

Financial Market in the Laboratory, an Experimental Analysis of some Stylized Facts[†]

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Abstract

This paper purports to provide experimental evidence explaining a number of stylized facts associated with the behaviour of financial returns, in particular, the fat tailed nature of their distribution and the persistence in their volatility. By means of a laboratory experiment, we will investigate the effect of *quantity* and *quality* of information, present in a financial market, upon its stylized facts, showing how both *quality* and *quantity* of information might have an impact on volatility clustering and the emergence of fat tail returns.

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I. Introduction

It is now well established (Mandelbrot, 1963; Fama, 1963) that changes in asset prices (returns) do not have a Normal distribution. In fact, if we assume that returns are normally distributed we have to accept an unrealistically high number of ‘outliers’. This leads us to reject the normality assumption.

There is abundant literature studying the empirical features of financial markets. Pagan (1996) provided an authoritative survey of these stylized facts and of the econometric techniques how to treat them.

There are also several empirical works that analyse the empirical regularity of those markets, i.e. de Vries (1994), Guillaume *et al.* (1997), and Lux and Ausloos (2002).

In the last couple of years, the study of behavioural models of dynamic interaction in financial markets – Beja and Goldman (1980), Day and Huang (1990), Lux and Marchesi (1999, 2000), Chen *et al.* (2001), Iori (2002), Farmer and Joshi (2002), LeBaron (2000), Gaunersdorfer and Hommes (2005), Gaunersdorfer, Hommes and Wagner (2000), Ariofovic and Gencay (2000) and Georges (2005) – has brought about a better understanding of some of the key stylized features of financial data, namely the *fat tails* of the distribution of returns and the *autoregressive dependence* in volatility. Some possible general explanations seem to emerge from this literature: first, volatility clustering and fat tails may emerge from indeterminacy in the equilibrium of the dynamics. In particular, with different strategies performing equally well in some kind of steady state, stochastic disturbances lead to continuously changing strategy configurations which at some point generate a burst of activity. This type of dynamics can be found already in Youssefmir and Huberman (1997) in the context of a resource exploitation model and can be identified in both the papers by Lux and Marchesi (1999, 2000).

The purpose of this paper is to verify if the above stylized facts can be reproduced in the laboratory, in order to better understand the reason why financial markets exhibit these features.

In the next section, we address the issue of informational efficiency market; in section III, we describe our data set. In section IV, we present the experimental design. Section V presents a theoretical solution of the model. In section VI, the experimental parameters are settled. Section VII presents some elementary statistics. In sections IIX-XIII, some stylised facts are analysed. Finally, in section XIV, we draw some conclusions.

II. Informational Efficiency of Markets

Various experimental studies attempted to analyse the role of information within financial markets. For the sake of clarity, we can categorize these studies into three groups:

- Studies addressing the issue of dissemination of information from a group of identical informed agents (insiders) to a group of identical un-informed agents.
- Studies addressing the issue of aggregation of different pieces of information owned by different traders and its dissemination.
- Studies addressing the issue of information's production.

Within the first line of research, Plott and Sunder (1982) studied the dissemination of information by running an experiment in which subjects can trade in each period a single unit of asset. The market institution employed was a double auction. Following the experiment, the authors found that, allowing traders for replications of the same tasks over the experiment time frame, markets' behaviour closely converged towards the predictions of the rational expectations theory, where traders decipher the state of the world by observing market phenomena.

This *approximate convergence* (i.e. convergence which occurs with a degree of volatility) was also present in an earlier experiment (Smith 1962), where convergence to equilibrium was characterised by a persistent noise. Moreover, in a recent work, Hey and Morone (2004) showed that whenever complexity increases noise increases as well.

With reference to the second group of works, there is clear evidence that information's aggregation problem depends dramatically on market features: information distribution, common knowledge, experience of subjects, number of assets and so on. For instance, Plott and Sunder (1988) designed an experiment on information aggregation in which traders were endowed with at least two assets in each period and the dividends of these assets were state-dependent. At the end of each period subjects received the realised dividend, but the information that they got in the trading period was noisy. The market institution was a double auction. The authors found that, first, whenever dividend varied across traders, the market could not aggregate information, and, second, that the market information aggregation process was inefficient.

The third type of approach was undertaken by several authors (Grossman and Stiglitz, 1980; Hellwig, 1980; Verrecchia, 1982; Sunder, 1992; Copeland and Friedman, 1991 and 1992) who developed noisy rational expectation models and addressed the issue of product of information by deriving equilibria in which asset markets only partially reveal information. In these models, the presence of noises impedes informed traders from recovering all the cost of acquiring information, hence generating an environment in which information is too costly. Morone and Morone (2005) added new insights into the existing literature by addressing the relationship between information and wealth distribution in a market context.

The extremely important aspect of expectation formation and learning in dynamic experimental markets with expectations feedback was partially addressed by Smith, Suchanek and Williams (1988). Recently, Hommes et al (1999, 2000) and Sonnemans et al (1999) tested for expectation formation in a cobweb model (for a survey see also Hommes, 2001).

As already mentioned, this paper purports to provide experimental evidence explaining a number of stylized facts associated with the behaviour of financial returns, in particular, the fat tailed nature of their distribution and the persistence in their volatility. By means of a laboratory experiment, we will investigate the effect of *quantity* and *quality* of information present in a financial market upon its stylized facts, showing how both *quality* and *quantity* of information

might have an impact on volatility clustering and the emergence of fat tail returns along the lines of the papers of Lux and Marchesi (1999, 2000).

III. Data description

The data used in this paper are obtained from two independent sources: experimental data (the experiment was run at the laboratory of EXEC at the University of York), and filed data (DAX real time series).

The experiment is based on at least two important strands of literature. The first of these strands is that of *herd behaviour* in a non-market context. The key references are Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992), both of which showed that herd behaviour may result from private information not publicly shared. More specifically, both of these papers showed that individuals, acting sequentially on the basis of private information and public knowledge about the behaviour of others, may end up choosing the socially undesirable option. For some experimental evidence see Anderson and Holt (1997), Allsopp and Hey (2000), and Fiore and Morone (2005). The second strand of literature motivating this paper is that of *information aggregation in market* contexts. A very early reference is the classic paper by Grossman and Stiglitz (1966) which showed that uninformed traders in a market context can become informed through the price in such a way that private information is aggregated correctly and efficiently. A summary of the progress of this strand of literature can be found in the paper by Plott (2000). A third, though less directly relevant, strand is that of the experimental economics literature, which suggests that the market may act as a sort of disciplining device on ‘irrational’ behaviour in individual contexts. This third strand reconciles, in a sense, the first two strands.

The experiment was programmed using the z-Tree¹ software of Urs Fischbacher (1999). It was piloted at the laboratory of ESSE at the University of Bari, and the main experiment, reported in this paper, was run at the laboratory of EXEC at the University of York. Significant changes were made between the pilot and the main experiment in the way that subjects were briefed. At York, a Power Point presentation, preset to run at a particular speed, was run on all subjects' computer screens. This was followed by a practice session in which particular subjects were asked to perform particular tasks (make a bid, make an ask, buy, sell, and buy one or more signals). The briefing period lasted some 40 minutes. An example of the Power Point presentation can be found at <http://www-users.york.ac.uk/~jdh1/papers/instructions.ppt>.

The four experimental data series are composed respectively by 1303, 1545, 813, and 1373 observations. We analysed also daily changes of the German share price index DAX over the time horizon 1959-1999. This is our benchmark to compare the experimental data to real data.

IV. The experimental design

As pointed out by Sunder (1995) “capital asset markets are distinguished from other markets by the informational role of price and by the duality of traders’ role: each trader may buy *and* sell asset(s) in exchange for money or some other numeraire commodity.” In order to capture this important feature we use a single-unit double-auction² mechanism in which agents are free at any time to make bids and ask, and to accept existing asks or bids. This market mechanism is symmetric in that both buyers and sellers can actively post and accept prices in a public manner. We adopted this trading procedure as it is well known from countless experiments (in simpler contexts) that this

¹ z-Tree (Zurich Toolbox for Readymade Economic Experiments) is a software for experimental economics. This software package allows to develop and to carry out economic experiments. In this program features that are needed in most experiments are generally defined. Among them are the communication between the computers, data saving, time display, profit calculation and tools for screen layout. A further strength of the program lies in its versatility: It can be used for a wide range of possible experiments such as public good experiments, structured bargaining experiments or markets - including double auctions and Dutch auctions (<http://www.iew.unizh.ch/ztree/index.php>).

² “Due to its impressively robust performance, the double auction is probably the most commonly used laboratory trading mechanism” (Holt, 1995).

mechanism reaches the competitive equilibrium quickly and efficiently (Smith, 1962, 1964; 1976; Holt, Langan and Villamil, 1986; Smith and Williams, 1989; Gode and Sunder, 1989, 1991; Davids and Williams, 1986; Ketcham, Smith and Williams, 1984; Davids, Harrison and Williams, 1993; Plott and Smith, 1978; Mestelman and Welland, 1992; Holt, 1995).

We have a market composed by n agents; each one is endowed with a quantity of experimental money and m units of an unspecified asset. This asset pays a dividend at the end of the trading period, but apart of the dividend it is worthless. This dividend is uncertain. There are two possible 'states of the world'- each with equal probability – either the dividend is some positive number d , or it is zero. At the beginning of each trading period the true state is determined by the experimenter – but not revealed to the agents. They can, however, buy signals – which are partially but not totally informative as to the true state of the world. These signals take either the value 1 or the value 0. More precisely the probability of getting a signal of 1 is p if the true state of the world is that the dividend is d ; the probability of getting a signal of 1 is q if the true state of the world is that the dividend is zero. More accurately, if a subject receives a signal 1, he will infer that the dividend will be with probability p equal to d and with probability q equal to 0, *vice versa* if he receives a signal 0 then he will infer that the asset will pay with probability q a dividend equal to d , and with probability p it will pay a dividend equal to 0 ($p > q$). In most respects this is identical to the Bikhchandani *et al* model, though the experimental set-up differs in two crucial respects.

- First, signals are costly – each signal costs an amount c .
- Secondly, agents can buy at any time during the trading period as many or as few signals as they want.

So information is not released sequentially and the number of signals per agent is not restricted to 1. Obviously, these differences change the nature of the solution to the model, but these changes are necessary in a market environment in which trading takes place continually throughout the trading period. Agents are informed of all the relevant parameters – the positive dividend d , the cost of buying a signal c , and the two probabilities p and q .

We had four different Treatments, Treatments 1 through 4 each one corresponding to a different quadrant in the following diagram:

Cost and quality of information	
Treatment 1 low cost/low quality	Treatment 2 high cost/low quality
Treatment 3 low cost/high quality	Treatment 4 high cost/high quality

Table 1: Signals' cost and quality in the four Treatments.

The Treatments differed in terms of the parameters we used. The key parameters of the experiment are the cost of buying a signal, c , and the two probabilities p and q . It seems natural to make these signal probabilities symmetric, so we put $p = 1 - q$. We then chose two values for the cost c , and two pairs of values for p and q . The four Treatments consisted of the four possible combinations implied. For each Treatment, we had a different Power Point briefing presentation – containing the correct parameters.

The payment mechanism that we used to motivate the subjects in our experiment was the obvious and natural mechanism: agents start with some experimental money and with m units of the asset. During the trading process they can increase or decrease the number of units of the asset that they own and, depending upon the prices at which they trade, their stock of experimental money will increase or decrease during the period. At the end of each market period the true dividend for that period is announced and the appropriate dividend distributed in experimental money to the asset owners at the end of the period. Accordingly, agents will end up with a stock of experimental money at the end of each trading period – which may be more or less than that with which they started that period. An agent's trading profit for any trading period is the difference between the final stock of experimental money and the initial stock. For the experiment as a whole the payment

to an agent is simply the sum of the profits over all trading periods of the experiment. There was a fixed rate of exchange between experimental money and real money.

Note that agents can make losses. To avoid some of the problems associated with subjects making real losses in experiments, we endowed all agents with a participation fee, which could be used (if the subject agreed) to offset losses. Once this participation fee was exhausted, any further losses had to be covered by the subjects themselves – some subjects chose this option, others chose to leave the experiment once they had exhausted their participation fee.

V. Theory

This experiment is close to the two literatures mentioned above (section III). However, it differs in crucial respects from both those literatures, so we cannot use those literatures to help us find the theoretical predictions of our model. However, we can say two things.

First, we can conclude that the price in the market ‘should’ converge to the true value of the dividend – *if the market correctly aggregates the information available to the agents*. This is the conclusion that would be reached by the branch of the literature starting with Grossman and Stiglitz (1976). If the true state of the world is that the dividend is d the price ‘should’ converge to d ; if the true state of the world is that the dividend is zero the price ‘should’ converge to zero. However, we note the nature of the theory of that branch of the literature – it is not a theory providing a description of the *process* by which the market converges, but rather a theory of the equilibrium state of the price in such a market. Thus, this branch of the economics literature does not tell us that the price *will* converge to the true value of the dividend. Moreover, we have no theory that tells us what is the optimal behaviour of agents in this experiment, though we can identify one possible equilibrium – in which no agent does anything. If all except one agent is doing nothing, then it is clearly optimal for the remaining agent to do nothing – for this remaining agent can neither buy nor

sell (because no one else is selling or buying) and so can only buy signals. But there is no point in buying signals as no use can be made of the knowledge gained.

So doing nothing is one possible symmetric equilibrium. We now argue that this is the *only* possible equilibrium in a world populated by risk-neutral agents. To demonstrate this, we begin by noting that the expected per-period payoff for any subject who does nothing must be $md/2$ because each subject is endowed with m units of the asset, each of which is worth either d or 0 with equal probability. Suppose now that some subject buys k signals. Because these signals are costly this subject must be expecting to make at least $md/2 + kc$ from trading the asset. Because the game is a constant sum game, this must imply that the remaining subjects must be averaging $md/2 - kc/(n-1)$ from trading the asset. As this is less than what they would get doing nothing, it is clearly better for them to do nothing – from which it follows that our first subject can not be making at least $md/2 + kc$ from trading the asset. In this case, the purchase of k signals can not be worthwhile. This can not be an equilibrium.

This would suggest that we would see no trade in a model in which all the agents are risk-neutral. Similar arguments would suggest that if all agents have the same beliefs about the future value of the dividend and if all agents are equally risk-averse then we would again observe no trade. However, if different agents have different beliefs or different attitudes to risk then some trade may be possible. Consider, for example, a situation in which individual A owns a unit of the asset and is more risk-averse than individual B. Suppose they have the same beliefs – that the probability is π that the dividend will be d . Then A would be happy to sell his or her unit at any price bigger than the P which satisfies the expression $u(P) = \pi u(d) + (1-\pi) u(0)$ (where $u(\cdot)$ is A's utility function, expressed relative to his or her present wealth) while B would be happy to buy this unit of the asset at any price less than the Q which satisfies the expression $v(Q) = \pi u(d-Q) + (1-\pi) v(-Q)$ (where $v(\cdot)$ is B's utility function, expressed relative to his or her present wealth). In general, we should be able to find a price which satisfies these two conditions – if B is less risk-averse than A.

Thus, if agents have the same beliefs but different attitudes to risk, some trade may be possible. The converse situation – in which agents have different beliefs but the same attitude to risk - is somewhat different. Suppose A and B know that they have the same attitude to risk, then if they can find a price at which one wants to buy and the other wants to sell, what can they infer? What they *must* infer is that they have different beliefs about the probability that the asset is valuable. Let π_A denote A's probability and π_B denote B's probability. Then if there exists a price at which A is happy to buy a unit and B is happy to sell a unit it must follow that $\pi_A > \pi_B$; that is, A must be more optimistic than B about the probability that the asset will be valuable. At this point, A must infer from the fact that B wants to sell that $\pi_A > \pi_B$, and B must infer the same from the fact that A wants to buy. If they each assume that the other is rational they may conclude that one or other or both of them is wrong.

But this provides a clue why we might observe trade: *everyone thinks that they have better information than the others about what the dividend is going to be*. Obviously, this is impossible, but we have already argued that anything other than a 'do-nothing' situation can not be an equilibrium in the usual sense used by economists. We should stress this point: apart from the buying of signals, the game is a constant sum game – there is a total of $mnd/2$ given by the experimenter to the n subjects each market period. Apart from the buying of signals, each subject makes on average $md/2$ each period. Subjects can guarantee this on average by doing nothing. However, the buying of signals is costly and simply makes the average per subject per period payoff less than $md/2$. So why would anyone buy signals? And why would anyone else trade with anyone who has bought signals? There seems to be no reason for any activity in our experiment.

So where does that leave us? It leaves us with some very simple predictions. First, if we believe in equilibrium theory in games (which is concerned more with the process than the outcome), then we would expect to see no activity at all. Second, if we believe in the predictions of the Grossman and Stiglitz branch of the literature (which is concerned with outcomes rather than processes), we would expect to see the price converging to the true dividend. But this leaves

unanswered the question as to *how* the market converges. If we believe it converges because everyone thinks that they are better at predicting the true future dividend, then we leave open also the possibility that it converges to the wrong value. But note the paradoxical nature of all of this. If an agent can predict the future value of the asset *and can trade on that information* (buying at a price less than d when the asset is going to be valuable and selling at a positive price when the asset is going to be worthless) the agent can make a profit. But this must be at some other agent's expense. As all agents know this, why might we observe any trade?

VI. Experiment Setting

We had $n = 15$ agents, each of whom was endowed with 10 Sterling Pounds of 'experimental' money (actually equivalent to real money as the exchange rate was one for one) and 10 units of the asset. The dividend on each unit of the asset was either ($d=$) 10p or 0p, it is randomly determined at the beginning of each period and it is constant in the period. The experiment consisted in 4 practice periods and 10 real periods. Players were paid only for the profits made over the 10 real periods. Each period lasted for 4 minutes; the whole experiment lasted a bit more than one hour and thirty minutes, including reading instructions and the subjects' payment.

As noted above, the key parameters are the cost of buying a signal c , and the two probabilities p and q . We took two values for c , 4p and 6p, and two pairs of values for p and q , which were set respectively as: $3/5$ and $2/5$, and $4/5$ and $1/5$. Combining these values produced four different Treatments, as described in Table 1. Subjects can buy information at any time when the market is open.

With regard to c , the signal's cost, we predicted that higher values would induce agents to buy less signals. This would increase the degree of fuzziness of the market system (due to a scarcity of information). In turn, this might result in more information available in the market. We might therefore expect less noise in better informed Treatments. As far as the two probabilities are concerned, as p rises and q falls, the signals become more reliable – hence, the quality of the

information present in the system improves – and this might produce some considerable effect on price volatility. We shall expect less volatility in Treatment 4 compared to Treatment 1. However, a comparison between Treatment 2 and Treatment 3 would not be as straightforward.

	$c = 4p$	$c = 6p$
$p = 3/5$ and $q = 2/5$	Treatment 1	Treatment 2
$p = 4/5$ and $q = 1/5$	Treatment 3	Treatment 4

Table 2: Quality and cost of information in the four Treatments

Subjects’ pay-off depends considerably upon the parameters choice, but evaluating a possible earning interval without information on agents’ strategy is a very complicated task.

A possible strategy is “doing nothing”. In this case, agents would get a dividend of 10p with probability 0.5 and a dividend of 0p with probability 0.5. Their expected pay-off will be 50p in each trading period, and hence, their overall pay-off will be £5 plus the £3 of participation fee. Thus, on average, subjects made £8 from participating in this experiment.³

However, there was a considerable variation around this average figure: some subjects gained less than their participation fee whereas others got paid a considerably higher sum than the participation fee plus the average dividend.

VII. Some elementary statistics

We have already noticed that empirical data in financial markets are not normally distributed. Table 1 reports some elementary statistics for the returns of the DAX and our four experimental Treatments.

³ Since the experiment is a zero-sum game for each subject, this will be the amount paid out on average.

It is clear that all five distributions exhibit excessive kurtosis. This implies that the experimental financial market, like real markets, exhibits more probability mass in the tails and in the centre compared to a Normal distribution. Additionally, the Bera –Jarque test for normality leads to a rejection of its null hypothesis.

	DAX	Treatment 1	Treatment 2	Treatment 3	Treatment 4
Mean	0	0	-0,001	-0,005	0
S.D.	0,005	0,252	0,213	0,418	0,261
Skewness	-0.3216	-0.0826	0.0437	0.0602	-0.0185
Kurtosis	10.4481	3.3944	8.8983	6.4489	14.0119
Bera-Jarque test	44908.312 (0.000)	626.579 (0.000)	5094.409 (0.000)	1407.571 (0.000)	11223.906 (0.000)

Table 3: Elementary statistics

Now, we will take a closer look at the statistical characteristics of our experimental data sets. More precisely, we will investigate whether and how the experimental market compares with the stylised facts observed in real financial markets: Unit Root, Fat Tail, Cluster Volatility and Autocorrelation.

VIII. Unit root property

“A realistic market should yield a dynamics which appears to be close to a random walk. We, therefore, perform a typical test for the presence of a unit root in both our experimental and filed time series using the Augmented Dickey-Fuller test” (Lux and Schornstein, 2002). The usual finding in financial markets is non-stationarity for the price time series and stationarity for its first difference, i.e. the returns. Table 4 reports the outcome of the Augmented Dickey-Fuller test. For each case, the time series have been divided into 10 sub-samples and the test has been run on each sub-sample.

Time Series	Range of λ		No. of rejections for one-sides test at 95% level	No. of rejections for two-sides test at 95% level
	Min	Max		
DAX	0,4148	1,0038	2 out of 10	2 out of 10
Treatment 1	-0,2788	0,2674	10 out of 10	10 out of 10
Treatment 2	-0,0891	0,3301	10 out of 10	10 out of 10
Treatment 3	-0,1763	0,9719	3 out of 10	3 out of 10
Treatment 4	-0,0724	0,9635	8 out of 10	7 out of 10

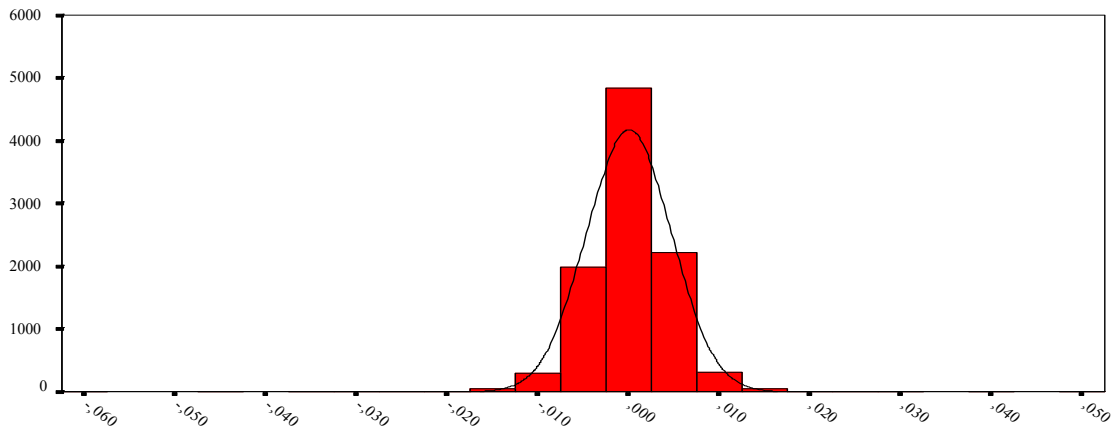
Table 4: Augmented Dickey-Fuller test

Now, we can try to categorise the four Treatments. It seems that we can divide them into two groups. Treatment 1 and Treatment 2 completely fail to exhibit non-stationarity in the price series. On the other hand, Treatment 4 has a unit root in two out of ten periods, and Treatment 3 has a unit root in seven out of ten periods. Note that the DAX exhibits a unit root in eight out of ten subsamples. This is an interesting result, since in Treatments 1 and 2 the quality of the signals is very poor (a signal is informative with probability 0.6 and is misleading with probability 0.4) and thus the aggregated information is not very informative. For this reason price fluctuates around the ‘un-informed’ expected price⁴. On the other hand, in Treatments 3 and 4, the quality of the information is higher (a signal is informative with probability 0.8 and is misleading with probability 0.2). Thus the price does not fluctuate around the ‘un-informed’ expected price but converges (in average) to the correct price (i.e. the true dividend).

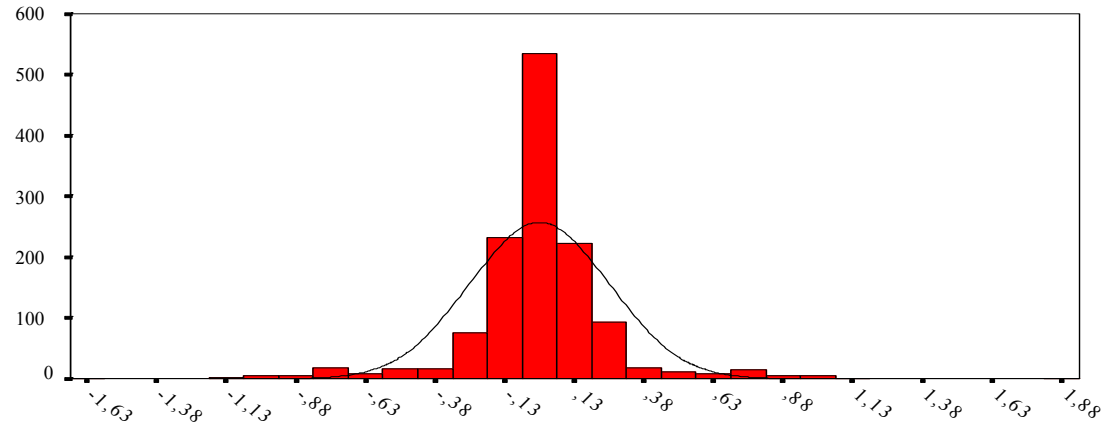
IX. Fat tail phenomenon

In section III, we reported that our four experimental financial markets (as well as the DAX) exhibit excessive kurtosis, and we noticed that return distributions exhibit more probability mass in the centre and in the tail of the distribution. In the following figures, it is clear what we meant by fat tails. In fact, it is possible to see how the distributions of the returns are leptokurtotic.

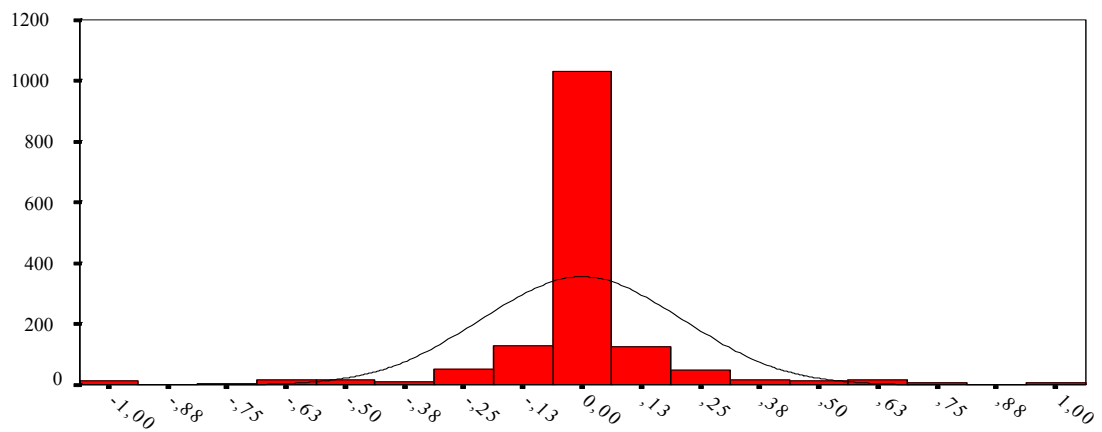
⁴ If all subjects are un-informed, i.e. there is no information at all in the market the asset expected price is simply $0.5 \times 10 + 0.5 \times 0$.



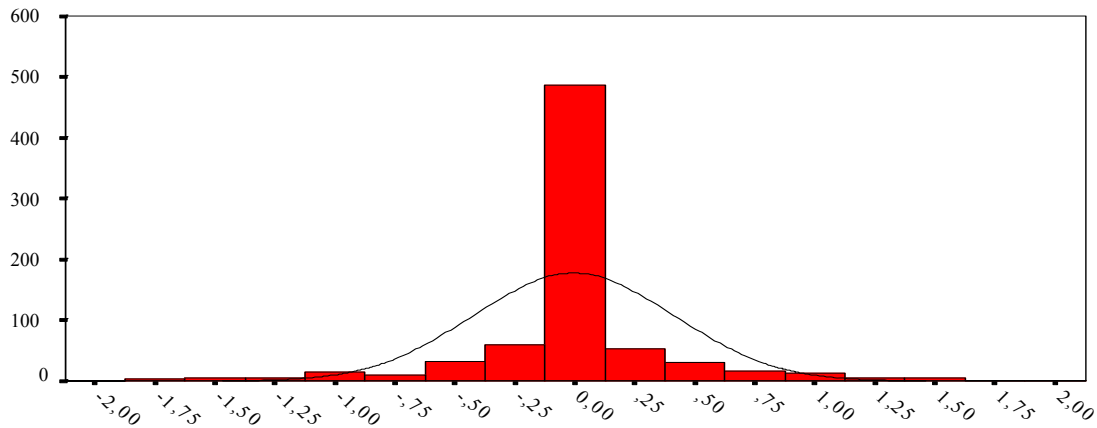
Returns' distribution in the DAX



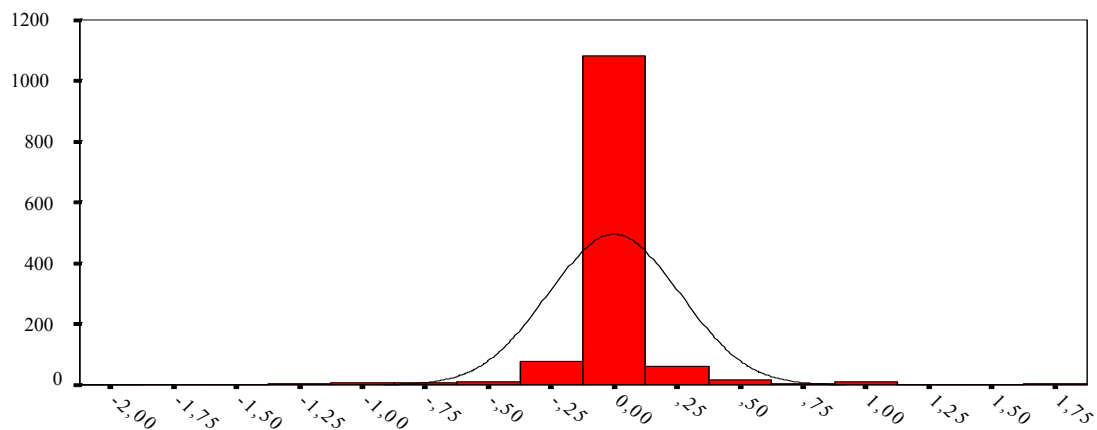
Returns' distribution in treatment 1



Returns' distribution in treatment 2



Returns' distribution of treatment 3



Returns' distribution of treatment 4

We have to note that the kurtosis is, in a certain sense, a poor measure of deviation from normality. For this reason, we need to refer to a sharper characterization of the empirical distribution. It is now well known that the distribution of returns belongs to the class of the ‘fat tail’ distributions. These distributions exhibit a hyperbolic decline of probability mass⁵.

	Treatment 1	Treatment 2	Treatment 3	Treatment 4	DAX ⁶
Hill 10%	2,527	1,898	2,320	1,013	2,945
Hill 5%	6,035	2,905	3,422	1,723	2,974
Hill 2.5%	7,684	6,177	4,930	5,937	3,105

Table 5: Hill estimators for the five distributions

⁵ For a more detailed analysis see Lux (1996).

⁶ The New York Stock Exchange Composite Index, the US Dollar – DM exchange rate and the price of gold exhibits similar figures, cf. Lux (1996).

The visual impression of fat tails is also confirmed by the above Table 5, which reports the Hill estimators for the five distributions (i.e. the four experimental Treatments and the DAX).

The Hill estimator is calculated according the following equation, where m is the number of observations in the corresponding tail of the distribution.

$$\alpha = \frac{m}{\sum_{i=1}^m (\ln x_{n-i+1} - \ln x_{n-m})}$$

The tail index gives us information about the “fatness” of the tails of the distributions. It is obtained ordering the return series in decreasing order and the last $i\%$ as the $i\%$ of the tail. In fact, given a tail index, the biggest integer number smaller than the tail index is the number of finite moments of the distribution. The Hill estimators estimate the tail index.

Empirical studies show that the Hill estimators usually lie in the range of [2.5, 5]. Examples include Koedijk, Schafgans and de Vries (1990), Jansen and de Vries (1991), Loretan and Phillips (1994), Longin(1996), Lux (2002) and Lux and Ausloos (2005).

From Table 4, we can argue that all our four Treatments look like a real financial market, even though for a tail size of 2.5% the tails seems not to be very fat. It is interesting to note that in Treatment 2 and Treatment 4 the bursts are so strong that even tail indices below 2 were found. Remembering that in Treatment 2 and 4 the cost of information was high compared to Treatment 1 and 3, it seems that markets in which the information is more expensive have larger price changes.

We can try to rank our four Treatments according to their tail index. Treatment 3 and Treatment 2 seem to be very good approximations of a real financial market, whereas Treatment 4 exhibits too fat tails at both 10% and 5% tail size and too thin tails at 2.5%. Treatment 1 exhibits realistic tails at 10% level but too thin tails at both 5% and 2.5% levels.

These results seem to be quite encouraging and the “rejection” of the fat tail hypothesis at 2.5% level could be related to the sample size (1303, 1545, 813, and 1373 in Treatment 1, 2, 3 and 4 respectively). It could be of some interest to note that Lux and Sornette (2002) demonstrated that

the prevalence of a rational bubble component would lead to an Hill tail index estimator smaller than 1, which would imply non existence of the mean and variance of the data.

X. Volatility clustering

Plotting the time series of returns it is immediately evident that the results of our experiment are different from previous experiments on financial markets (Forsythe and Russell, 1990; Forsythe, Palfrey and Plott, 1982; Plott and Sunder, 1982; Plott and Sunder 1988; Redrawn and Sunder, 1992; Friedman, Harrison and Salmon, 1984). In fact, we do not have the usual fast convergence to equilibrium, but we see periods of tranquillity interrupted by periods of turbulence. The time series plotted in the figures 7 and 8 below are quite similar to the empirical ones. Periods of quiescence and turbulence tend to cluster together.

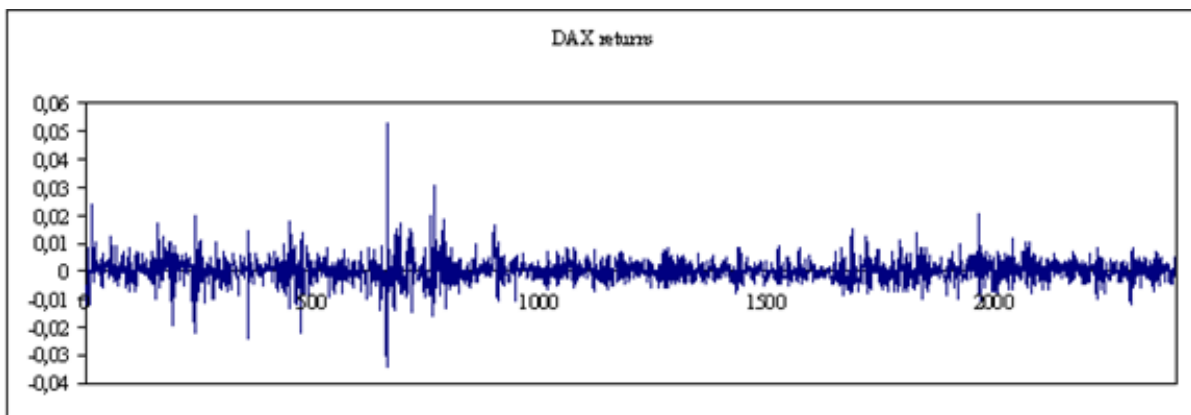


Figure 6

This fact was already pointed out by Mandeldrot (1963), but it was by and large neglected until recently. The volatility cluster regularity (which is particularly clear in figures 6, 7, 8, 9 and 10) suggests that there is autocorrelation in the scale of the process, i.e. in the second moments.

Also the figure below exhibits clustered volatility in the returns. Treatment 1 and Treatment 2 seem to capture this phenomenon pretty well.

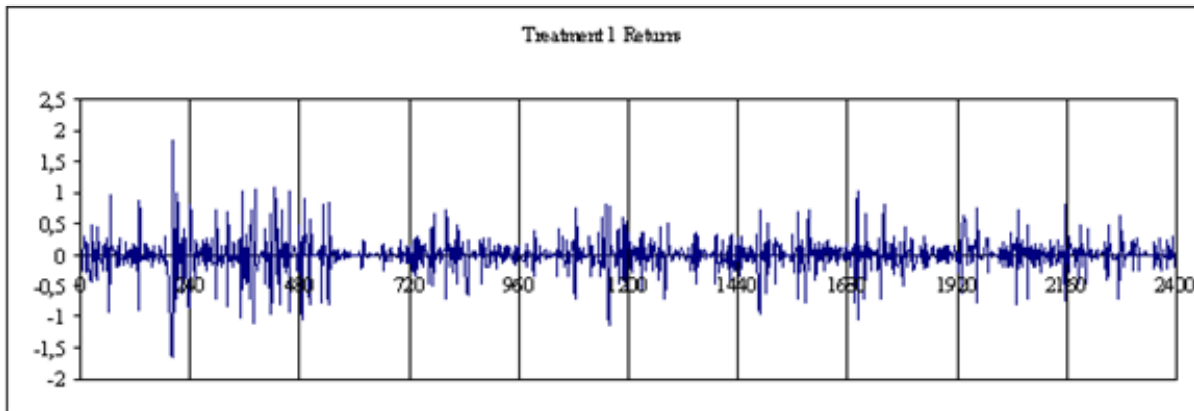


Figure 7

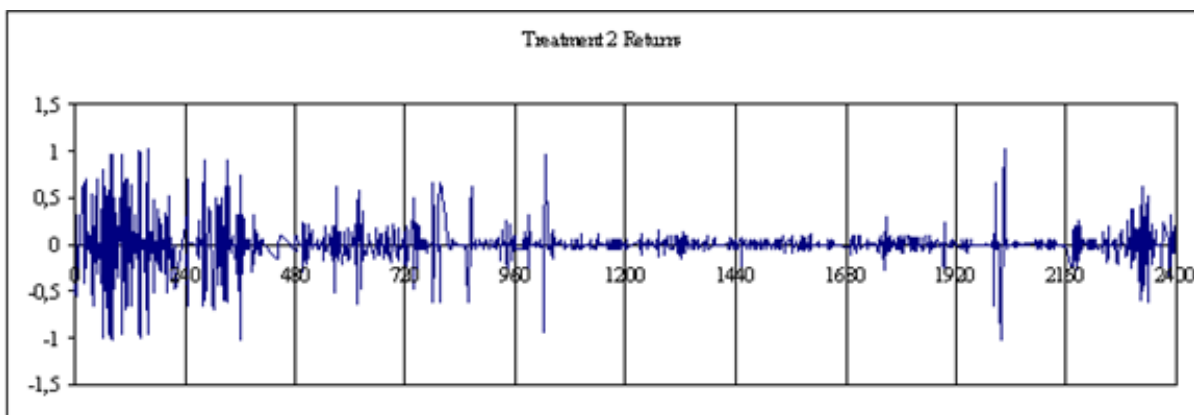


Figure 8

On the other hand, Treatment 3 and Treatment 4, even though they exhibit clustered volatility, seem to be different from a real financial market. A simple explanation could be that, because the quality of information in the market is higher compared to Treatment 1 and Treatment 2, the *invisible hand is less trembling*⁷.

⁷ With the notion of trembling we refer to the trembling hand used by Harsanyi and Selten for modelling equilibrium perfection. A player who wants to play one action might through a slip of the hand take another. That is, players could make uncorrected mistakes (tremble) that lead to unexpected events.

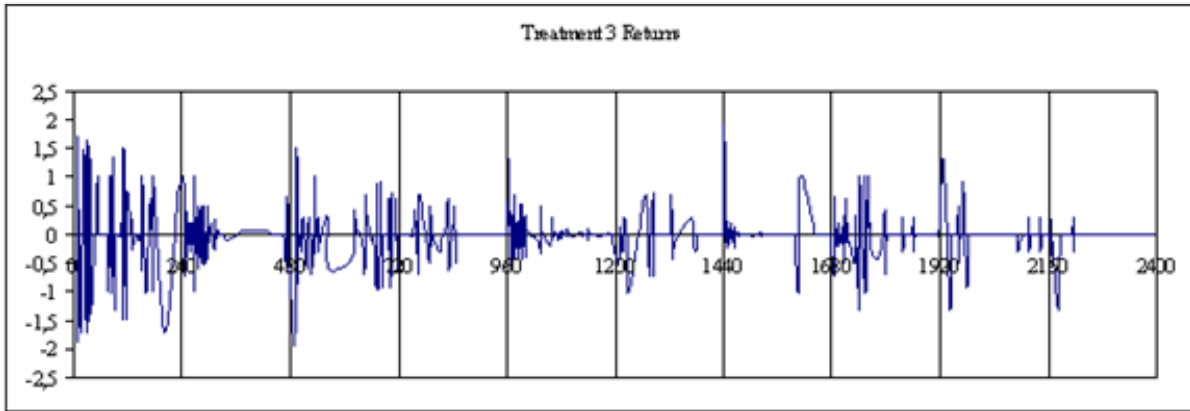


Figure 9

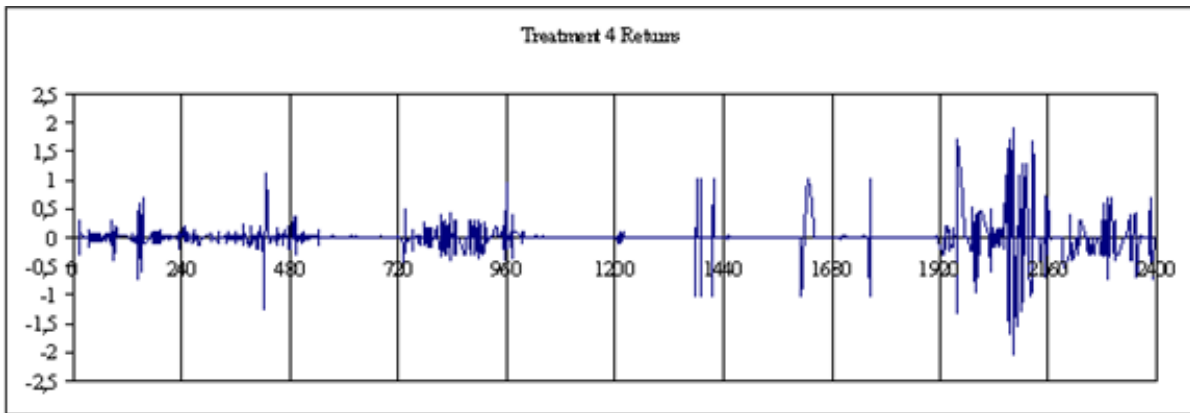


Figure 10

There is abundant literature that studies this phenomenon. Gaunersdorfer and Hommes (2005) proposed a dynamical system with two attractors, a stable steady state and a state limit cycle. When the system is buffed with dynamic noise, irregular switching between close to steady state fluctuations with small price changes and almost periodic fluctuations with large price changes occurs that clustered volatility emerges from stochastic dynamics with multiple attractors.

XI. Absence of autocorrelation

Autocorrelation is often insignificant in raw returns, but highly significant in the volatility measures, i.e. squared returns and absolute returns. The absence of autocorrelation is a very well-known fact in financial data.

In figures 11-13, we plotted the autocorrelation functions of the DAX raw returns, squared returns and absolute returns in the period 1959-1999 (daily observations). For each time series we computed the autocorrelation functions for 100 lags. In figure 11, it is evident that the autocorrelation of raw returns is not significantly different from zero.

Dax autocorrelation function of raw returns

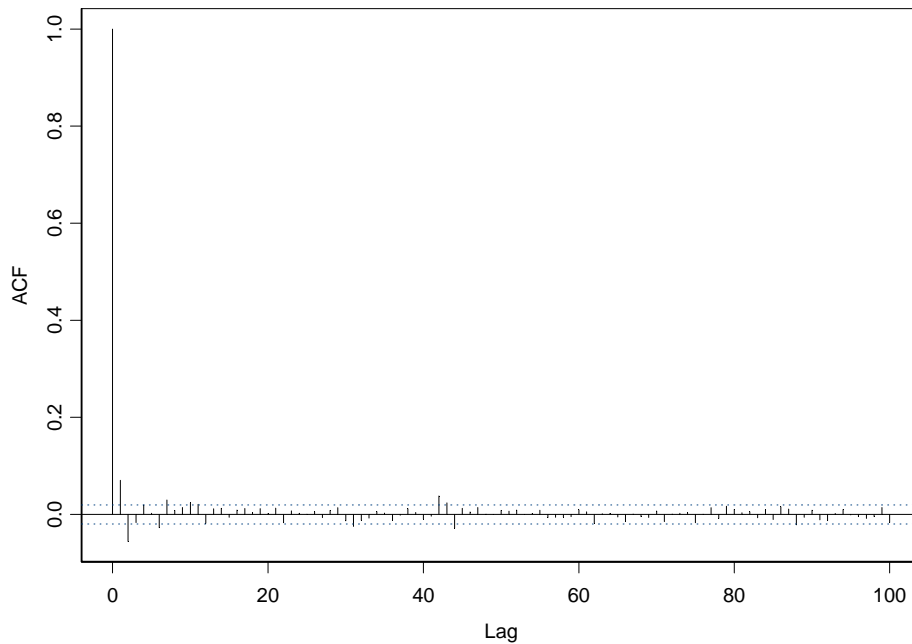


Figure 11

On the other hand, considering the squared returns, we can observe a very long autocorrelation, and it is even larger in the case of absolute returns.

Dax autocorrelation function of squared returns

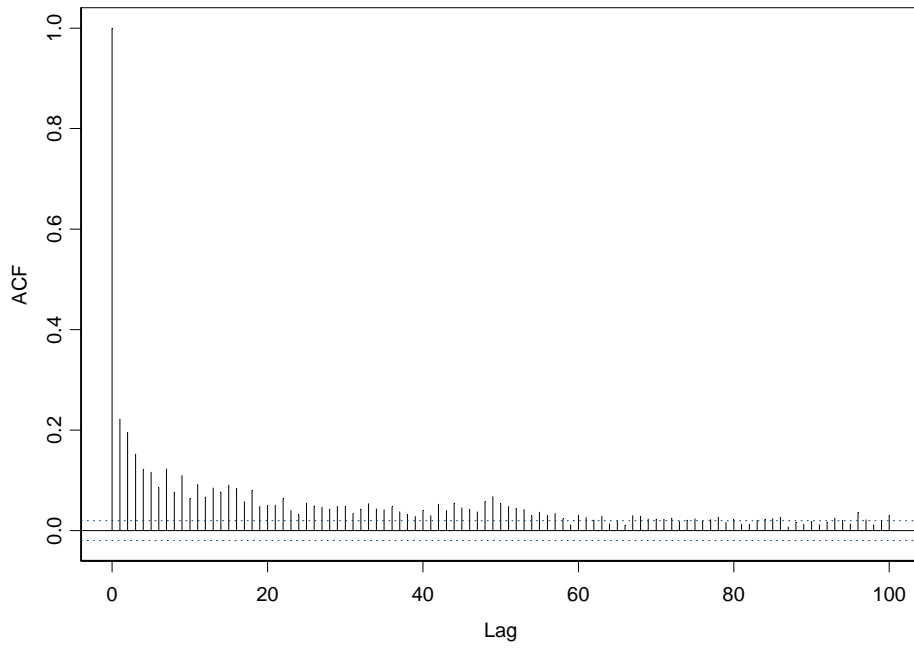


Figure 12

For squared and absolute values the temporal independence is strongly rejected.

Dax autocorrelation function of absolute returns

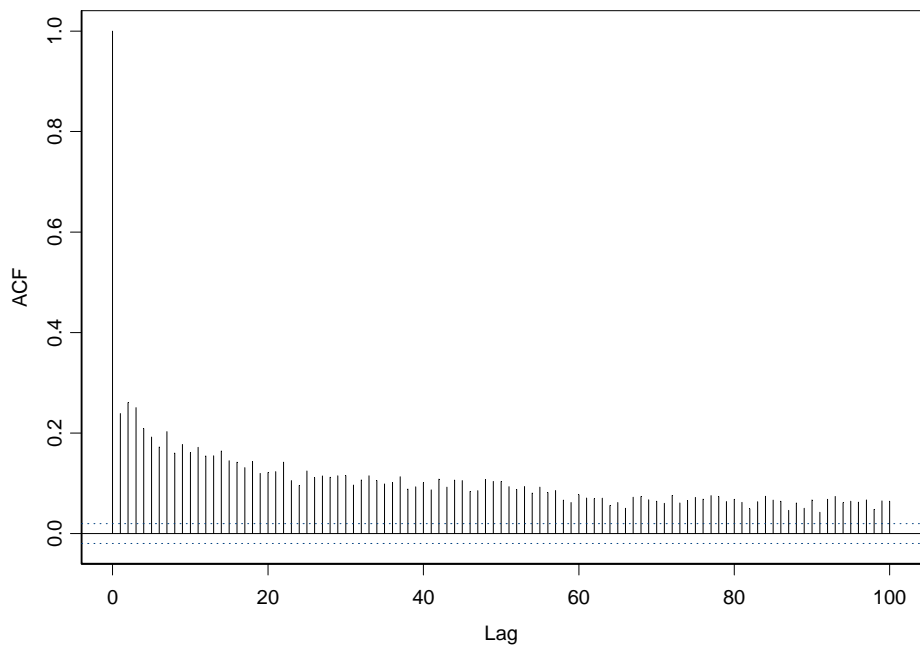


Figure 13

These results are common to all financial markets. In figures 14-16 we report the autocorrelation of returns for Treatment 1 of our experiment. It is evident that returns, squared returns and absolute returns exhibit, temporal independence contrary to financial market empirical evidence.

Treatment 1: autocorrelation function of raw returns

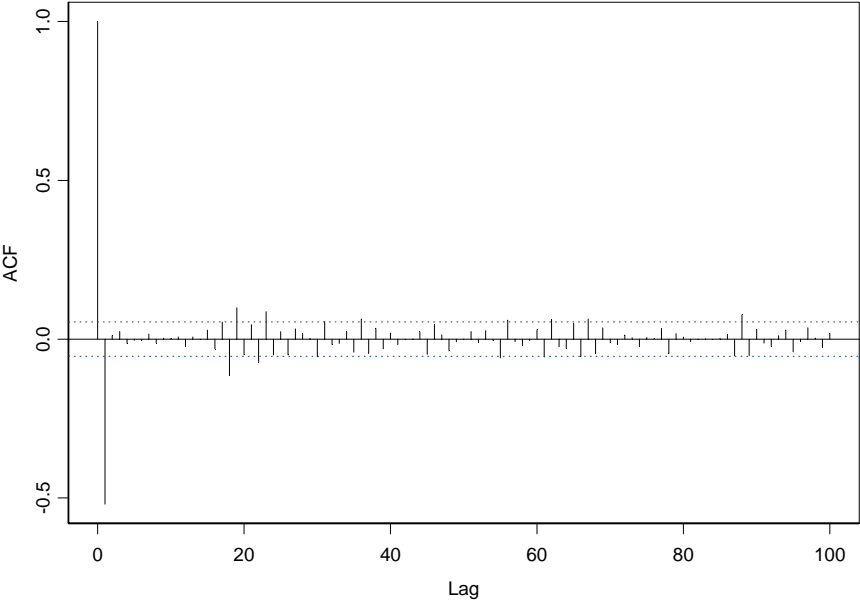


Figure 14

Treatment 1: autocorrelation function of squared data

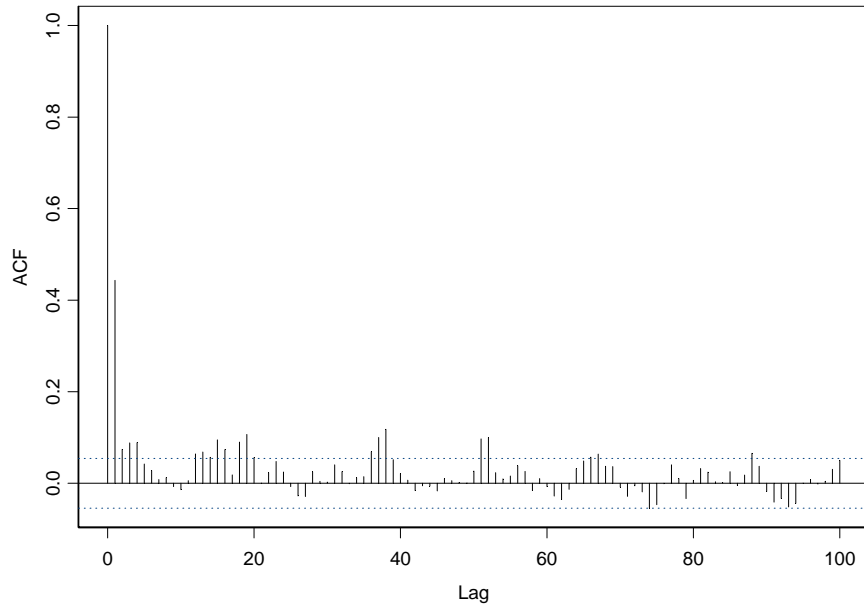


Figure 15

Treatment 1: autocorrelation function of absolute returns

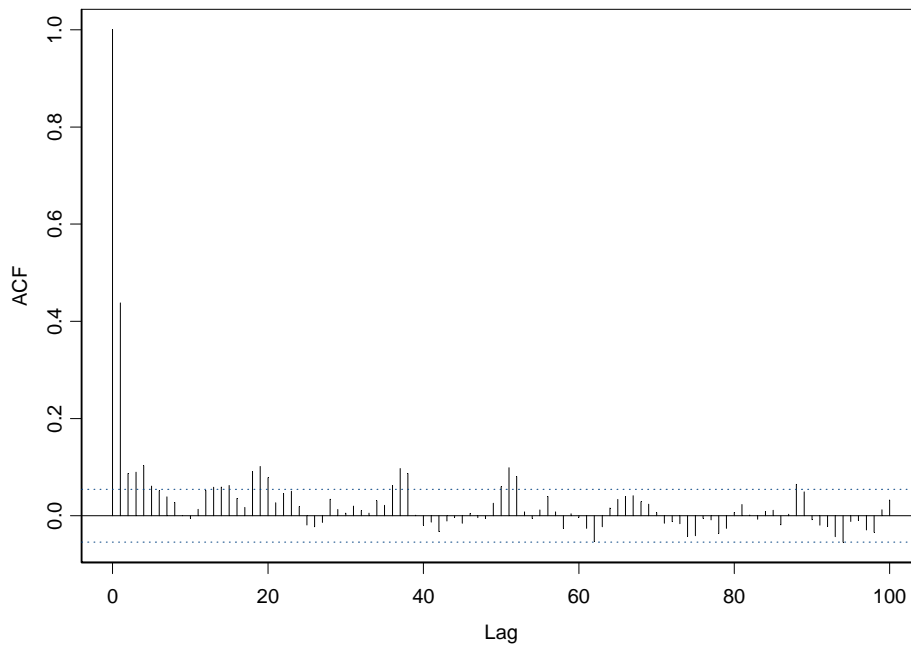


Figure 16

Treatment 2 (figures 17-19) exhibits autocorrelation functions with features typically characterizing financial markets. In fact, the raw data are completely uncorrelated, the squared returns have very long correlation and the absolute returns exhibit longer and higher autocorrelation.

Treatment 2: autocorrelation function of raw returns

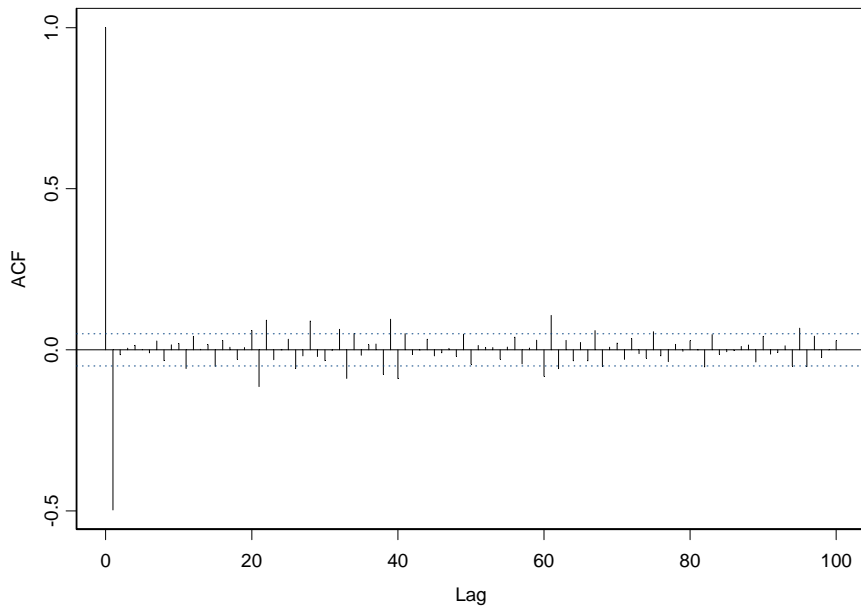


Figure 17

Treatment 2: autocorrelation function of squared returns

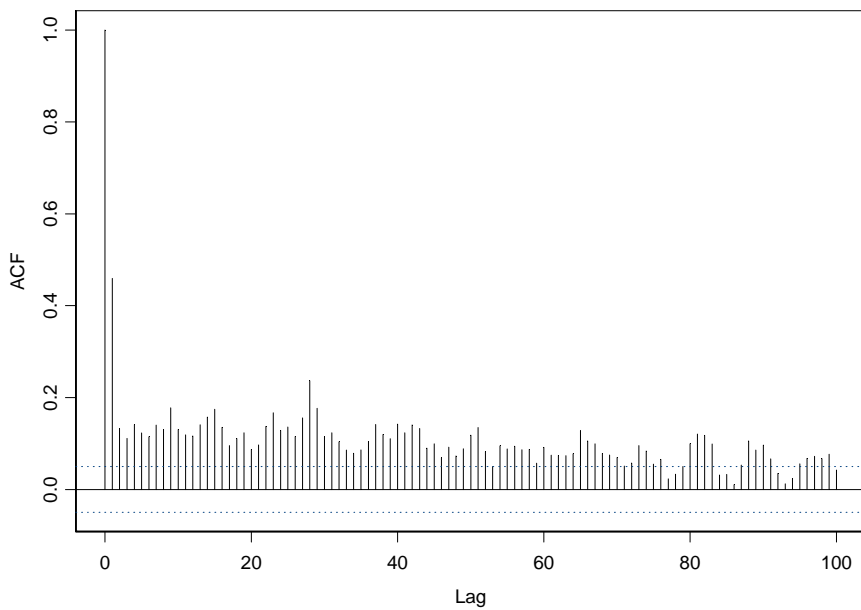


Figure 18

Treatment 2: autocorrelation function of absolute returns

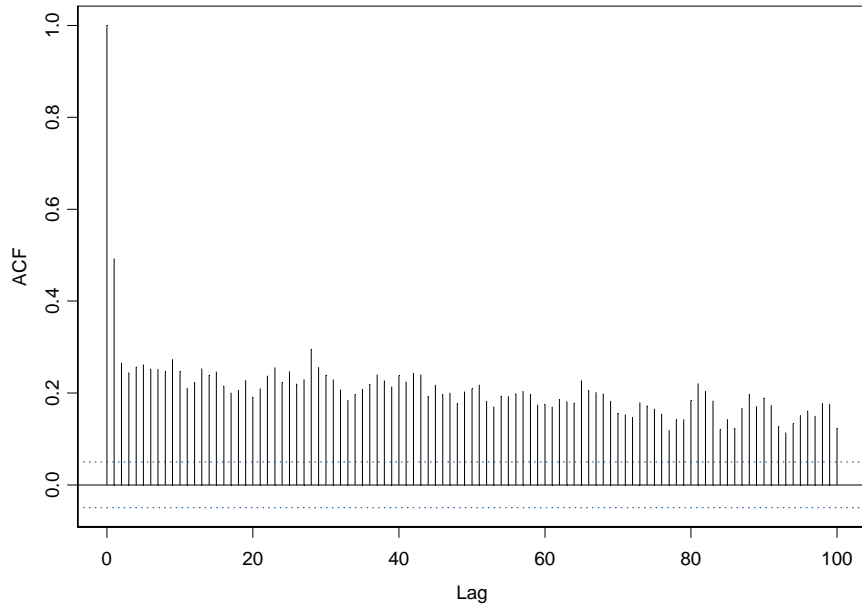


Figure 19

Treatment 3: autocorrelation function of raw returns

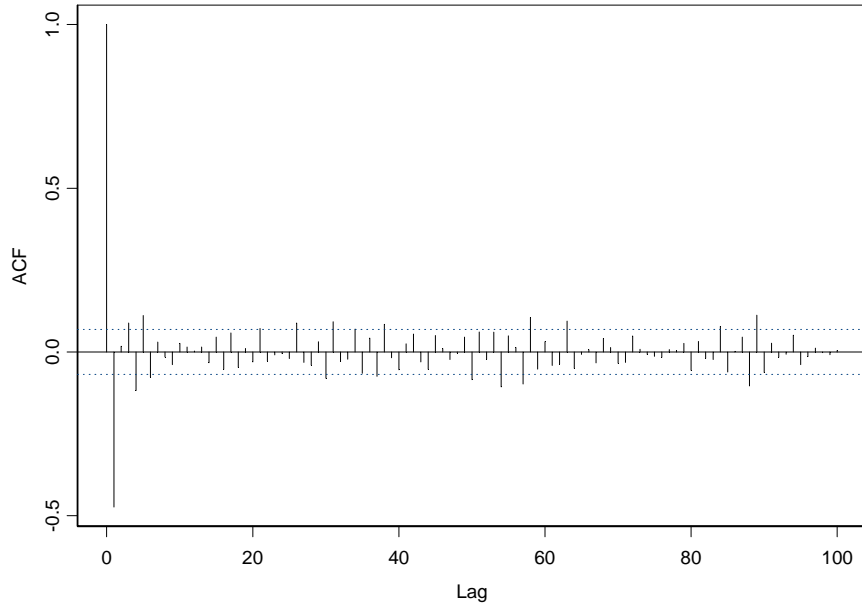


Figure 20

Treatment 3: autocorrelation function of squared data

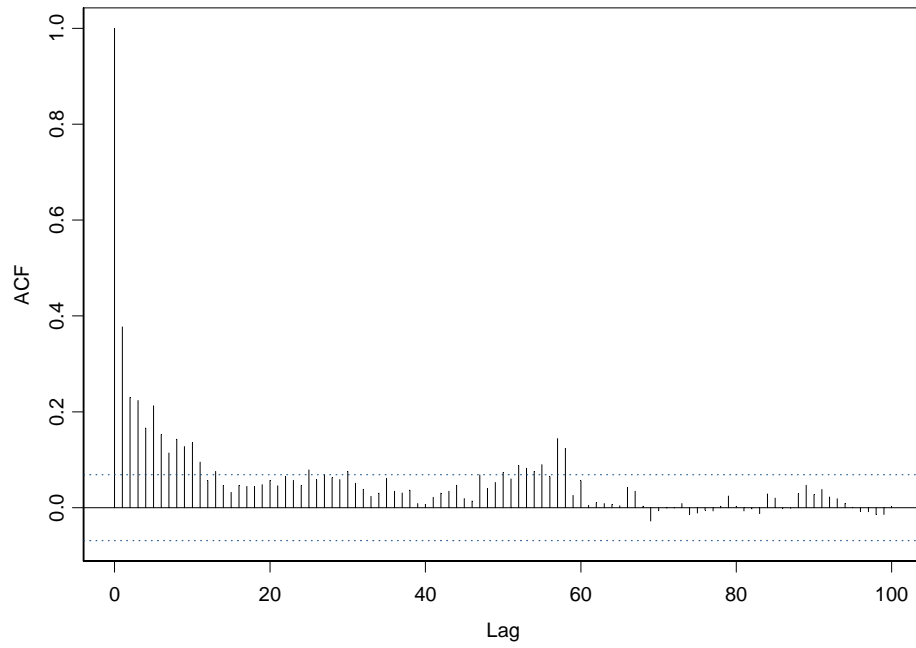


Figure 21

Treatment 3: autocorrelation function of squared data

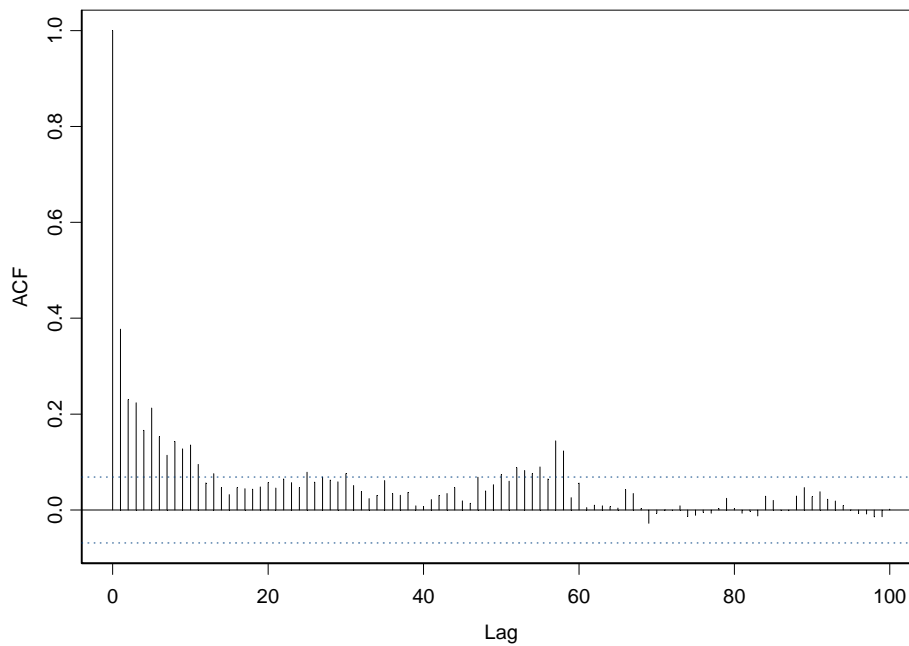


Figure 22

Treatment 3: autocorrelation function of absolute return

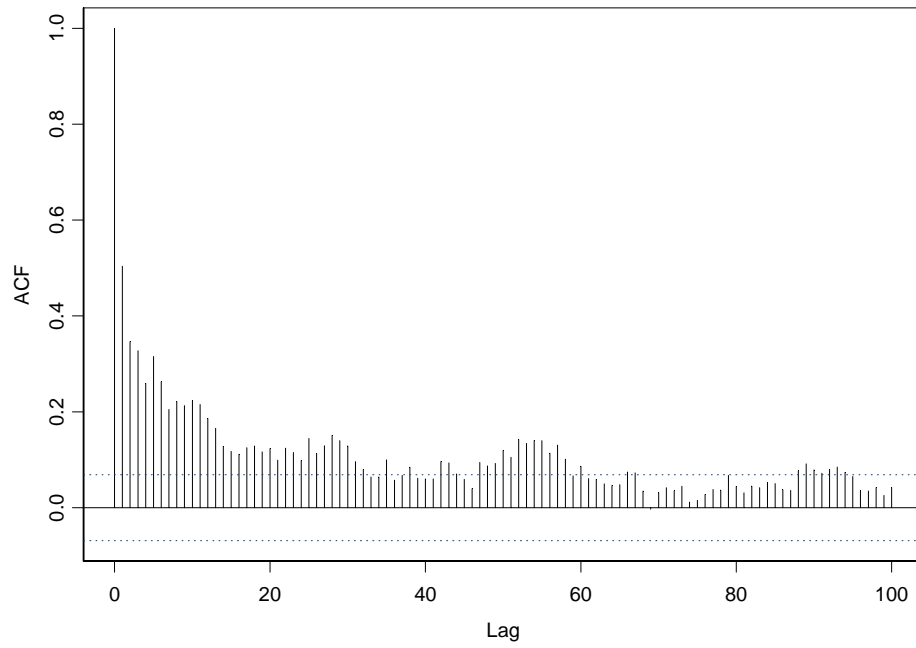


Figure 23

Treatment 4: autocorrelation function of raw data

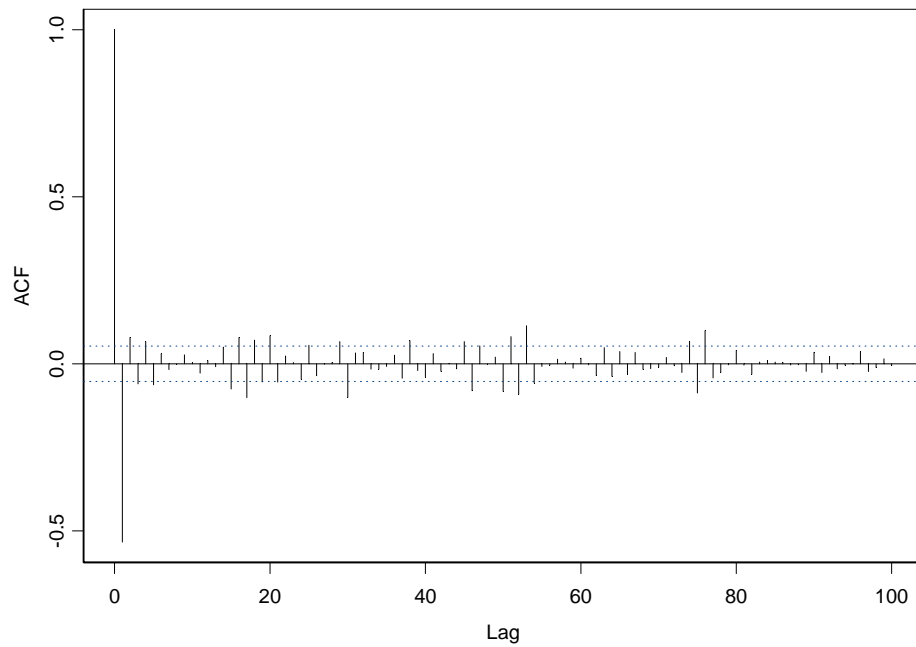


Figure 24

Treatment 4: autocorrelation function of squared data

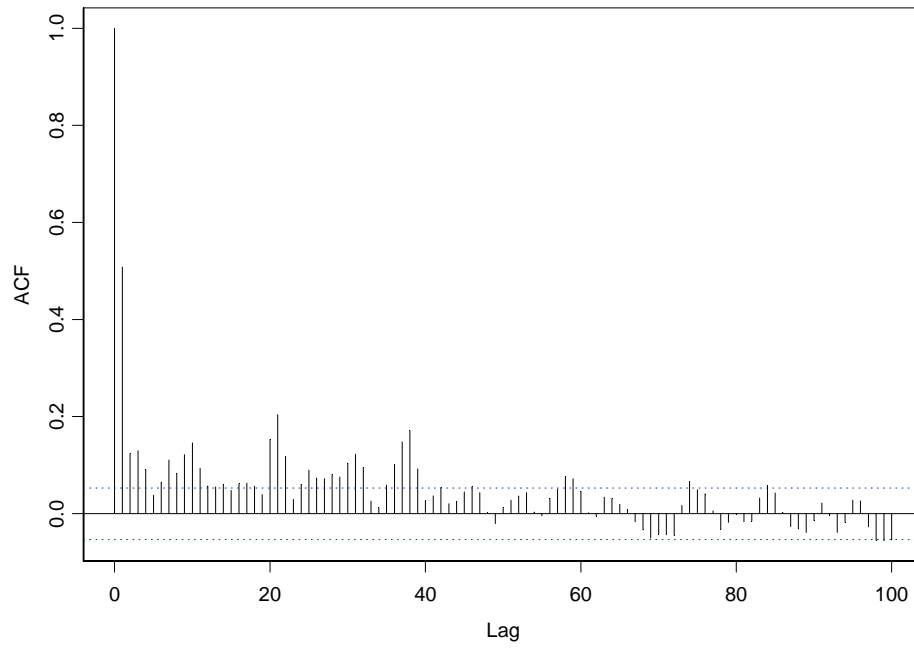


Figure 25

Treatment 4: autocorrelation function of absolute data

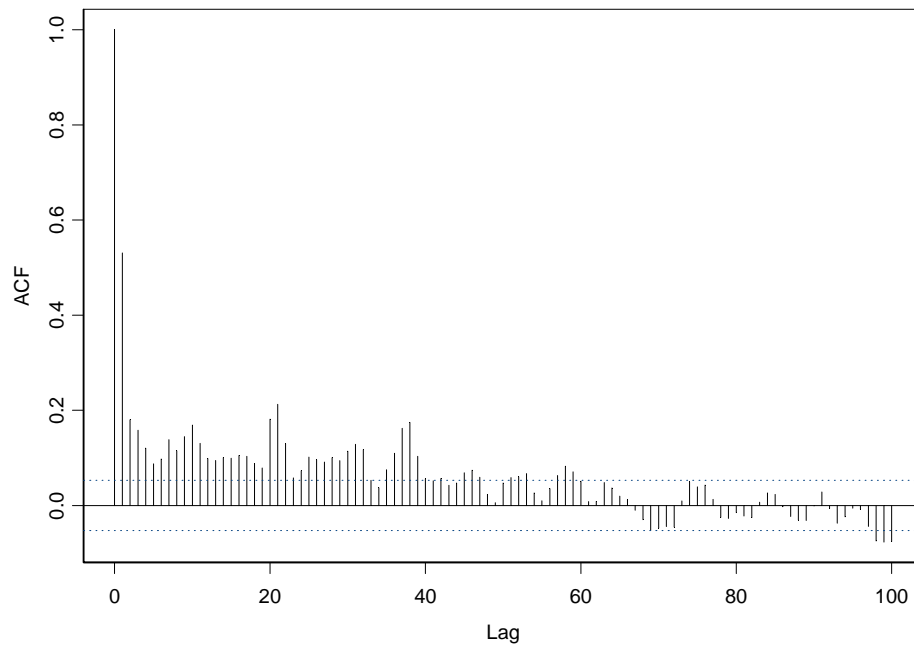


Figure 26

Also Treatment 3 and 4 (figures 20-22 and figures 23-25 respectively) have this feature but it is weaker compared to Treatment 2.

To investigate better the autocorrelation structure, we applied the Box-Ljung test (Table 6) to the autocorrelations up to lags 8, 12, 16 for the raw data as well as the squared and the absolute returns.

	Treatment 1			Treatment 2			Treatment 3			Treatment 4			DAX		
	8 lags	12 lags	16 lags	8 lags	12 lags	16 lags	8 lags	12 lags	16 lags	8 lags	12 lags	16 lags	8 lags	12 lags	16 lags
Raw returns	353.56	287.26	295.73	386.51	504.60	1073.42	216.72	305.89	658.69	418.19	444.48	554.08	102.46	1641.57	3590.50
Significance	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Squared returns	354.32	292.99	299.50	395.44	623.38	1431.08	218.70	344.46	803.71	420.43	510.93	659.40	117.41	1926.47	4682.58
Significance	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Absolute returns	356.73	322.26	315.29	401.30	769.32	1785.33	218.70	344.46	803.71	440.36	528.58	714.63	121.41	2204.23	5592.22
Significance	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 6: Box-Ljung test

It is clear that we have to reject the null hypothesis that the absolute values of the autocorrelation coefficients are not significantly different from zero at both the 0.05 and 0.01 significance level (for raw, and for squared and absolute returns as well) .

The volatility autocorrelation in real financial time series decays hyperbolically. In order to check if the experimental data exhibit long term memory, we used the nonparametric estimator of Geweke and Porter-Hudak (1983) in which the null hypothesis of no long memory persistence is tested against the alternative of long memory process. GPH test provides an estimate of the fractional integration parameter d . The estimated d 's are also list in Table 7.

	Treatment 1			Treatment 2			Treatment 3			Treatment 4			DAX		
	Raw	Absolute	Squared	Raw	Absolute	Squared	Raw	Absolute	Squared	Raw	Absolute	Squared	Raw	Absolute	Squared
Est. d	-0.4046	0.2788	0.2904	-0.1961	0.5376	0.5706	-0.0060	0.3632	0.1861	0.0298	0.4571	0.2819	0.0510	0.3434	0.1912
sd	0.1253	0.1253	0.1253	0.1194	0.1194	0.1194	0.1462	0.1462	0.1462	0.1233	0.1233	0.1233	0.0698	0.0698	0.0698
t-stat	-3.2285	2.2249	2.3176	-1.6419	4.5022	4.7789	-0.0410	2.4837	1.2726	0.2422	3.7091	2.2870	0.7305	4.9167	2.7374
p-value	0.0012	0.0261	0.0205	0.1006	0.0000	0.0000	0.9673	0.0130	0.2032	0.8087	0.0002	0.0222	0.4651	0.0000	0.0062

Table 7: GPH test

XII. Comparing the four Treatments

We now compare more systematically the four Treatments, in order to summarize the effect of quality and quantity on information in our experimental financial markets. In table 8 is reported a summary of all the statistical analysis carried out in this paper. Treatment 1 is characterised by a lot of noise and volatility and a great deal of market activity. Treatment 2 is also characterised by a lot of noise and volatility and a great deal of activity too. In Treatment 3, there is generally less noise and much less volatility compares to the first two Treatments. In Treatment 4, compared to Treatment 3, there was much more heterogeneity in the time series, though there was a similar form of nervousness at the beginning of the Treatment. Probably, the most natural way to investigate the effect of quality of information is to compare Treatment 1 with Treatment 3 and to compare Treatment 2 with Treatment 4. Concerning nonstationarity the price series of Treatment 1 fail to exhibit nonstationarity while the price series of Treatment 3 exhibits nonstationarity. We got similar findings comparing Treatments 2 and 4: Treatment 2 seems stationary while Treatment 4 seems nonstationary.

Treatment	Signal		Volume	Signal	Nonstationarity	Hill	Volatility clustering	Autocorrelation		
	Quality	Cost						Raw	Squared	Absolute
T1	Low	Low	1303	278	No	Too thin	Yes	Uncorrelated	Uncorrelated	Uncorrelated
T2	Low	High	1545	806	No	Ok	Yes	Uncorrelated	Correlated	Correlated
T3	High	Low	813	266	Yes	Ok	No	Uncorrelated	Correlated	Correlated
T4	High	High	1373	423	Yes	Too fat	No	Uncorrelated	Correlated	Correlated

Table 8: Summary of statistical analysis

Treatment 1 (see table 5) exhibits realistic tails at 10% level but it is too thin at both 5% and 2.5%. Treatment 2 and Treatment 3 have a very realistic tail index at all levels. Finally, in Treatment 4 the Hill index is too fat at 5% and 10% and too thin at 2.5%.

It seems reasonable to state that information play an important role. Markets with better or more information result in a (non realistic) random walk like behaviour (i.e. less fat tails and less clustering).

XIII. Conclusion

In this paper, we investigate the characteristics of an experimental financial market, and we compare it to a real one. The paper is focused on the fat tail property of returns and on cluster volatility. Our experimental markets exhibit important and well known features: excess kurtosis (see table 3), and in accordance with this the Hill coefficients lie (almost for all the Treatments) in the appropriate interval. Concerning volatility clustering Treatment 2, 3 and 4 exhibit uncorrelated squared and absolute return.

Our results suggest that the volatility of prices is lower (with the implication that herding is less likely) when the quality and quantity of information in the market is higher. The quality of information is a function of the noisiness of the signal (which is exogenous in our experiment), whereas the quantity of information (the number of signals purchased) is endogenous and seems to be a rather complicated function of the cost and noisiness of signals, and of the behaviour of the other participants in the experiment (which depends, *inter alia*, on their attitude to risk). What we are saying suggests that the sharing of information in this manner in a market context might lead to fewer herds – and less activity.

First, we obtain results concerning the relationship between information quality and market efficiency:

- If the quality of the information is low, market seems to fail to aggregate information and the price fluctuate around the un-informed price.
- If the quality of the information is high, the invisible hand seems to work ‘properly’.

Second, we obtain results relating the cost of information to the leptokurtosis of the returns distribution:

- The more expensive the information, the more leptokurtic the returns distribution are.

We, finally, have evidence that dissemination and aggregation of information through the trading mechanism is possible, but it is no longer defensible to argue that rational expectations can be achieved instantaneously, or precisely.

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