paper will be relevant to their successors. This assumption expresses the basic tenet of evolutionary epistemology (Campbell 1974, Simonton 1997, 1999): science develops through blind variation of ideas and selective retention, where retention in the present model means using ideas and results from previous researchers.

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[4] See Broad and Wade (1982) for the view that opportunistic behavior in science is much more prevalent than the idealized picture drawn by Popper (1959), Merton (1973) and others allows, and Hull (1988: 313-317) for a detailed criticism of Broad and Wade's argument and evidence.

Blind variation of ideas does not mean that any wild idea is published. The selection process has many stages; a new idea has to pass most of them before it is written down by a researcher. The present paper is concerned with the last, social stage, which interacts with prior stages through the selection of inputs for a new paper.

The problem is to explain why being in a certain category gives a paper a greater chance of earning citations. Even if all researchers prefer to produce X-papers, this alone creates no incentive to *use and cite* predominantly papers in category X. But if X-papers are not cited more than other papers, there is no reason for researchers interested in earning citations to aim at producing X-papers in the first place. In order to explain an X-preference, some link between input and output quality must be assumed.

Let us consider an example of a possibly relevant quality norm. For instance, theoretical papers may be mathematically correct or, more generally, deductively sound, meaning that the paper is consistent and that the results follow deductively from the assumptions (category X), or not (category Y). Why might a researcher who wants to produce a deductively sound paper cite deductively sound papers? Remember that, in the present model, a paper is cited only if it is used. Using a paper means that at least some part of its argument is included into one's own paper. If this argument is not valid or its premises are inconsistent, the paper using the argument will be deductively unsound unless the problem is a missing premise and the new paper happens to contain the missing premise, so that the argument is completed by chance.

Hence, in the case of deductive soundness, using papers that are deductively unsound makes it more likely that one's own paper is deductively unsound (contamination). Whether using deductively sound papers increases the chances of writing a deductively sound paper (support) is unclear. At least in a paper breaking new ground, the risk of combining inconsistent premises or of not including a necessary premise may be quite the same whether premises are taken from deductively sound papers or whether they are invented from scratch. On the other hand, using standard arguments means that mathematical errors are less likely to creep in.

This example illustrates two points. First, it may be difficult to check whether a paper falls into a certain category or not. Second, using a certain kind of papers increases, at least for some categories, the probability that one's own paper falls into the same category.

The model assumes that the quality of a paper can be ascertained by its readers; it is only concerned with the second and more important point. It is argued that the basic mechanism for the emergence of quality norms is a self-fulfilling expectations equilibrium based on a specific property of the production function in scientific research: quality is hereditary.

If using and citing papers saves labor, researchers will tend to use as much papers as possible, unless there is a negative effect on quality. Hence, selective citation presupposes that low-quality papers *contaminate* the production of high quality: researchers will only hesitate to cite low-quality papers if this increases their risk of producing a low-quality paper. Thus, if papers and labor are substitutes, an X-preference equilibrium where X-papers are high quality presupposes contamination by low-quality Y-papers.

If, on the other hand, papers and labor are complements, researchers will tend to use as few papers as possible. Selective citation then presupposes that high-quality papers *support* the production of high quality: researchers will only be prepared to cite high-quality papers if this increases their chance of producing a high-quality paper.

Contamination and support, then, are the two relevant cases of hereditary quality. Depending on whether labor and papers are substitutes or complements, one or the other must be present for a self-fulfilling expectations equilibrium where researchers prefer to cite high-quality papers because they believe that high-quality papers have a higher chance of earning citations.

3 A Linear Model of Scientific Competition

All researchers are identical and write exactly one paper. All papers are published. Researchers maximize a utility function $u(R, L) = R - hL$, where R is the present value of the expected rank of the researcher, L is labor invested in research, and $h > 0$.

The rank of a researcher is the share of papers in the citation statistics cited less often than the researcher's own paper. The utility of individual researchers depends on their rank in the citation statistics for all times after publication of their paper. Future ranks are discounted with a constant rate. Researchers are risk neutral with respect to rank and consider the present value of their expected ranks (or just their expected rank, for short).

The labor costs of writing a paper have a fixed component, which is independent of citations, and a variable component, which depends only on citations. We assume that fixed costs are sunk once the researcher has assessed the papers relevant to his project and calculated his expected rank; hence, we can neglect them. Variable costs may decrease or increase with the number of citations. We consider both cases in a linear specification.

Researchers generate a continuous flow of research papers. There exists a pool of recent papers, which are read by every researcher. Papers no longer in the pool are obsolete, that is, unavailable or hopelessly outdated. There is a constant obsolescence rate, that is, a rate with which papers from the pool become obsolete and drop out of the pool. We assume a stationary pool size.

We distinguish several categories of papers. We consider the flow of citations under the assumption that citation behavior is given and can be characterized by a citation rate, which is independent from the number of citations papers have already earned but differ between papers from different categories. The assumption of given citation behavior will be correct in finite neighborhoods of the model's stationary states.

For each researcher, a set of papers from the pool of non-obsolete papers turns out to be relevant. The set of relevant papers contains all papers with ideas and results the researcher could use. If he decides to use any of them, he always cites the corresponding paper. There is no restriction on the number of references. Relevance is independent of the number of citations or the category of a paper. Only relevant papers can be used; hence, relevance is necessary but not sufficient for citation. For each researcher, the set of relevant papers has the same size; however, the composition may differ.

After evaluating the relevant papers, a researcher writes a new paper, which is added to the pool. Upon publication, the paper's category becomes known. Depending on its category, the paper may have different chances to be used and cited.

We assume that the complete process is deterministic. However, researchers cannot predict the fate of their papers better than by using subjective probabilities equal to the citation rates and the obsolescence rate observed in the aggregate. Researchers have rational expectations with respect to these rates.

We consider only open-loop equilibria, that is, equilibria where researchers' citation behavior is independent of the history or present state. Given the interpretation of the model, open-loop equilibria are the most plausible candidates for focal points. After all, we consider coordination on a rule for evaluating scientific papers. While exogenous events, for instance, new methodological arguments, may certainly shift the focal point, it is not plausible that researchers adopt a rule, or believe that others adopt a rule, that makes the evaluation depend on the state of the system or, even worse, on the past citation behavior of other scientists. While such dependencies may result from a learning process, they are highly implausible as aspects of equilibrium behavior.

The model is presented in three steps. First, the dynamics of citations is considered when there is just one category of papers. This model is then extended to a k-category model. Finally, we close the model by considering researcher behavior.

3.1 The One-Category Model

3.1.1 The Flow of Papers and Citations

Let $G_i(t)$ be the mass of non-obsolete papers with $i = 0, 1, 2, \ldots$ citations at time t. Let $\sum_{i=0}^{\infty} G_i(t) = N(t)$ be the size of the pool of papers. Let $\dot{N}(t) = 1 - \gamma N$, where γ is the obsolescence rate, that is, the rate with which papers from the pool become obsolete and leave the pool. The stationary pool size is $N = 1/\gamma$.

Let $g_i(t) = G_i(t)/N$ the share of papers with i citations. Citation behavior can be characterized by a citation rate c, which is independent from the number of citations papers have already earned. The same independence holds for the obsolescence rate. Dynamic adjustment is described by

$$\begin{aligned} \dot{g}_0(t) &= \gamma - (c+\gamma)g_0(t) \\ \dot{g}_n(t) &= cg_{n-1}(t) - (c+\gamma)g_n(t) \text{ for all } n > 0 \end{aligned} \tag{1}$$

or, in matrix notation,

$$\dot{\mathbf{g}}(t) = \Gamma \mathbf{g}(t) + \gamma \mathbf{e}_1 \,, \tag{2}$$

where $\mathbf{e}_1$ is the first unit vector and

$$\Gamma := \begin{pmatrix} -(c+\gamma) & 0 & 0 & \ldots \\ c & -(c+\gamma) & 0 & \ldots \\ 0 & c & -(c+\gamma) & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \tag{3}$$

The solution of (2) is[5]

$$\mathbf{g}(t) - \mathbf{g}^* = \mathrm{e}^{\Gamma t} \left[\mathbf{g}(0) - \mathbf{g}^* \right] , \tag{4}$$

where $\mathbf{g}^* = -\gamma \Gamma^{-1} \mathbf{e}_1$ is the stationary solution. Its components are

$$g_n^* = (1-\theta)\theta^n \,, \tag{5}$$

with

$$\theta := \frac{c}{\gamma + c} \,. \tag{6}$$

We have $\mathrm{e}^{\Gamma t} = \Phi(t,c)\mathrm{e}^{-(c+\gamma)t}$ with

$$\Phi(t,c) = \begin{pmatrix} \phi_0(t,c) & 0 & 0 & \ldots \\ \phi_1(t,c) & \phi_0(t,c) & 0 & \ldots \\ \phi_2(t,c) & \phi_1(t,c) & \phi_0(t,c) & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{7}$$

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[5] See Tu (1994: 85). The fact that the dimension of the system (2) is infinite does not invalidate the solution: differentiating the solution leads back to the system.

and

$$\phi_n(t,c) := \frac{t^n c^n}{n!} \,. \tag{8}$$

Let $\mathbf{1}$ be the vector of 1s. Since $|\mathbf{g}(0) - \mathbf{g}^*| \leq \mathbf{1}$, and since the row sum of $\mathrm{e}^{\Gamma t}$ is strictly smaller than $\mathrm{e}^{-\gamma t}$, we have $\left|\mathrm{e}^{\Gamma t}\left[\mathbf{g}(0) - \mathbf{g}^*\right]\right| \leq \mathrm{e}^{-\gamma t}\mathbf{1}$ and, therefore,

$$|\mathbf{g}(t) - \mathbf{g}^*| \leq \mathrm{e}^{-\gamma t}\mathbf{1} \,. \tag{9}$$

This implies asymptotic stability of the system at $\mathbf{g}^*$.

The percentage rank $r_n(t)$ of a paper with $n > 0$ citations is the share of papers with fewer citations, that is, $r_n(t) = \sum_{i=0}^{n-1} g_i(t)$. The rank of a paper without citations or an obsolete paper is 0. The dynamic adjustment of ranks is described by

$$\begin{aligned} \dot{r}_1(t) &= \gamma - (c+\gamma) r_1(t) \\ \dot{r}_n(t) &= \gamma + c r_{n-1}(t) - (c+\gamma) r_n(t) \text{ for all } n > 1 \end{aligned} \tag{10}$$

or, in matrix notation,

$$\dot{\mathbf{r}}(t) = \Gamma \mathbf{r}(t) + \gamma \mathbf{1} \,. \tag{11}$$

The solution of this system is

$$\mathbf{r}(t) - \mathbf{r}^* = \mathrm{e}^{\Gamma t}\left[\mathbf{r}(0) - \mathbf{r}^*\right] \,, \tag{12}$$

where $\mathbf{r}^* = -\gamma \Gamma^{-1}\mathbf{1}$ is the vector of equilibrium values with components

$$r_n^* = 1 - \theta^n \,. \tag{13}$$

Since $|\mathbf{r}(0) - \mathbf{r}^*| \leq \mathbf{1}$, and since the row sum of $\mathrm{e}^{\Gamma t}$ is strictly smaller than $\mathrm{e}^{-\gamma t}$, we have $\left|\mathrm{e}^{\Gamma t}\left[\mathbf{r}(0) - \mathbf{r}^*\right]\right| \leq \mathrm{e}^{-\gamma t}\mathbf{1}$ and, therefore,

$$|\mathbf{r}(t) - \mathbf{r}^*| \leq \mathrm{e}^{-\gamma t}\mathbf{1} \,, \tag{14}$$

implying asymptotic stability of the system at $\mathbf{r}^*$.

3.1.2 Expectations

Consider a paper published in t. The paper's possible states at time $s \geq t$ are $S(s) = -1, 0, 1, \ldots$, where $S = -1$ means that the paper is obsolete while $S = i \geq 0$ means that the paper is not obsolete and has i citations.

Formally, expectations of researchers publishing at time 0 can be described by a homogeneous Markov chain in continuous time with transition probabilities $p_{i\to j}(s-t) = P\left(S(s) = j | S(t) = i\right)$ for $s \geq t \geq 0$ (cf. Grimmett and Stirzaker 1992: section 6.9). The following assumptions hold, where "$\approx$" indicates a linear approximation for small time intervals h.

1. An obsolete paper stays obsolete: $p_{-1\to-1}(h) = 1$ and $p_{-1\to i}(h) = 0$ for all $i \geq 0$.

2. In the time interval h, a non-obsolete paper becomes obsolete with probability γh: $p_{i\to-1}(h) \approx \gamma h$ for all $i \geq 0$.

3. In the time interval h, a non-obsolete paper is cited once with probability ch: $p_{i\to i+1}(h) \approx ch$ for all $i \geq 0$.

4. In the time interval h, the probability that a paper is cited more than once is 0: $p_{i\to j}(h) \approx 0$ for all $i, j \geq 0$ with $j \neq i+1$.

5. In the time interval h, then, the probability for a paper to remain in the same state is $1-(\gamma+c)h$: $p_{i\to i}(h) \approx 1-(c+\gamma)h$ for all $i \geq 0$.

The probability that there is some change in the state of a non-obsolete paper during h is approximately $(c+\gamma)h$. The probability of a citation conditional on a change in status is $\theta = c/(c+\gamma)$. Hence, θ can be interpreted as the probability that the next change in the status of a non-obsolete paper —which occurs with probability 1—is a citation, while $1-\theta$ is the paper's probability of becoming obsolete with the next change in status.

The assumptions above determine the generator

$$P = \begin{pmatrix} 0 & \gamma\mathbf{1}^t \\ \mathbf{0} & \Gamma \end{pmatrix} \tag{15}$$

of the Markov chain, where $\mathbf{0}$ is a vector of 0s. The matrices of transition probabilities are denoted by $P_h = \left(p_{i\to j}(h)\right)_{ji}$ and given by

$$P_h = \mathrm{e}^{Ph} = \begin{pmatrix} 1 & \left[1-\mathrm{e}^{-\gamma h}\right]\mathbf{1}^t \\ \mathbf{0} & \Phi(c,h)\mathrm{e}^{-(c+\gamma)h} \end{pmatrix} \tag{16}$$

with $P_0 = I$, P_h stochastic (entries nonnegative, column sums equal to 1), and $P_{h+g} = P_h P_g$ for all $g, h \geq 0$ (Chapman-Kolmogorov equation).

Since each paper starts in state 0, expectations of researchers can be characterized by the probabilities $p_n(h) := p_{0\to n}(h)$, which are given in the second column of P_h:

$$\begin{aligned} p_{-1}(h) &= 1-\mathrm{e}^{-\gamma h} \\ p_n(h) &= \phi_n(h,c)\mathrm{e}^{-(c+\gamma)h} = \frac{c^n h^n}{n!}\mathrm{e}^{-(c+\gamma)h} \text{ for all } n \geq 0 \end{aligned} \tag{17}$$

Thus, expectations are described by a Poisson process that has been modified by adding an absorbing state, obsolescence, which is reached with probability 1. The probability of obsolescence after n citations is $g_n^* = \theta^n(1-\theta)$.

The stationary state of the dynamical system (1) is reached if the proportion of papers with n citations equals the subjective probability of a paper to earn n citations. The subjective probability of earning citations depends only on the citation and obsolescence rates, not on the state of the dynamical system.

3.1.3 Expected Rank

Given a discount rate $\delta \geq 0$, the present value of the expected rank (or just expected rank, for short) is

$$\begin{aligned} R(t) &= \int_0^\infty \sum_{n=1}^\infty p_n(h) r_n(t+h)\, \mathrm{e}^{-\delta h} dh \\ &= \int_0^\infty \sum_{n=1}^\infty \phi_n(h,c) r_n(t+h)\, \mathrm{e}^{-(\delta+c+\gamma)h} dh\,. \end{aligned} \tag{18}$$

The expected rank develops according to

$$\dot{R}(t) = \int_0^\infty \sum_{n=1}^\infty \phi_n(h,c) \dot{r}_n(t+h)\, \mathrm{e}^{-(\delta+c+\gamma)h} dh\,. \tag{19}$$

Since $D_1 \phi_n(h,c) = c\phi_{n-1}(h,c)$, partial integration yields

$$\dot{R}(t) = (\delta+c+\gamma)R(t) - c\int_0^\infty \sum_{n=1}^\infty \phi_{n-1}(h,c) r_n(t+h)\, \mathrm{e}^{-(\delta+c+\gamma)h} dh\,. \tag{20}$$

Using $r_1 = g_0$ and, for $n > 1$, $r_n = r_{n-1} + g_{n-1}$, we find

$$\sum_{n=1}^\infty \phi_{n-1} r_n = \sum_{n=1}^\infty \phi_n r_n + \sum_{n=0}^\infty \phi_n g_n\,. \tag{21}$$

Hence, the integral in (20) can be written as

$$\int_0^\infty \sum_{n=1}^\infty \phi_{n-1}(h,c) r_n(t+h)\, \mathrm{e}^{-(\delta+c+\gamma)h} dh = R(t) + E(t)\,, \tag{22}$$

where

$$\begin{aligned} E(t) &= \int_0^\infty \sum_{n=0}^\infty \phi_n(h,c) g_n(t+h)\, \mathrm{e}^{-(\delta+c+\gamma)h} dh \\ &= \int_0^\infty \sum_{n=0}^\infty p_n(h) g_n(t+h)\, \mathrm{e}^{-\delta h} dh\,, \end{aligned} \tag{23}$$

yielding

$$\dot{R}(t) = (\delta + \gamma)R(t) - cE(t)\,. \tag{24}$$

The variable E is the present value of the expected share of papers with the same number of citations (or expected share of rivals, for short).

In order to understand (24), remember that the probabilities determining the citations for a paper are fixed since we assume that the citation rate c and the obsolescence rate γ are given. A change in expected status $R(t)$ reflects the fact that a paper published at time t achieves different ranks even if it makes the same career in terms of citations than a paper published earlier or later. That is, changes in expected status result from the status changes of other papers.

There are three sources of change in expected status. First, new papers flowing into the pool at rate γ have zero citations and therefore raise the rank of each paper with a least one citation. Second, discounting implies that future ranks increase in value at the discount rate δ as they are approached in time. Third, the more papers with the same number of citations exist, the more papers move up in parallel when a paper earns a citation; hence, the expected share of rivals diminishes the effect of citations on rank.

In a stationary equilibrium, the expected share of rivals is constant. We have

$$\begin{aligned} E^* &= \int_0^\infty \sum_{n=0}^\infty \phi_n(h,c) g_n^* \, \mathrm{e}^{-(\delta+c+\gamma)h} dh \\ &= \sum_{n=0}^\infty \frac{c^n g_n^*}{n!} \int_0^\infty h^n \, \mathrm{e}^{-(\delta+c+\gamma)h} dh \\ &= \sum_{n=0}^\infty \frac{c^n g_n^*}{(\delta+c+\gamma)^{n+1}} \\ &= \frac{\gamma}{\gamma(\delta+2c+\gamma)+\delta c}\,, \end{aligned} \tag{25}$$

which is decreasing in c. For the stationary value of the expected rank, we find

$$R^* = \frac{c}{\delta+\gamma} E^*\,, \tag{26}$$

which rises with c.

A stationary state, then, will be *ex ante* inefficient from the researchers' point of view as long as the citation rate could be raised.

3.2 The k-Categories Model

The model of the last subsection is extended by distinguishing k categories of papers that may differ with respect to their citation rates.

3.2.1 The Flow of Papers and Citations

We focus on papers in one category, X. Let the share of X-papers in the pool of non-obsolete papers at time t be $g_x(t)$. The citation rate for X-papers is c_x. The rate at which new X-papers enter the pool is $p_x(t)\gamma$, where $p_x(t)$ is the entry share for category X. All entry shares are nonnegative and add to 1. The obsolescence rate is the same for all categories, namely, γ. As before, the size of the pool is constant at $N = 1/\gamma$.

Let $G_{xn}(t) = g_{xn}(t)N, n = 0, 1, \ldots, \infty$ with $\sum_{n=0}^{\infty} g_{xn}(t) = g_x(t)$ be the mass of X-papers in the pool that at time t have already been cited i times. The dynamical system is

$$\dot{\mathbf{g}}_x(t) = \Gamma_x \mathbf{g}_x(t) + p_x(t)\gamma \mathbf{e}_1 \tag{27}$$

where

$$\Gamma_x := \begin{pmatrix} -(c_x+\gamma) & 0 & 0 & \cdots \\ c_x & -(c_x+\gamma) & 0 & \cdots \\ 0 & c_x & -(c_x+\gamma) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{28}$$

For each category, we have such a dynamical system. Adjustment of the complete system is driven by the conditions of paper production. The use of papers as inputs determines the composition of the output, that is, the entry shares p_z. We assume a linear relation:

$$\mathbf{p} = \mathbf{a} + BC\mathbf{g} \tag{29}$$

The vector $\mathbf{a} \in \mathbb{R}^k$ is nonnegative with $\mathbf{1}^t\mathbf{a} = 1$. The vector $\mathbf{g} \in \mathbb{R}^k$ is the vector of category shares g_z. The diagonal of the matrix $C \in \mathbb{R}^k \times \mathbb{R}^k$ is given by the citation rates c_z; the off-diagonal elements are 0. Hence, $C\mathbf{g}$ is the flow of citations in each category:

$$C\mathbf{g} = \begin{pmatrix} c_x & 0 & \cdots \\ 0 & c_y & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} g_x \\ g_y \\ \vdots \end{pmatrix} = \begin{pmatrix} c_x g_x \\ c_y g_y \\ \vdots \end{pmatrix} \tag{30}$$

The matrix $B \in \mathbb{R}^k \times \mathbb{R}^k$ satisfies $\mathbf{1}^t B = \mathbf{0}$; moreover, its elements are absolutely small enough such that $\mathbf{p}$ is nonnegative. A special case results if we additionally assume $B\mathbf{1} = 0$, which means that papers of the different categories neutralize each other.

We first consider adjustment of the entry shares $\mathbf{p}$. From (27) and the corresponding equations for the other categories, we find

$$\dot{\mathbf{g}} = \gamma(\mathbf{p} - \mathbf{g})\,. \tag{31}$$

Inserting according to (29) yields

$$\dot{\mathbf{g}} = \gamma\left[\mathbf{a} - (I - BC)\mathbf{g}\right]\,. \tag{32}$$

From (32), we find the stationary value $\mathbf{g}^* = (I - BC)^{-1}\mathbf{a}$. The solution of (32) is

$$\mathbf{g}(t) = \mathrm{e}^{-\gamma(I-BC)t}\left[\mathbf{g}(0) - \mathbf{g}^*\right] + \mathbf{g}^*\,. \tag{33}$$

Stability is ensured if the eigenvalues of $I - BC$ have positive real parts; thus, stability requirements put restrictions on BC and, therefore, on the technology as well as on behavior.

The stationary entry shares are $\mathbf{p}^* = \mathbf{a} + BC\mathbf{g}^*$. From $(I - BC)(I - BC)^{-1} = I$, we get $I + BC(I - BC)^{-1} = I$ and thus

$$\mathbf{p}^* = \mathbf{a} + BC(I - BC)^{-1}\mathbf{a} = (I - BC)^{-1}\mathbf{a} = \mathbf{g}^*\,,$$

which proves consistency with (31). Inserting (33) into (29) yields an equation for the adjustment of the entry shares.

We consider adjustment within category X under the assumption that the entry shares are close enough to their stationary values so that they can assumed to be constant:

$$\dot{\mathbf{g}}_x(t) = \Gamma_x \mathbf{g}_x(t) + p_x^* \gamma \mathbf{e}_1 \tag{34}$$

The solution of this system is

$$\mathbf{g}_x(t) - \mathbf{g}_x^* = \mathrm{e}^{\Gamma_x t}\left[\mathbf{g}_x(0) - \mathbf{g}_x^*\right] \tag{35}$$

with stationary values $\mathbf{g}_x^* = -p_x \gamma \Gamma_x^{-1} \mathbf{e}_1$ or

$$g_{xn}^* = p_x^*(1 - \theta_x)\theta_x^n\,, \tag{36}$$

where

$$\theta_x := \frac{c_x}{\gamma + c_x}\,. \tag{37}$$

Asymptotic stability at $\mathbf{g}_x^*$ follows as before in the one-category model.

In order to describe the dynamics of ranks, we introduce the rank components $r_{xn}(t) = \sum_{i=0}^{n-1} g_{xi}(t)$. Let g_n be the sum over all k categories of shares of papers with n citations. The rank $r_n(t)$ of a paper with n citations at time t is the sum of the rank components over all k categories:

$$r_n(t) = \sum_z r_{zn}(t) = \sum_z \sum_{i=0}^{n-1} g_{zi}(t) = \sum_{i=0}^{n-1} \sum_z g_{zi}(t) = \sum_{i=0}^{n-1} g_i(t) \tag{38}$$

The dynamic adjustment of rank components is described by

$$\dot{\mathbf{r}}_x(t) = \Gamma_x \mathbf{r}_x(t) + p_x^* \gamma \mathbf{1} \,. \tag{39}$$

The solution of this system is

$$\mathbf{r}_x(t) - \mathbf{r}_x^* = \mathrm{e}^{\Gamma_x t} \left[\mathbf{r}_x(0) - \mathbf{r}_x^* \right] \tag{40}$$

with stationary values $\mathbf{r}_x^* = -p_x \gamma \Gamma_x^{-1} \mathbf{1}$ or

$$r_{xn}^* = p_x^* (1 - \theta_x^n) \,. \tag{41}$$

Asymptotic stability of the system at $\mathbf{r}_x^*$ follows as before.

3.2.2 Expected Rank

The model of expectations formation is the same as before except that the transition probabilities depend on the citation rate in the relevant category. The expected rank in category X (or expected category rank, for short) is

$$R_x(t) = \int_0^\infty \sum_{n=1}^\infty \phi_n(h, c_x) r_n(t+h) \, \mathrm{e}^{-(\delta + c_x + \gamma)h} dh \,. \tag{42}$$

Since a higher citation rate means a higher probability of earning citations during any given time interval, the ordering of categories according to expected category ranks is the same as the ordering according to citation rates: $R_x(t) > R_y(t)$ if and only if $c_x > c_y$.

The expected category rank develops according to

$$\dot{R}_x(t) = \int_0^\infty \sum_{n=1}^\infty \phi_n(h, c_x) \dot{r}_n(t+h) \, \mathrm{e}^{-(\delta + c_x + \gamma)h} dh \,. \tag{43}$$

Partial integration yields

$$\dot{R}_x(t) = (\delta + c_x + \gamma) R_x(t) - c_x \int_0^\infty \sum_{n=1}^\infty \phi_{n-1}(h, c_x) r_n(t+h) \, \mathrm{e}^{-(\delta + c_x + \gamma)h} dh \,. \tag{44}$$

Again, we have

$$\int_0^\infty \sum_{n=1}^\infty \phi_{n-1}(h, c_x) r_n(t+h)\, \mathrm{e}^{-(\delta+c_x+\gamma)h} dh = R_x(t) + E_x(t)\,, \tag{45}$$

where

$$E_x(t) = \int_0^\infty \sum_{n=0}^\infty \phi_n(h, c_x) g_n(t+h)\, \mathrm{e}^{-(\delta+c_x+\gamma)h} dh \tag{46}$$

is, for an X-paper, the expected number of rivals. Hence, we have

$$\dot{R}_x(t) = (\delta+\gamma) R_x(t) - c_x E_x(t)\,. \tag{47}$$

Analyzing E_x, we find

$$E_x(t) = \sum_z E_{xz}(t) \tag{48}$$

with

$$E_{xz}(t) := \int_0^\infty \sum_{n=0}^\infty \phi_n(h, c_x) g_{zn}(t+h)\, \mathrm{e}^{-(\delta+c_x+\gamma)h} dh\,. \tag{49}$$

In a stationary state, we have

$$\begin{aligned} E^*_{xz} &= \int_0^\infty \sum_{n=0}^\infty \phi_n(h, c_x) g^*_{zn}\, \mathrm{e}^{-(\delta+c_x+\gamma)h} dh \\ &= \frac{p^*_z \gamma}{\gamma(\delta + c_x + c_z + \gamma) + \delta c_z}\,, \end{aligned} \tag{50}$$

and

$$R^*_x = \frac{c_x}{\delta+\gamma} \sum_z E^*_{xz}\,. \tag{51}$$

Thus, the stationary-state expected category rank rises with a rising citation rate of the respective category and falls with rising citation rates in other categories.

3.3 Researcher Behavior and Equilibrium

As already explained, researchers maximize a utility function $u\,(R, L) = R - hL$, where R is the expected rank of the researcher, L is labor invested in research, and $h > 0$. The overall flow of relevant papers at time t is αN, $\alpha > 0$. Relevance is independent of the number n of citations or the category

Z of a paper. The flow of relevant Z-papers with n citations at time t is $\alpha g_{zn}(t)$. Only relevant papers can be used; hence, relevance is necessary but not sufficient for citation.

For each researcher, the set of relevant papers has the same size; however, the composition may differ. Hence, we can describe a researcher's sample of relevant papers by the shares of papers from each category. Let $s_z \geq 0$ be the share of relevant Z-papers, where $\sum_z s_z = 1$. For each category Z, the researcher chooses the share $q_z \in [0,1]$ of the relevant Z-papers he wants to use and cite. The overall share of relevant papers used and cited is $q \in [0,1]$. The contribution of each category to q is $\beta_z = q_z s_z \in [0, s_z]$. Thus, the citation share from the sample is $q = \sum_z q_z s_z = \sum_z \beta_z = \mathbf{1}^t \boldsymbol{\beta}$.

The variable labor costs of writing a paper may decrease or increase with the number of citations. We consider both cases in a linear specification, that is, marginal costs are constant: either $L = 1 - q$ (citations save labor) or $L = q$ (citations cost labor). Since $q = \mathbf{1}^t \boldsymbol{\beta}$, we write $L = L(\boldsymbol{\beta})$ with $D_z L = \pm 1$. The sign of the derivative is, of course, the same for all z; we have $D_z L = -1$ if citations save labor and $D_z L = 1$ if citations cost labor.

The probability of producing a Z-paper is determined by the linear production function (29) already introduced before:

$$\mathbf{p}\left(\boldsymbol{\beta}\right) = \mathbf{a} + B\boldsymbol{\beta}\,. \tag{52}$$

Aggregation of $\boldsymbol{\beta}$ over all researchers yields $C\mathbf{g}$, which leads to (29).

If there is no connection between the category composition of the input and the output, all entries of the matrix B are 0. We then have $\mathbf{p} = \mathbf{a}$: there is no hereditariness of quality, and therefore no reason to be selective in using and citing papers.

If the matrix B is non-zero, the citation decisions determine the expected rank of the researcher. From the perspective of an individual researcher, the expected category ranks R_z are exogenous. Let $\mathbf{R}$ denote the vector of the expected category ranks. The expected rank over all categories is

$$R\left(\boldsymbol{\beta}\right) = \mathbf{p}^t\left(\boldsymbol{\beta}\right)\mathbf{R} = \mathbf{a}^t\mathbf{R} + \boldsymbol{\beta}^t B^t \mathbf{R}\,. \tag{53}$$

Accordingly, a researcher maximizes

$$u\left(\boldsymbol{\beta}\right) = R\left(\boldsymbol{\beta}\right) - hL\left(\boldsymbol{\beta}\right)\,, \tag{54}$$

where $\beta_z \in (0, s_z)$. Let Δ_z be the zth element of the column vector $B^t\mathbf{R}$. We have $D_z u = \Delta_z - hD_z L$. Obviously, all relevant Z-papers are cited if $D_z u > 0$, while no Z-papers are cited if $D_z u < 0$. We ignore unlikely constellations resulting in $D_z u = 0$, where the researcher would be indifferent between citing and not citing Z-papers.

Since $B\mathbf{1} = 0$, we also have $\sum_z \Delta_z = 0$. If $\Delta_z > 0$, citing Z-papers contributes positively to the expected rank, while $\Delta_z > 0$ means that citing Z-papers reduces the expected rank. Hence, Δ_z is the relevant measure of quality from a researcher's point of view. Specifically, we call Z-papers high quality if $\Delta_z > 0$ and low quality if $\Delta_z \leq 0$.

Obviously, quality in this sense results from the technology of paper production, which determines B, and expectations about aggregate citation behavior, which determine $\mathbf{R}$. Thus, quality is not determined by the chances of earning citations alone. The quality of a paper is its contribution to the production of a paper that has high chances of earning citations. Papers with the same chances of earning citations may nevertheless differ in their quality.

Assume first that Z-papers are high quality ($\Delta_z > 0$) and that the citations save labor ($D_z L = -1$). In this case, $D_z u > 0$; hence, $\beta_z = s_z$ is optimal. If citations save labor, all high-quality papers are cited. Low-quality papers, on the other hand, are only cited if their quality is not too low: if $\Delta_z < -h$, we have $D_z u < 0$, and Z-papers are not cited.

Assume now that Z-papers are low quality ($\Delta_z < 0$) and that the citations cost labor ($D_z L = -1$). In this case, $D_z u < 0$; hence, $\beta_z = 0$ is optimal. If citations cost labor, no low-quality paper is cited. High-quality papers, on the other hand, are only cited if their quality is high enough: if $\Delta_z > h$, we have $D_z u > 0$, and all relevant Z-papers are cited.

If h is very high, labor-cost considerations can dominate the citation decisions, and it may be optimal to cite all relevant papers (in the case where citations save labor) or no paper at all (in the case where citations cost labor). In all other cases, there is a cutoff point on the quality scale. Below the cutoff point, no papers are cited; above the cutoff point, all relevant papers are cited.

Since these cutoff points are identical for all researchers, the citation rates for all papers at time t are either equal to 0 or equal to the relevance rate $\alpha > 0$. In accordance with the analysis of citation flows, we consider only cases where citation behavior does not change during adjustment. This will be true in neighborhoods of stationary states defined by the condition that expected category ranks $R_z(t)$ no longer change much. As long as changes are small enough to ensure that Δ_z remains above or below the cutoff point for each category z, citation behavior remains constant and adjustment to the stationary state is asymptotically stable.

Computation of equilibria proceeds as follows. First, one assumes a set of categories that are cited. Thus, let $\mathbb{C}$ be the set of cited categories, that is, $c_z = \alpha$ if $z \in \mathbb{C}$ and $c_z = 0$ if $z \notin \mathbb{C}$. It follows that $R_z = \rho > 0$ if $z \in \mathbb{C}$ and $R_z = 0$ if $z \notin \mathbb{C}$. Given $\mathbb{C}$, we can compute the stationary entry shares

$\mathbf{p}^*$, the expected rank R, and the values Δ_z. As a last step, we need to check whether the cited categories have a quality above the cutoff point and the other categories have a quality below the cutoff point.

In order to illustrate the possibilities for self-fulfilling expectations equilibria, we consider special cases. We assume two or three categories named X, Y and, if relevant, Z, which correspond to the first, second and, if relevant, third component of the different vectors.

A Symmetric Two-Categories Model: There are only two categories, X and Y. We assume symmetry in the sense that

$$B = \begin{pmatrix} b & -b \\ -b & b \end{pmatrix},$$

where $b > 0$. Note that $B\mathbf{1} = 0$ (neutralization). We assume first that citations are labor saving. Three equilibria are possible.

1. Indiscriminate citation: Researchers expect all relevant papers to be cited. Hence, $c_x = c_y = \alpha$ and $R_x = R_y > 0$. We have $\Delta_x = \Delta_y = 0 > -h$. Therefore, expectations are self-fulfilling: researchers who expect all relevant papers to be cited cite all relevant papers.

2. X-Preference: Researchers expect only X-papers to be cited. Hence, $c_x = \alpha$, $c_y = 0$ and $\rho = R_x > R_y = 0$. We have $\Delta_x = \rho$ and $\Delta_y = -\rho$. If $-\rho < -h$ or $\rho > h$, expectations are self-fulfilling: researchers who expect only X-papers to be cited cite only X-papers. If $\rho < h$, this equilibrium is not possible.

3. Y-Preference: An Y-preference equilibrium is possible iff an X-preference equilibrium is possible.

4. No citation: Researchers expect no papers to be cited. Hence, $c_x = c_y = 0$ and $R_x = R_y = 0$. We have $\Delta_x = \Delta_y = 0 > -h$. Therefore, expectations are not self-fulfilling: researchers who expect no papers to be cited cite all relevant papers. Such an equilibrium is not possible.

If citations cost labor, indiscriminate citation becomes impossible while no citation is possible.

Thus, the production possibilities as described by the matrix B rule out some conceivable equilibria as inconsistent with rational expectations.

In order to demonstrate the computation of equilibria, we assume no discounting ($\delta = 0$), $a_x = 0.6$, $a_y = 0.4$, $b = 0.2$, and $\alpha = 1$. This leaves two free parameters, h and γ. We consider the question under which parameter constellations a stable X-preference equilibrium is possible.

An X-preference equilibrium is characterized by $c_x = \alpha = 1$ and $c_y = 0$. Since

$$I - BC = \begin{pmatrix} 0.8 & 0 \\ 0.2 & 1 \end{pmatrix}$$

has eigenvalues 1.0 and 0.8, stability is ensured.

The stationary values of $\mathbf{g} = \mathbf{p}$ are given by

$$\begin{aligned} \begin{pmatrix} g_x^* \\ g_y^* \end{pmatrix} = (I - BC)^{-1}\mathbf{a} &= \begin{pmatrix} 0.8 & 0 \\ 0.2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} \\ &= \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix} = \begin{pmatrix} p_x^* \\ p_y^* \end{pmatrix} . \end{aligned}$$

An X-preference equilibrium is characterized by $R_x = \rho > 0$ and $R_y = 0$. Since $\delta = 0$, we have

$$\rho = \frac{\alpha}{\gamma}\frac{p_x^*}{\gamma + 2\alpha} + \frac{\alpha}{\gamma}\frac{p_y^*}{\gamma + \alpha} = \frac{1.25 + \gamma}{\gamma(\gamma + 1)(\gamma + 2)} .$$

Paper qualities are

$$\begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix} = B \begin{pmatrix} \rho \\ 0 \end{pmatrix} = \frac{0.25 + 0.2\gamma}{\gamma(\gamma + 1)(\gamma + 2)} \begin{pmatrix} 1 \\ -1 \end{pmatrix} .$$

Thus, if the obsolescence rate γ is small enough, the X-preference equilibrium is possible because $\Delta_x > h$.

A Symmetric Three-Categories Model: There are three categories, X, Y, and Z. We assume symmetry in the sense that

$$B = \begin{pmatrix} 1 & -b & b-1 \\ -b & 2b & -b \\ b-1 & -b & 1 \end{pmatrix}$$

with $b \in [0, 0.5]$. Specifically, we again have $B\mathbf{1} = 0$ (neutralization). With $b = 0.5$, there is symmetry between all categories. With $b < 0.5$, category Y stands out. It is a simple exercise to consider the different kinds of equilibria. The main point of this example is to show that citation preferences are not the same as quality in equilibrium.

Consider a case where, in equilibrium, only X- and Y-papers are cited and where $b < 0.25$. Thus, we have $c_x = c_y = \alpha$ and $c_z = 0$ and, correspondingly,

$R_x = R_y = \rho > R_z = 0$. This results in $\Delta_x = (1-b)\rho > b\rho = \Delta_y$, while $\Delta_z = -\rho$. Thus, although all relevant X- and Y-papers are cited, X-papers are better papers from the researchers' point of view.

This has implications for comparative-static results. We consider a numerical example. Again, we assume no discounting ($\delta = 0$). Let, moreover, $\alpha = 0.5$, $b = 0.1$ $a_x = 0.6$, $a_y = 0.3$, and $a_z = 0.1$. This leaves the same two free parameters as before: h and γ. We consider the question under which parameter constellations a stable X, Y-preference equilibrium is possible.

We have

$$I - BC = \begin{pmatrix} 0.5 & 0.05 & 0 \\ 0.05 & 0.9 & 0 \\ 0.45 & 0.05 & 1 \end{pmatrix}$$

with three eigenvalues approximately equal to 0.49, 1.00 and 0.91. Again, stability is ensured.

The stationary values of $\mathbf{g}$ or $\mathbf{p}$ are given by $(I - BC)^{-1}\mathbf{a}$, which yields $p_x^* \approx 0.17$, $p_y^* \approx 0.32$ and $p_z^* \approx 0.51$.

Since $\delta = 0$, we have

$$\rho = \frac{\alpha}{\gamma} \frac{p_x^* + p_y^*}{\gamma + 2\alpha} + \frac{\alpha}{\gamma} \frac{p_z^*}{\gamma + \alpha} \approx \frac{0.5\gamma + 0.38}{\gamma(\gamma + 0.5)(\gamma + 1)} .$$

Paper qualities are

$$\begin{pmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{pmatrix} = B \begin{pmatrix} \rho \\ \rho \\ 0 \end{pmatrix} \approx \frac{0.5\gamma + 0.38}{\gamma(\gamma + 0.5)(\gamma + 1)} \begin{pmatrix} 0.9 \\ 0.1 \\ 0 \end{pmatrix} .$$

Thus, if the obsolescence rate γ is small enough, the X, Y-preference equilibrium is possible because $\Delta_x > \Delta_y > h$.

However, if Δ_y and h are close, a small increase in the obsolescence rate can make the equilibrium impossible. As can easily be shown, a stable X-preference equilibrium remains possible, which is characterized by a higher value of ρ but by a smaller expected rank, because the probability of producing an X-paper is now approximately 0.20, while before the probability of producing an X- or a Y-paper was approximately 0.49. The quality scale changes drastically: Y-papers drop from the high-quality to the low-quality segment since Δ_y changes its sign.

4 Discussion

4.1 Summary and Interpretation

The model of scientific competition proposed in this paper shows how an endogenous product quality scale can emerge in scientific competition through self-fulfilling expectations. However, not any kind of quality norm can be established in this way. Unless participants believe that the quality of the scientific input, which consists of ideas and results from other researchers, has an influence on the quality of the output of a researcher, there is no mechanism leading to the establishment of a quality norm. A quality norm can only be established in scientific competition if quality is hereditary in the production process.

While scientific competition shows some similarities to competition on markets, there are two important differences.[6]

First, the basic transaction in science is not an exchange (*no-exchange condition*). A researcher decides unilaterally whether to use ideas and results published by another researcher. While use requires payment in the form of a citation, there is no cooperation problem because it is in a researcher's own interest to refer to the literature. Thus, the basic cooperation problem in anonymous markets illustrated by the so-called trust game, where one party to an exchange has to deliver first and can therefore be exploited by the second party, is not fundamental to scientific competition.[7] Of course, cooperation is possible; the obvious example is joint authorship. Nevertheless, the competitive framework also works without cooperation.

Second, production decisions are not governed by the evaluations of final consumers (*producer sovereignty*). From the perspective of researchers doing basic research, their output serves only as an input in scientific production. The preferences of the consumers of science—that is, those who use the output of science for some other purpose than basic research—have no influence on this process of production for the sake of production. The consumers of science profit from scientific progress or may sometimes be hurt by it, but from the perspective of scientists engaged in basic research, these are external effects. The value of the products of science in the eyes of the producers is not determined by the demand of final consumers but by the demand of

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[6] On differences and similarities between competition in science and on markets, see Walstad (2002), who emphasizes the first of the two differences but does not mention, or analyze, the second. Note that Walstad subsumes science under a broader concept of market.

[7] On the trust game(s), together with experimental results, see, e.g., Camerer (2003: 83-100).

other producers.

Producer sovereignty, the absence of exchange, and the assumption of hereditary quality imply that the game of science is a sequential beauty contest.[8]

Producer sovereignty implies that the results of research have no fundamental value within science. Instead, everybody tries to guess what the next researcher would like to make use of. Since there is no exchange, production proceeds on the basis of this guess; there exist no contracts fixing a price for papers satisfying some quality norm.

There is a certain similarity to stock markets, where a similar beauty contest also leads to the coordination on evaluation norms (Pratten 1993). However, the need to coordinate on evaluation norms is more obvious on stock markets, where, after all, shares of stock are exchanged, so that buyer and seller have to find a common price at which both are willing to make a deal. In science, there is no exchange. Every researcher produces a unique product which is a common pool resource appropriated by nobody and used by everybody who wants to do so. Hence, there is no need to find a price. Without further assumptions, the guessing game in science is not a beauty contest. The coordination problem that turns science into a beauty contest is created by hereditary quality, which forges a link between input quality and output quality.

The equilibrium in scientific competition is not *ex ante* efficient from the researchers' point of view. Researchers would be better off if they did cooperate in order to raise the citation rates. The present paper has abstracted from this possibility. However, in section 3's model, the possibility of intertemporal cooperation exists at least in principle. By making use of the folk theorem, one could explain intertemporal citation cartels. Such cartels can improve the situation for the participants, at the cost of reducing average quality. Whether this is bad or not for society as a whole depends, of course, on the quality scale itself. There is no guarantee that the quality scale established in science reflects the preferences of the consumers of science.

4.2 Extension: Citation Rules

The observance of norms regulating citation behavior must of course be explained. Even if ideas and results from previously published papers are used, they need not necessarily be cited. The incentives to cite, it seems, are quite different from the incentives to use other researchers' ideas and results.

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[8] On simultaneous beauty contests, together with experimental results, see, e.g., Camerer (2003: 209-218).

However, it can be shown that there is some connection.

The most important citation rules are the rule to refer to the literature at all, and the priority norm of citing the first published paper containing some idea or result. The following three explanations are based on informal extensions of section 3's model.

First, and obviously, citations reduce transaction costs because they are a means of abbreviation. In order to write a paper for a specialist audience, it is necessary to concentrate on the new points. Specialists, whose reading list is usually too long anyway, do not want to read a presentation that is overly long just because it presents well-known ideas and results as if they were unheard of. Moreover, plagiarizing well-known ideas or results is pointless anyway in an environment where the priority rule is observed because readers cannot be deceived into citing the plagiarist. Hence, well-known ideas or results are explained in few words and accompanied by appropriate references.

Second, citation is an important element of the propaganda for one's own ideas.[9] Papers need to explain why the new ideas they present are important for other researchers. Since different contributions to the same topic are, to use an example by Polanyi (1969), pieces of the same puzzle, it is clear that contributors need to explain where their pieces fit. A piece that cannot fit (a piece not belonging into the picture, so to speak) is considered low quality; a fitting piece is high quality. The regulative idea in the background is getting the picture right (that is, realism or truth). Obviously, quality is hereditary in this context: a researcher who accepts non-fitting pieces has a increased risk of supplying a piece that fits with these wrong pieces instead with the right pieces (contamination).

Researchers therefore cite previous contributions in order to explain how their contributions fit. Such explanations aim at making clear that the problem situation, the picture as it stands, has changed. They contain an implicit warning that tries to create a self-fulfilling expectation: almost everybody will take note of my contribution; hence, no contribution ignoring my contribution will fit into the picture any longer; hence, you should also take note of my contribution *and cite it,* so that others know that your contribution fits.

Third, observing citation rules gives a signal of (at least, basic) competence in a field. Given that time is scarce, nobody wants to read the papers of incompetent authors. Three necessary conditions must hold for a signaling explanation of citation rules. First, and trivially, violations of the rule must be detectable by quickly scanning a paper. Second, rule following must be

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[9] This is a more extended version of Hull's (1988: e.g., 310) explanation, who argues that researchers cite others in support for their own work.

more difficult for the incompetent than for the competent researcher; this follows from the logic of signaling games. Third, there must be a reason why researchers prefer to read papers of competent authors only. The simplest explanation is that papers by incompetent authors have a much higher probability to be low quality; reading a paper leads to a more reliable estimate of its quality but is also more costly, which explains why a paper that looks like low quality is not even read.

The first two arguments, citations as abbreviations and citations as propaganda, can explain why researchers refer to the literature at all. However, they cannot explain why researchers observe the priority rule of citing the original source instead of another paper containing the same idea, say, the paper containing the best presentation or the paper by the most influential colleague.

The signaling argument, on the other hand, supports the priority rule because the correct citation of original sources is one indicator of basic competence. Additionally, the priority rule is also useful in other ways, which makes it a focal point among citation rules. It establishes an incentive to publish, and to publish fast (Merton 1973); and it is fair from an *ex ante* point of view since it gives nobody a privileged starting position in the competition for status.

Establishing citation rules is a coordination game. Given that the norms are respected by almost everybody, it is in one's own interest to do the same. Thus, the rules are incentive compatible. The reasons for the acceptance of citation rules can, especially with respect to the important priority rule, be explained by an extension of section 3's model.

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