

# How Much to Pay in Cash? Employee Retention via Stock Options\*

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*(First Version April 2004)*

August 2008

## Abstract

We model deferred compensation as a share of an uncertain future profit granted by a financially constrained employer to her employee in mutual agreement. Deferred compensation serves as a retention mechanism, helping the employer to avoid bankruptcy. The optimal combination of cash and deferred payments that a firm can use to retain qualified personnel depends on the cost of new credit and bankruptcy risk: If interest rates are greater (smaller) than the ex-ante odds of bankruptcy, the employer will to defer compensation (pay in cash) to the employee. The employee always improves his position in the labor market if imminent bankruptcy is avoided.

*Keywords:* Deferred Compensation, Employee Retention, Nash Bargaining

*JEL-Classification:* J32, J33, M12, M5

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\*We gratefully acknowledge the constructive comments and suggestions given to us by Sabine Böckem, Werner Güth, Burkhard Hehenkamp, Wolfgang Leininger, participants of the ESA European meeting 2003, IZA Summer School 2004, IXth SMYE 2004, EALE 2007, IAB seminar 2008 and our colleagues in Dortmund, Jena and Trier. The opinions contained or implied in this paper are not necessarily endorsed by any of the institutions to which the authors have been affiliated. The sole responsibility for any errors is ours.

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Urged by cash constraints, firms often seek to renegotiate labor contracts and design compensation schemes that may allow both to reduce current payment obligations and to retain qualified personnel.<sup>1</sup> One common solution is to defer part of previously agreed-upon compensation payments. This may help not only to avert bankruptcy in the short run, but also to induce employees to stay in their jobs until the firm recovers liquidity. Literature on deferred compensation (see, e.g., Lazear, 1990 and 1998; Prendergast, 1993) has stressed this retention role in the context of incentive contracts.<sup>2</sup> Against this backdrop, a special emphasis is made on stock options (e.g., Core and Guay, 2001; Hall and Murphy, 2003; Oyer, 2004; Oyer and Schaefer, 2005).<sup>3</sup>

Whereas the studies mentioned above are mostly restricted to empirical analysis of the issue, the current paper presents a comprehensive theoretical model of bargaining on employee retention via deferred compensation. Furthermore, we analyze the issue of retention in the context of liquidity constraints (i.e., cash constraints), and there are at least three reasons for doing that. First, it appears to be the most conventional context, since employee retention in this case is vitally important for the firm's survival. Second, as it is argued in the literature (see, e.g., Curme and Kahn, 1990; Askildsen and Ireland, 2003a and 2003b; Friebel and Matros, 2005), the risk of bankruptcy can dramatically affect agreement on deferred compensation that might never materialize. The liquidity constraint context, therefore, naturally incorporates the concept of risk into the analysis. Finally, the extensive literature on bilateral contracting under liquidity constraints (see e.g., Sappington, 1983; Dewatripont and Tirole, 1994; Hart, 1995 and 2001; Che and Gale, 1998; Lewis and Sappington, 2000 and 2001; Inderst and Mueller, 2004; Tirole, 2005) provides a well understood framework for our analysis.

Deferring and/or delaying payment of wages by a dominant firm that

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<sup>1</sup>As an illustration, The Economist 2003 noted that “the stock market seems to be betting on imminent mass bankruptcies” of “most of the American airlines” and that the only remedy was “to renegotiate their labor contracts before they go bust” (March 22nd-28th, p.57-58).

<sup>2</sup>Other common effects mentioned in this literature are related to *motivation* and *selection*.

<sup>3</sup>E.g., according to Oyer (2004), stock options may be under certain circumstances the most efficient form of deferred compensation.

faces inelastic supply of unskilled labor could be analyzed in the framework of ultimatum bargaining within a monopsonic labor market. This resembles, for instance, the situation frequently observed in mining or heavy industries of *hinterland* regions of Eastern European countries, where wage arrears are not unusual.<sup>4</sup> Our intention here, however, is to model the individual behavior of high-skilled, non-substitutable employees who are likely to receive several attractive job offers on the labor market. Therefore, we use a cooperative Nash bargaining setting to determine what share of the firm's uncertain (future) profits goes to the employee. We define this share as *deferred compensation* and show that it can provide at least the same retention incentives as cash payments. Our theoretical approach, therefore, offers a stylized bargaining model over a compensation package that includes both immediate cash and deferred compensation.

The paper is organized as follows: In Section 1, we present the theoretical model; in Section 2, we state the bargaining problem and present the solution; in Section 3, we examine whether there is an optimal combination of cash and deferred compensation that ensures employee retention and firm survival. Section 4 concludes with a short summary of our results.

## 1 The model

Consider a two-stage game in which the employment relationship between a firm and its employee is at risk of breaking up due to the firm's initial lack of finance.<sup>5</sup> In particular, suppose that the firm (she,  $F$ ) faces a liquidity constraint (e.g., because the payments from customers or the revenues from some investment project did not arrive on time)<sup>6</sup> and is not able to pay the salary she already owes to her only employee (he,  $E$ ). In the first stage of the game, the firm has two possibilities: either to shut down immediately,

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<sup>4</sup>See, e.g., Earle and Sabirianova (2002), Friebel and Guriev (2005).

<sup>5</sup>Our model follows the conventional assumptions (e.g., about the timing of the game, crediting, borrowing, pay-outs) of Aghion and Bolton (1992), Dewatripont and Tirole (1994, A), Hart (1995, chapter 5), Inderst and Mueller (2004), Tirole (2005, chapter 3).

<sup>6</sup>The timing (or the sequence) of payments is usually considered as a main precondition for liquidity constraints, see Tirole (2005, p.199).

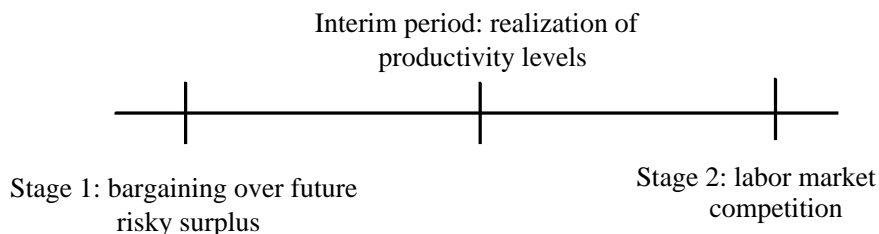


Figure 1: Time sequence of the game

or to engage in further debt to cover at least part of her delinquent payroll. The first possibility terminates the relationship by declaring bankruptcy, and yields a payoff normalized to zero in the two stages of the game,  $U_1^F = U_2^F = 0$ . The alternative is to continue the employment relationship until the second stage of the game, when a sufficiently high revenue is expected. In this case,  $F$  could engage in additional debt to obtain some cash from a bank, the government, etc., and re-negotiate the labor contract in order to dissuade  $E$  from leaving the firm before future revenue (or the lack of it) is observed.<sup>7</sup>

The sequence of the decision process is shown in Figure 1. It includes the two decision stages mentioned above, as well as an interim stage in which some random event occurs. The analysis of stage 1 begins at the point of time when  $E$  is already an employee of  $F$ . Here we assume that  $E$  has already delivered work effort, but due to unforeseen circumstances (e.g., payment delays by customers), the firm is short of cash to pay the employee's salary, regardless of  $E$ 's past productivity. As a result, the only decisions to be made in stage 1 concerns the re-negotiation of the original salary,  $w_1$ . In particular, bargaining in stage 1 is about a new compensation scheme that specifies the fraction (percentage) of future profits that the  $F$  can offer  $E$  in lieu of immediate cash payments. In the interim stage, some random

<sup>7</sup>The financing patterns where pay-outs to creditors are made from future revenue (monetary flow) are known as “retentions”, and defined as “unsecured” (see, e.g, Tirole 2005, p.80, p.95).

state of the world (i.e., the value that  $E$  can potentially create in alternative employments) is realized.<sup>8</sup> Stage 2 is a labor market stage, in which  $E$  can choose among several job offers. We proceed now to describe each stage in detail.

## 1.1 Stage 1: Renegotiating the labor contract

The game begins with both parties bargaining about a compensation scheme  $b(\alpha)$  such that, if  $F$  pays only a fraction  $\alpha$  of the salary  $w_1$  to the employee in the first period, she is obliged to give him a fraction  $1 - b(\alpha)$  of the second period profits, which are denoted by  $S^*$  and defined as the difference between the firm's revenues and the employee's salary (see Section 3.3 for more details).<sup>9</sup> This deferred compensation is lost if the employee leaves the firm. In what follows, we show that the equity-compensation scheme  $b(\cdot)$  that results from the salary re-negotiation is an increasing function of  $\alpha$ , meaning that it is possible to substitute deferred compensation for immediate cash payments in order to retain the employee. Moreover, if the outcome of the re-negotiation includes some cash payment in stage 1 (i.e., if  $\alpha > 0$ ), it is understood that it must be financed by a credit that the firm obtains at interest rate  $r \in \mathbb{R}$ . Therefore, paying a fraction  $\alpha \in [0, 1]$  of  $w_1$  to the employee in stage 1 results in an additional liability of the firm equal to  $(\alpha w_1)r$ .

If the re-negotiation of  $w_1$  fails, it is assumed that  $E$  leaves the firm without being paid and  $F$  goes bankrupt, both obtaining a zero utility level in the first period:  $U_1^E = U_1^F = 0$ . In contrast, if an agreement is reached in stage 1, the employee obtains

$$U_1^E = \alpha w_1,$$

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<sup>8</sup>See, e.g., assumptions about realization of a random state of nature in Sappington (1983), or Aghion and Bolton (1992).

<sup>9</sup> $b(\alpha)$  and  $1 - b(\alpha)$  represent the second-stage profit shares of the firm and the employee, respectively.

while the firm ends up with a total liability of

$$U_1^F = -\alpha w_1(1+r),$$

to be paid in the second stage of the game.

Although the employee is non-substitutable for the firm,  $F$  is not the only employment opportunity for  $E$ : As a highly qualified employee,  $E$  could easily find an alternative job on the labor market, where there are other  $n$  *ex ante* identical firms (indexed by the set  $\mathcal{L} = \{1, \dots, n\}$ ) ready to make competitive salary offers  $w_2^i$ ,  $i \in \mathcal{L}$ , in the second stage, depending on the different productivity levels  $(p_1, \dots, p_n)$  that the employee could attain by working for each of the  $n$  firms. Although these future productivity levels are uncertain during stage 1, it is common knowledge that  $\forall i \in \mathcal{L}$ ,  $p_i \sim \text{Uniform}[0, P]$  and that the productivity that  $E$  will be able to attain in the second stage at  $F$ , if their partnership is preserved, is  $p_F \sim \text{Uniform}[0, P_F]$ , with  $P_F > P > 0$ . Assuming  $P_F > P$  means that  $E$  has already acquired some firm-specific abilities in the first stage, which are not transferable to other firms. If the partnership is preserved (at least until the second stage labor - market competition),  $F$  is more likely to be a more productive employment opportunity for  $E$  than any of the other  $n$  firms. Thus,  $F$  has a better chance to compete for the employee in the second stage. More specifically, the *ex ante* probability that  $F$  will be able to profitably overbid the salary offers made by the other  $n$  firms in the second stage increases with  $P_F - P$ . It therefore may be advantageous for  $F$  to retain  $E$  even at the cost of additional debts.<sup>10</sup>

## 1.2 Interim stage: Realization of productivity levels

Before stage 2 begins, there is an interim stage at which the employee's productivity in each firm (i.e., the realized values of the  $p_i$ 's) are observed by them. Firm-specific levels of productivity are observed before production

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<sup>10</sup>Whether the employee stays with the firm or not is an endogenous result of the re-negotiations.

takes place when, for instance, companies receive orders in advance.<sup>11</sup> We write  $p_{(1)} = \max_{i \in \mathcal{L}} p_i$  for the highest productivity level among the  $n$  competing firms, and denote by  $i^* = \arg \max_{i \in \mathcal{L}} p_i$  the most productive one. Similarly, we let  $p_{(2)} = \max_{i \in \mathcal{L} \setminus i^*} p_i$  be the second highest productivity level among all competitors.

### 1.3 Stage 2: Labor market competition

Stage 2 of the game begins after the potential productivity of the employee for each of the existing  $(n + 1)$  firms becomes commonly known, allowing them to make him salary offers under Bertrand competition.<sup>12</sup> Thus, each firm except the most productive one offers a salary equal to its own productivity level. Only the most productive of the  $n + 1$  firms offers a salary (namely the employee's opportunity-cost salary,  $w_2$ ), which is equal to the second-order statistic of the sample of *all* productivities (including the productivity of firm  $F$ ) and below its own productivity:

$$w_2 = \begin{cases} p_{(1)} & , \text{ if } p_F \geq p_{(1)} \geq p_{(2)} \\ p_F & , \text{ if } p_{(1)} > p_F \geq p_{(2)} \\ p_{(2)} & , \text{ if } p_{(1)} > p_{(2)} > p_F. \end{cases} \quad (1)$$

Here it is important to stress that  $E$ 's share  $1 - b(\alpha)$  of profits is a fraction of his productivity in firm  $F$ ,  $p_F$ , minus his opportunity-cost salary,  $w_2$ , conditioned on  $E$  being hired by  $F$ . Therefore,  $E$  will always prefer firm  $F$  to be able to bid in the labor market, since this can only increase his expected competitive salary,  $Ew_2$ . In other words,  $E$  has an interest in helping  $F$  avoid bankruptcy in the first stage, regardless of who finally hires him in stage 2.

Assuming that the employee always accepts the highest salary offer in the second period, two types of employment are open to him:

1. If his initial employer  $F$  survives bankruptcy and becomes the most

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<sup>11</sup>This is usual practice in the production of software and consulting services.

<sup>12</sup>Note that the assumption of common knowledge regarding the realization of  $(p_1, \dots, p_n)$  is made for convenience only. Our results would remain qualitatively unchanged if the wage offers were the result of a first-price auction with private values  $p_i$ .

productive firm in stage 2 (i.e., if  $p_F \geq p_{(1)}$ ), she employs him, offering a second period salary equal to  $w_2 = p_{(1)}$  according to (1). Additionally,  $E$  is entitled to a share  $(1 - b(\alpha))$  of the second stage profits  $S$ , as agreed upon during the contract re-negotiation process in stage 1, where

$$S = p_F - p_{(1)}.$$

2. In case that  $F$  does not become the most productive firm (i.e., if  $p_F < p_{(1)}$ ),  $E$  is hired by the firm  $i^*$  with salary  $w_2 = \max \{p_F, p_{(2)}\}$ , and any deferred compensation promises held by the employee become void.

Therefore, at the end of stage 2 the employee receives his second period salary  $w_2$  and, if his initial employer  $F$  turns out to be most productive firm, he also receives a share  $(1 - b(\alpha))S$  of the surplus. Only in this latter case does  $F$  receive a payoff equal to  $b(\alpha)S$ . Put differently,  $F$ 's utility in the second stage is equal to

$$U_2^F = b(\alpha)S^*,$$

where,  $S^* = \max \{0, S\}$ , while the utility of  $E$  in stage 2 is given by

$$U_2^E = w_2 + [1 - b(\alpha)] S^*,$$

with

$$w_2 = \begin{cases} p_{(1)} & , \text{ if } S^* > 0 \\ \max \{p_F, p_{(2)}\} & , \text{ otherwise.} \end{cases}$$

## 2 The bargaining outcome

In this section, we present the cooperative bargaining solution for stage 1 of the game, assuming that both  $E$  and  $F$  have equal bargaining power. Specifically, we apply the Nash bargaining solution to the problem of finding the share  $b(\alpha)$  of future (uncertain) profits that the firm would have to offer to the employee (given a fixed immediate cash payment  $\alpha w_1$ ) in order to retain him.

The utility of the firm and the employee for two periods can now be



written as

$$EU^F = E [U_1^F + U_2^F] = E [-\alpha w_1(1 + r) + b(\alpha)S^*]$$

and

$$EU^E = E [U_1^E + U_2^E] = E [\alpha w_1 + w_2 + (1 - b(\alpha))S^*],$$

respectively.

Note that, since the productivity levels of stage 2 are still unknown in stage 1, an agreement about a combination of cash and deferred compensation must be reached considering *expected* utilities. Thus, the bargaining problem in the next section is solved by taking into account both the probability that  $F$  becomes the most productive firm in the future and the expected size of the bargaining surplus.

## 2.1 Bargaining setting

The bargaining setting in our model is characterized by two important features: First, we assume that utility is not *completely* transferable between the firm and the employee; and second, we allow for the possibility of *bankruptcy*, which means that liabilities acquired by the firm in stage 1 can only be paid back to the creditor in full if the firm makes enough profits in stage 2.

The *nontransferability assumption* captures the idea that payments made to the employee in the first stage,  $\alpha w_1$ , as well as additional expected gain in the second stage salary,  $\Delta E w_2$  (see expression (4)), are both not transferable to the firm.<sup>13</sup> This means that what the firm can obtain in stage 2 is at most equal to the expected value of the joint surplus,  $b(\alpha)ES^* \leq ES^*$ , implying that the total expected utility of the firm is constrained by

$$EU^F \leq ES^* - \alpha w_1(1 + r)\theta, \quad (2)$$

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<sup>13</sup>This is along the same vein as in Aghion and Bolton's (1992) assumption about non-transferability, but in our context it has rather a monetary connotation.

where

$$\theta = \Pr(p_F > p_{(1)}) = 1 - \frac{n}{n+1} \frac{P}{P_F}$$

is the probability of  $F$  being the best employment opportunity for  $E$  (the most productive firm) in stage 2, and

$$ES^* = \frac{(P_F - P)^2}{2P_F} + \frac{(n+3)}{(n+1)(n+2)} \cdot \frac{P^2}{P_F}$$

is the expected value of profits in that stage (see Appendix 1).

The *limited liability assumption*,  $EU^F \geq 0$ , requires the introduction of an exogenous actor (e.g., a bank or a government) that is willing to give credit to the firm during stage 1, knowing that this credit will become unrecoverable if the firm is not able to hire the employee in stage 2 (an event which occurs with probability  $1 - \theta > 0$ ), and that the credit is recoverable only up to the realized value of  $b(\alpha)S^*$ . In other words, while the employee receives  $\alpha w_1$  in stage 1 with certainty, the firm pays back to the creditor  $\alpha w_1(1+r)$  in stage 2 only with probability  $\theta < 1$ , and this payment is subject to the limited liability constraint of the firm.<sup>14</sup>

Assuming for a moment that the limited liability constraint is not binding ( $EU^F > 0$ ), and denoting the difference between the amount received from the creditor and the expected payback as

$$\beta(\alpha) \equiv \alpha w_1(1 - (1+r)\theta),$$

it is possible to distinguish three cases, depending on the value of the interest rate  $r$ :<sup>15</sup>

1. If  $\alpha w_1 > \alpha w_1(1+r)\theta \Rightarrow r < \left(\frac{1-\theta}{\theta}\right)$ , the interest rate is such that stage 2's expected refund is lower than what the employee received in stage 1, i.e.,  $\beta(\alpha)$  can be interpreted as an increase in the agreement surplus since the creditor is providing funds in excess to what the firm will pay

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<sup>14</sup>As Innes (1990, p.45) notes, “[a] firm’s liability to its security-holders is limited to firm assets and profits”.

<sup>15</sup>We follow here the standard approach and assume a perfectly competitive credit market with risk-neutral creditors, see e.g., Tirole (2005, p.115), Hart (1995).

back in expected value.

2. If  $\alpha w_1 < \alpha w_1(1+r)\theta \Rightarrow r > \left(\frac{1-\theta}{\theta}\right)$ , the interest rate is such that stage 2's expected refund is higher than what the employee received in stage 1, i.e., the resulting negative value of  $\beta(\alpha)$  decreases the agreement surplus.

3. If  $\alpha w_1 = \alpha w_1(1+r)\theta$ , the interest rate  $r^*$  that exactly matches  $F$ 's odds of bankruptcy

$$r^* = \left(\frac{1-\theta}{\theta}\right), \quad (3)$$

can be easily shown to be the unique competitive interest rate at which creditors make neither losses nor profits in expected value since  $(1+r^*)\theta = 1$ , or  $\beta(\alpha) = 0$ .<sup>16</sup>

As explained in Section 1.3, another particular feature of our model is the fact that the competitive salary expected by the employer in stage 2,  $Ew_2$ , depends on the success of the agreement in stage 1. Defining  $k = 1$  if bargaining succeeds, and  $k = 0$  otherwise, it is possible to show that (see Appendix 1):

$$E(w_2|k) = \begin{cases} \left(\frac{n}{n+1} - \frac{n}{(n+1)(n+2)} \cdot \frac{P}{P_F}\right) P, & \text{if } k = 1, \\ \left(\frac{n-1}{n+1}\right) P, & \text{otherwise.} \end{cases}$$

Hence,

$$\Delta Ew_2 \equiv E(w_2|1) - E(w_2|0) > 0 \quad (4)$$

is the additional gain in the joint surplus corresponding to the employee's direct interest in helping the firm to survive.

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<sup>16</sup>Note that, according to expression (3), a firm with higher probability of success  $\theta$  should be able to obtain financial support at a lower interest rate.

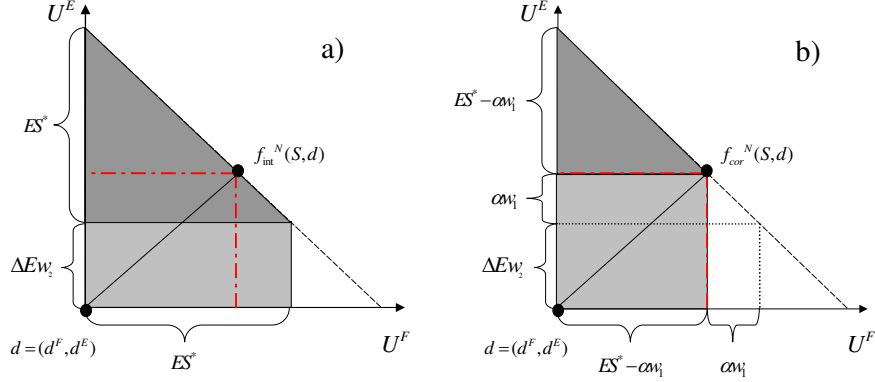


Figure 2: Nash bargaining solution for  $\beta(\alpha) = 0$  ( $r = r^*$ ). The shaded area is the bargaining set: a) Cash payments are equal to zero; b) Cash payments are equal to  $\alpha w_1$ .

## 2.2 Bargaining solution

We can now state the bargaining problem faced by  $E$  and  $F$  in the canonical form  $(B_\alpha, d)$ , where  $B_\alpha$  is the set of feasible agreements (bargaining set) and  $d = (d^E, d^F) = (E(w_2|k=0), 0)$  is the conflict payoff. Taking into account the *nontransferability* and *limited liability* assumptions, and defining  $\tilde{U}^j \equiv E(U^j - d^j)$ ,  $j = E, F$ , we have

$$B_\alpha = \left\{ (\tilde{U}^E, \tilde{U}^F) : \tilde{U}^E + \tilde{U}^F \leq ES^* + \Delta E w_2 + \beta(\alpha), \tilde{U}^F \leq ES^* - \alpha w_1(1+r)\theta \right\}.$$

Then the Nash bargaining solution with symmetric bargaining power

$$f^N(B_\alpha, d) = \arg \max_{(\tilde{U}^E, \tilde{U}^F) \in B_\alpha} \tilde{U}^E \cdot \tilde{U}^F$$

results in the following two cases:

1. Internal solution:

$$\tilde{U}_{int}^E = \tilde{U}_{int}^F = \frac{ES^* + \Delta E w_2 + \beta(\alpha)}{2}, \quad (5)$$

implying

$$b(\alpha)_{int} = \frac{1}{2} + \frac{\Delta Ew_2 + \alpha w_1(1 + (1+r)\theta)}{2ES^*}$$

2. Corner solution:

$$\tilde{U}_{cor}^E = \Delta Ew_2 + \alpha w_1, \quad \text{and} \quad \tilde{U}_{cor}^F = ES^* - \alpha w_1(1+r)\theta \quad (6)$$

which implies  $b(\alpha)_{cor} = 1$ .

See graphical representation of the bargaining solutions for  $\beta(\alpha) = 0$  on Figure 2.

Whether the bargaining problem results in a corner solution or in an internal one, depends on the size of the first stage cash payments,  $\alpha w_1$ , or more precisely - on the size of  $\alpha$ . Substituting (5) in (6), it is straightforward to obtain the threshold value of  $\alpha$  where  $\tilde{U}_{int}$  turns into  $\tilde{U}_{cor}$

$$\alpha^* \equiv \frac{ES^* - \Delta Ew_2}{w_1(1 + (1+r)\theta)}.$$

To put it differently

$$\tilde{U}^{E,F} = \begin{cases} \tilde{U}_{int}^{E,F} & , \text{ if } \alpha \leq \alpha^* \\ \tilde{U}_{cor}^{E,F} & , \text{ otherwise.} \end{cases}$$

The solution, therefore, can be summarized by the compensation schedule<sup>17</sup>

$$b^*(\alpha) = \begin{cases} \frac{1}{2} + \frac{\Delta Ew_2 + \alpha w_1(1 + (1+r)\theta)}{2ES^*} & , \text{ if } \alpha \in [0, \alpha^*] \\ 1 & , \text{ otherwise.} \end{cases} \quad (7)$$

In order to illustrate our bargaining solution numerically, we construct the example depicted on Figure 3.<sup>18</sup> The example illustrates that only for small

<sup>17</sup>Note, it is easy to show that  $\frac{1}{2} + \frac{\Delta Ew_2 + \alpha w_1(1 + (1+r)\theta)}{2ES^*} \leq 1, \forall \alpha \in [0, \alpha^*]$ .

<sup>18</sup>The example is calculated according to the following parameters:  $n = 1, p_F \sim \text{Uniform}[0, 200]$  and  $p_1 \sim \text{Uniform}[0, 100]$ ; we calculated the probability that  $F$  will become the most productive firm in the second stage with  $\theta = \frac{3}{4}$ ; the expected value for the joint surplus equals to  $ES^* = 58\frac{1}{3}$ ; the employee's second period salary in case  $F$  stays in the market at the second period is  $Ew_2 = 41\frac{2}{3}$ , and  $Ep_{(2)} = 0$  otherwise. For this example we

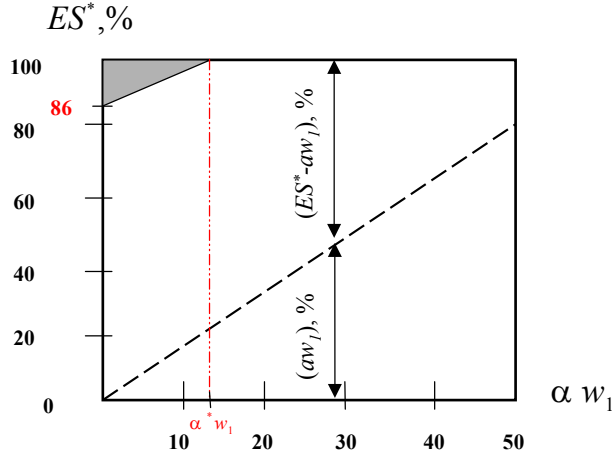


Figure 3: Predicted bargaining outcome according to the example parameters. The shaded area represents the employee's share.

values of cash payments ( $\alpha w_1$ ) can the employee expect to receive a part of the profit. Moreover, the share of the profit he receives is fairly small. The biggest share in our example amounts to approximately 14% of the profit, and it decreases (up to zero) as the cash payments increase. The following section discusses the bargaining outcome and its implication for retention in more detail.

### 3 Optimal combination of cash and deferred payments

We have shown that bargain can result in either an *internal solution* or a *corner solution*. Following expression (7), the internal solution means that the part of the profit that the employee can obtain,  $(1 - b^*(\alpha))ES^*$ , is a function of  $\alpha \in [0, \alpha^*]$ . On the other hand, the corner solution indicates that if the cash payments exceed  $\alpha^*$ , the whole stage-two profit is taken by

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also have chosen six consequent values of cash payments,  $\alpha w_1 = \{0, 10, 20, 30, 40, 50\}$  (see Appendix 1).

the firm. Therefore, the employee receives a share of the profit only if its expected value exceeds the cash payment

$$(1 - b^*(\alpha))ES^* \geq \alpha w_1,$$

which is true  $\forall \alpha \in [0, \alpha^*]$ .

As a result, all values  $\alpha \in [0, \alpha^*]$  would yield to the employee and the employer the same expected utility (under the assumption of risk neutrality), given the solution schedule  $b^*(\alpha)$ . Alternatively, the employee is indifferent to receive (and the firm to give) any amount of payments up to  $\alpha^*$ , either as a first stage cash ( $\alpha w_1$ ) or as a second stage part of the profit ( $(1 - b^*(\alpha))ES^*$ ). Moreover, these payments also *provide the same incentive* for the employee to stay with the firm at least until the productivity levels of the second stage become common knowledge. In this context a reasonable question arises: Is there an optimal value of  $\alpha$  which can help the firm to retain the employee and to avert bankruptcy? To answer it, we consider the consequences of different values of  $\alpha$ , both from the firm's and the creditor's perspective.

The cost of credit in stage 1 is a key determinant of the firm's expected utility. In particular, from expression (5) it is clear that the utility of the firm is a monotonically increasing (or decreasing) function of  $\alpha$  if the interest rate is  $r < r^*$  (or  $r > r^*$ ).<sup>19</sup> Additionally, from equation (6), it is readily evident that cash payments higher than  $\alpha^* w_1$  always decrease the firm's expected utility.<sup>20</sup> Thus, from the viewpoint of the firm, the preferred value of  $\alpha$  is  $\alpha^F = \alpha^*$  if  $r < r^*$ , and  $\alpha^F = 0$  if  $r > r^*$ . If the interest rate is equal to competitive value  $r^*$ , the firm is indifferent between any value of  $\alpha$  within the range  $[0, \alpha^*]$  (see Figure 4).

Furthermore, the firm's expected creditworthiness (i.e., its expected ability to pay at the end of stage 2) is given by  $b^*(\alpha)ES^*$ . It is straightforward to show that, for all values  $\alpha \in [0, \alpha^*]$  the *limited liability assumption* holds

$$b(\alpha)ES^* \geq \alpha w_1(1 + r)\theta, \quad (8)$$

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<sup>19</sup>This and the following result we obtain as long as the firm's limited liability constraint is not binding ( $EU^F > 0$ ).

<sup>20</sup>We neglect the case  $r \leq -1$ .

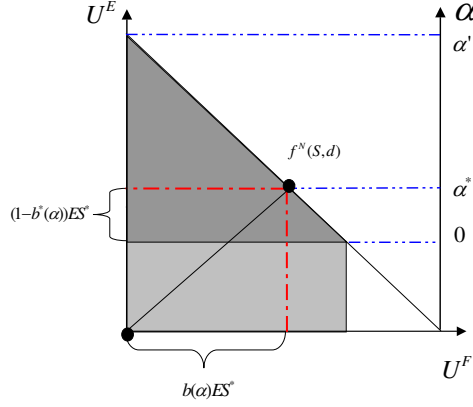


Figure 4: Different  $\alpha$  in relation to the bargaining set.

meaning that all cash payments to the employee that lead to an internal solution in the bargaining stage will (in expectation) allow the firm to avoid bankruptcy. Recalling that  $b^*(\alpha) = 1$  for all  $\alpha \geq \alpha^*$  and using (2), one can similarly show that there is a threshold value

$$\alpha' \equiv \frac{ES^*}{w_1(1+r)\theta} > \alpha^*$$

above which the firm is not expected to make enough profits to pay back the full amount of the credit taken out in stage 1. This is due to the fact that, given a corner solution, a higher value of  $\alpha$  only increases the firm's liabilities but not its expected ability to pay. For this reason, the creditor should not lend an amount higher than  $\alpha'w_1$ , regardless of the value of  $r$ , since a loan of this magnitude is likely to lead the firm into bankruptcy (see Figure 4). Finally, it should be clear by now that, whereas the creditor's return is identically equal to zero at the competitive interest rate  $r^*$ , it is increasing (or decreasing) in  $\alpha \in [0, \alpha']$  in case that  $r > r^*$  (or if  $r < r^*$ ). Therefore, a profit-maximizing creditor should lend money up to  $\alpha'$  as long as the interest rate is higher than (or equal to) the firm's odds of bankruptcy. Otherwise, the creditor should not provide any credit (see Figure 5).



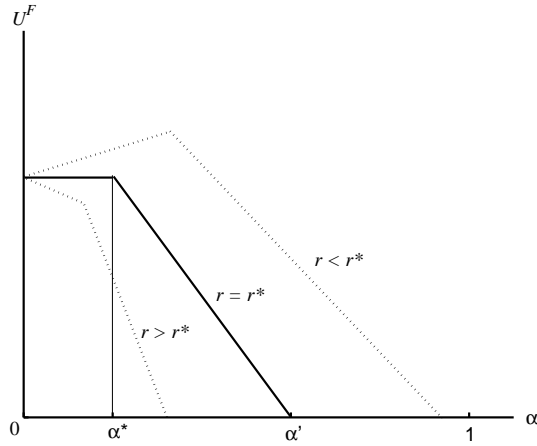


Figure 5: Firm's expected utility as a function of  $\alpha$ .

## 4 Concluding remarks

The results of this work show that it is possible to re-negotiate the initial contract – i.e., by means of deferred compensation – in order to keep the employee and try to avert bankruptcy. Indeed, in some cases we can define an optimal amount of cash that a firm may offer to an employee, together with a corresponding share of deferred compensation (e.g., stock options), in order to prevent him from leaving.

The optimal combination of cash and deferred compensation derived in our model crucially depends on how the firm's odds of failure compare to the interest rate of available credit. If both are equal, the firm provides a “combined” compensation package. If the latter is lower than the former, then it will be profitable for the firm to take on further liabilities in order to make a cash payment to the employee, while offering nothing in terms of deferred compensation. In contrast, when the interest rate is higher than the firm's odds of failure, then the firm should offer a payment consisting only of, e.g., stock options. This payment is greater, the less important it is for the employee that the firm survives (i.e., the smaller the improvement in the employee's expected opportunity-cost salary that results from  $F$  being able to bid for  $E$  in the labor market). In its turn this relates to the number and

quality of the alternative job offers that an employee can expect to receive in the future.

The paper has been inspired by specific cases of start-ups with liquidity constraints. Nevertheless, we believe that using a simple two-stage structure and the axiomatic Nash bargaining solution makes our model flexible enough to provide insights in the issues of a more general interest.

## Appendix 1

The productivity levels of the competing firm are *iid* Uniform(0,  $P$ ) random variables. Thus, the productivity of the most productive firm,  $p_{(1)}$ , has a density function  $f(p_{(1)}) = \frac{np_{(1)}^{n-1}}{P^n}$ . On the other hand, the productivity of the firm  $F$ , denoted by  $p_F$ , has density  $f(p_F) = \frac{1}{P_F}$ , with  $P < P_F$ . Since all productivity levels are independent, the joint density of  $p_F$  and  $p_{(1)}$  is given by

$$f(p_F, p_{(1)}) = \frac{1}{P_F} \frac{np_{(1)}^{n-1}}{P^n}.$$

The probability that firm  $F$  is more productive than any other firm is equal to

$$\begin{aligned} \theta = \Pr(p_F > p_{(1)}) &= \int_0^P \int_y^{P_F} f(x, y) dx dy \\ &= \frac{n}{P_F P^n} \int_0^P y^{n-1} (P_F - y) dy \\ &= \frac{n}{P_F P^n} \left[ \frac{P_F P^n}{n} - \frac{P^{n+1}}{n+1} \right] \\ &= 1 - \frac{P}{P_F} \cdot \frac{n}{n+1}. \end{aligned}$$

Note that  $\lim_{n \rightarrow \infty} \theta = 1 - \frac{P}{P_F}$ .

To calculate the expected value of profits,  $ES^*$ , where  $S^* = \max\{0, S\}$  with  $S = p_F - p_{(1)}$ , we make use of the following

**Lemma 1**  $S$  is a random variable with density function

$$f_S(s) = \begin{cases} \frac{1}{P_F} - \frac{(-s)^n}{P_F P^n} & , \text{ if } -P \leq s \leq 0 \\ \frac{1}{P_F} & , \text{ if } 0 < s \leq P_F - P \\ \frac{(P_F - s)^n}{P_F P^n} & , \text{ if } P_F - P < s \leq P_F. \end{cases}$$

**Proof.** Define the bivariate transformation  $S = U(p_F, p_{(1)}) = p_F - p_{(1)}$

and  $T = V(p_F, p_{(1)}) = p_F + p_{(1)}$  with Jacobian

$$J = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}.$$

Then, the joint density of  $S$  and  $T$  is given by

$$\begin{aligned} f_{S,T}(s, t) &= |J| f_{X,Y}(U^{-1}(s, t), V^{-1}(s, t)) \\ &= \frac{n}{2^n P_F P^n} (t - s)^{n-1}, \end{aligned}$$

with support  $S \in [-P, P_F]$  and

$$T \in \begin{cases} [-S, 2P + S] & , \text{ if } -P \leq S \leq 0 \\ [S, 2P + S] & , \text{ if } 0 < S \leq P_F - P \\ [S, 2P_F - S] & , \text{ if } P_F - P < S \leq P_F \end{cases}$$

Integrating with respect to  $T$  the marginal density  $f_S(s)$  is obtained. ■

Thus, taking expectations,

$$ES^* = \frac{(P_F - P)^2}{2P_F} + \frac{P^2}{P_F(n+1)} \left[ 1 + \frac{1}{(n+2)} \right].$$

We now calculate the expected value of the employee's opportunity-cost wage, given that renegotiation succeeds,  $E(w_2|k=1)$ . Since its value is equal to the second-order statistic of the sample of *all* productivities (including the productivity of firm  $F$ ), it is possible to prove the following:

**Lemma 2** *The opportunity-cost wage is distributed as*

$$F_{w_2}(x|k=1) = \left(\frac{x}{P}\right)^{n-1} \left[ \frac{x}{P} + n \left(1 - \frac{x}{P}\right) \left(\frac{x}{P_F}\right) \right].$$

**Proof.** We offer only a sketch of the proof, while referring to Casella and Berger (1990, Theorem 5.5.2) for its underlying logic. For any real value  $x$ ,

define the random variable  $Y_n$  as the number of firms other than  $F$ , whose productivity turns out to be less than  $x$ . Recall that these productivities are *iid* Uniform $[0, P]$ , so that  $Y_n \sim \text{Binomial}(n, \frac{x}{P})$ . Also, define  $Y_F$  as a Bernoulli variable with  $\Pr(Y_F = 1) = \frac{x}{P}$ . The employee's opportunity-cost wage is the second-order statistic of the whole sample of productivities (which includes  $n + 1$  numbers). Thus, its distribution is given by  $F_{w_2}(x|k = 1) = \Pr(W = n) + \Pr(W = n + 1)$ , where  $W = Y_c + Y_F$ . ■

Using Lemma 2, the expected value of the employee's opportunity-cost wage is equal to

$$E(w_2|k = 1) = \left[ \frac{n}{n+1} - \frac{n}{(n+1)(n+2)} \cdot \frac{P}{P} \right] P.$$

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