Are preferences incomplete? An experimental study using flexible choices

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Abstract

Completeness, the most commonly assumed axiom in preference theory, has not received much attention from the experimental literature. Indeed, incomplete preferences model a *cognitive* phenomenon (an agent's inability to compare alternatives), and therefore cannot be directly revealed through choice behavior. Implementing a solution to this methodological issue recently proposed by Danan (2003), we build an experimental protocol involving choices among menus of lotteries, and reveal cognitive preferences' incompleteness by means of the concept of *preference for flexibility*. Our experimental protocol is designed to assess the descriptive validity of the completeness axiom, as well as to relate its possible violations to lotteries' riskiness. Two-thirds of the subjects whose choices reveal preferences in accordance with the underlying theory exhibit a strictly positive measure of incompleteness. The observed average measure of incompleteness equals approximately 17 percent and it is significantly greater than 10 percent. We do not find a significant relationship between a lottery's *riskiness* and its cognitive comparability with certain payoffs.

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1 Introduction

An individual's preferences are said to be *complete* if given any two alternatives x and x', either x is weakly preferred to x' or x' is weakly preferred to x. Paradoxically, completeness is by far the most commonly assumed axiom in preferences theory (it is implied by the existence of a utility function), while there is an old tradition of considering it as intuitively questionable (e.g. von Neumann and Morgenstern 1944). Furthermore, this axiom has appeared to be technically disposable in decision theory,[1] as well as in general equilibrium theory.[2]

A descriptive test of the completeness axiom has long been precluded by the methodological critique often addressed to the theory of incomplete preferences. Motivated by the methodology of *revealed preferences*, this critique simply consists in noting that an agent's preferences are generally observed through her choice behavior: the agent is said to prefer x to x' if and only if she *chooses* x over x'. Clearly, this *behavioral* interpretation of preferences rules out incompleteness as the agent is always forced to choose between x and x'. The interpretation of preferences under which incompleteness is intuitively appealing is a *cognitive* one: x is preferred to x' if and only if the agent *desires* x more than x'. This cognitive interpretation plays a fundamental role in economics, as it is naturally linked to welfare analysis (while the behavioral interpretation is naturally linked to equilibrium determination).[3] However, it makes preferences unobservable according to the revealed preferences methodology, and hence any theory of incomplete preferences is descriptively intestable, so it is said.

Recently, Danan (2003) has proposed a solution to this methodological issue. Starting from the aforementioned distinction between an agent's *behavioral preferences* and her *cognitive preferences*, he has introduced the *BC-preferences* model, which *derives* the latter from the former, thereby enabling to observe cognitive preferences without resorting to non-behavioral data. To understand why this derivation is not trivial, note that there are two ways in which an agent's tastes might not agree with her choice behavior:

— *incomplete tastes*. The agent chooses an alternative without knowing which alternative she desires the most,

— *unobservable indifference*. The agent chooses an alternative while desiring

[1] E.g. Aumann (1962), Bewley (1986), Dubra, Maccheroni, and Ok (2001), and Ok (2002).

[2] E.g. Mas-Collel (1974), Fon and Otani (1979), and Rigotti and Shannon (2001).

[3] The distinction between these two concepts of preferences was emphasized by Sen (1973).

both alternatives equally.

The key idea of the BC-preferences model consists in extending the set of alternatives to the power set of *menus* over these alternatives, and assuming that the agent is unable to compare x and x' if and only if she prefers the menu $\{x, x'\}$ to both x and x', i.e. if she has a *preference for flexibility* in the sense of Koopmans (1964).[4] This assumption, named *learning-then-acting*, is extensively discussed in Danan (2003); its intuitive justification is that choosing the menu $\{x, x'\}$ enables the agent to postpone her choice between x and x', thereby giving herself a chance to learn about her tastes before she has to choose.

As for unobservable indifference, the solution relies on a classical device consisting in adding *monetary payoffs* to the alternatives (e.g. Savage 1954). Indifference between x and x' is then detected by the fact that each of these two alternatives, if augmented by an arbitrarily small monetary *bonus*, is chosen over the other. This methodology is more rigorous than allowing individuals to directly express indifference by selecting x and x' simultaneously. Indeed, if one is indifferent between x and x', then one has no incentive to select both x and x' instead of, say, x alone (Danan 2002, Danan 2003).

In this paper we report results from an experiment designed to test the descriptive validity of the completeness axiom. Our experimental protocol is based on the BC-preferences model: we implement the "flexible choices" (choices between menus) which enables us to derive subjects' cognitive preferences, and hence to measure their incompleteness.[5] As alternatives, we use *elementary lotteries*, i.e. lotteries with monetary outcomes modelling the toss of a fair coin. The simplicity of these lotteries aims at leaving aside complexity matters as much as possible so that the observed incomparability would arguably come from the agent's inability to determine her cognitive attitude towards risk. We consider six different lotteries with risk indexes ranging from 4 to 40 which enables us to investigate whether and if so how incompleteness is influenced by lotteries' *riskiness*.

Our results clearly show that, among the behavioral preferences which are compatible with the BC-preferences model, cognitive incompleteness is the rule rather than the exception. Indeed, two-thirds of the subjects whose choices reveal

[4]The menu $\{x, x'\}$ is interpreted as the commitment to choose between x and x' at some given later date.

[5]Since we add monetary payoffs to the alternatives, unobservable indifference is, by definition, a "null measure" phenomenon (as is one point in a continuum of monetary amounts), and hence its measurement is not a relevant issue.

preferences in accordance with the BC-preferences model exhibit a strictly positive measure of incompleteness. The observed average measure of incompleteness equals approximately 17 percent and it is significantly greater than 10 percent. We do not find a significant relationship between a lottery's *riskiness* and its cognitive comparability with certain prizes.

As far as we know, related literature is made up by a single, recent and independent work by Eliaz and Ok (2003). Like in the BC-preferences model, they propose a way of deriving cognitive preferences from choice behavior. They then conduct two experiments, and find that allowing for preferences incompleteness enables to explain about 35 % more observed choices. Their approach, however, has two drawbacks compared to the present one. First, from a theoretical viewpoint, they detect incompleteness by intransitivity of choice behavior, a methodology which is normatively questionable since it automatically renders incomplete preferences vulnerable to "money pumps" (see Danan 2002). Second, they detect indifference by allowing subjects to select several alternatives at once (in which case an outside individual selects an alternative within the selected set), and hence their experimental methodology is questionable in regard of unobservable indifference, as explained above. Since our approach, on the contrary, relies on the "monetary *bonus*" device, it enables us to gather purely behavioral data by forcing subjects to select one, and only one alternative.

The paper is organized as follows. Section 2 introduces the theoretical framework and presents a simple version of the BC-preferences model. We describe the design of our experiment in Section 3, and Section 4 reports the experimental results. Section 5 closes the paper with a discussion about the insights and the limitations of our work. Short proofs are to be found in the Appendix.

2 Theoretical framework

Consider an agent and a set $\mathcal{X}$ of *alternatives*. As usual, the agent's *preferences* over $\mathcal{X}$ are modelled by a binary relation $\succsim$ on $\mathcal{X}$, with $x \succsim x'$ being interpreted as " x is *weakly preferred* to x' ". From $\succsim$, we define the binary relations $\succ$, $\sim$, and $\bowtie$ on $\mathcal{X}$ by:

$$x \succ x' \Leftrightarrow x \succsim x' \not\succsim x, \qquad x \sim x' \Leftrightarrow x \succsim x' \succsim x, \qquad x \bowtie x' \Leftrightarrow x \not\succsim x' \not\succsim x.$$

$x \succ x'$ is interpreted as " x is *strictly preferred* to x' ", $x \sim x'$ as " x and x' are *indifferent* ", and $x \bowtie x'$ as " x and x' are *incomparable* ". Say that $\succsim$ is:

— *reflexive* if $\forall x, x' \in \mathcal{X}$, $x = x' \Rightarrow x \sim x'$,

— *antisymmetric* if $\forall x, x' \in \mathcal{X}$, $x \sim x' \Rightarrow x = x'$,

— *complete* if $\nexists x, x' \in \mathcal{X}$ such that $x \bowtie x'$.

Clearly, completeness implies reflexivity.

As explained in the introduction, the agent is endowed with two different kinds of preferences: *behavioral preferences* and *cognitive preferences*. The binary relations modelling them are respectively denoted by $\succsim_B$ and $\succsim_C$, with $x \succsim_B x'$ being interpreted as " the agent *chooses* x over x' " and $x \succsim_C x'$ as " the agent *desires* x at least as much as x' ". By definition, behavioral preferences are complete (since the agent can be forced to choose among any two alternatives) and antisymmetric (since choosing means selecting one and only one alternative). Cognitive preferences need not be complete (since the agent might not know which among two alternatives she desires more) nor antisymmetric (since she might desire two alternatives equally), still they are reflexive by definition (i.e. two equal alternatives are equally desired). These properties are summed up in the following definition.

Definition 1. *A* ***behavioral preference relation*** *is a complete and antisymmetric binary relation. A* ***cognitive preference relation*** *is a reflexive binary relation.*

Cognitive preferences are not defined in terms of choice behavior, and hence are not directly observable, according to the *behavioral methodology* prevailing in economic theory. The *BC-preferences model* (Danan 2003) solves this methodological issue by proposing a way of deriving cognitive preferences from behavioral preferences. We shall now present a simple version of this model.

2.1 A simple version of the BC-preferences model

The model proceeds by imposing sensible conditions on cognitive preferences, and then showing that these conditions characterize a unique way of deriving cognitive preferences from behavioral preferences. In order to be able to state these conditions, we shall assume the existence of a partition Φ of $\mathcal{X}$ such that each $\phi \in \Phi$ is isomorphic to $\mathbb{R}$, and interpret each $\phi \in \Phi$ as a set of alternatives that are equal up to the adjunction of a *monetary payoff*. This assumption is compelling in economic settings, in particular it is satisfied if alternatives are commodity bundles or monetary payoffs, or lotteries/acts over commodity bundles/monetary payoffs. We shall denote by $x+\varepsilon$ the alternative yielded by the adjunction of the monetary payoff ε to the alternative x (thus $x \in \phi \Leftrightarrow x+\varepsilon \in \phi$). We shall then extend $\mathcal{X}$ to the set $\mathcal{P}(\mathcal{X})$ of nonempty, finite subsets of $\mathcal{X}$, and interpret each $X \in \mathcal{P}(\mathcal{X})$ as a *menu* (or opportunity set), i.e. as the commitment to choose an *option* $x \in X$ at some fixed later date. Thus the larger the menu, the weaker the commitment, i.e. the higher the *flexibility*. We shall define the adjunction of a monetary payoff $\varepsilon \in \mathbb{R}$ to a menu $X \in \mathcal{P}(\mathcal{X})$ by $X+\varepsilon = \{x+\varepsilon : x \in X\}$. In this framework, the following uniqueness result follows from Danan's (2003) Theorem 5:

Theorem 2. *Let $\succsim_B$ be a behavioral preference relation on $\mathcal{P}(\mathcal{X})$. Then there exists at most one cognitive preference relation on $\mathcal{P}(\mathcal{X})$ such that $\forall X, X' \in \mathcal{P}(\mathcal{X})$,*

2a. $X \succ_C X' \Rightarrow X \succ_B X'$,
2b. $X \bowtie_C X' \Leftrightarrow [X \cup X' \succ_C X$ *and* $X \cup X' \succ_C X']$,
2c. $X \succsim_C X' \Leftrightarrow [\forall \varepsilon > 0,\ X+\varepsilon \succ_C X']$,
2d. $X \succ_C X' \Rightarrow [\exists \varepsilon > 0$ *such that* $X \succsim_B X'+\varepsilon]$,
2e. $X \succ_C X' \Rightarrow X \cup X' \succ_C X'$.

Proof. See Appendix 5 □

We refer the reader to Danan (2003) for detailed discussions of Conditions 2a to 2d (2b is the learning-then-acting condition). Condition 2e asserts that if the agent desires X strictly more than X', then she desires $X \cup X'$ (which gives her the opportunity to choose an option in X) strictly more than X' (which does not). It is only imposed to simplify subsequent analysis. To grasp the intuition behind Theorem 2, one can check that given Conditions 2a, 2c, and 2d, Condition 2b is equivalent to:

$$X \bowtie_C X' \Leftrightarrow [\exists \varepsilon, \varepsilon' > 0 \text{ such that } [X \cup X' \succsim_B X+\varepsilon \text{ and } X \cup X' \succsim_B X'+\varepsilon']],$$

which enables to behaviorally identify cognitive incomparability. Once this is done, it only remains, by Condition 2a, to behaviorally disentangle between cognitive indifference and cognitive strict preference. To this end, simply note that Conditions 2a and 2c imply that:

$$X \sim_C X' \Rightarrow [\forall \varepsilon > 0,\ X'+\varepsilon \succ_B X],$$

and hence by Condition 2d, $X \sim_C X'$ and $X \succ_C X'$ have incompatible behavioral implications.

The next step of the BC-preferences model consists in characterizing the behavioral preference relations for which there *exists* a cognitive preference relation satisfying the properties of Theorem 2, and providing an explicit behavioral definition of this cognitive preference relation. While Danan's (2003) Theorem 6 provides a general answer to this question, we shall here restrict attention to behavioral preferences satisfying the following properties:

Definition 3. *A behavioral preference relation* $\succsim_B$ *on* $\mathcal{P}(\mathcal{X})$ *is* ***admissible*** *if there exists a function* $m : \mathcal{P}(\mathcal{X}) \times \mathcal{P}(\mathcal{X}) \to \mathbb{R} \cup \{-\infty, +\infty\}$ *such that* $\forall X, X' \in \mathcal{P}(\mathcal{X})$, $\forall \varepsilon > 0$,

3a. $X' + m(X, X') + \varepsilon \succ_B X \succ_B X' + m(X, X') - \varepsilon$,

3b. $m(X, X') > 0 \Leftrightarrow m(X', X) < 0$,

3c. $m(X \cup X', X) > 0 \Rightarrow [m(X \cup (X' + \varepsilon), X) > 0$ *and* $m((X - \varepsilon) \cup X', X - \varepsilon) > 0]$.

We interpret $m(X, X')$ as the *monetary equivalent* of X w.r.t. X'. This interpretation is justified by Condition 3a, which asserts that the adjunction of a monetary payoff to X' makes it chosen over X if and only if this payoff is higher than $m(X, X')$ (roughly speaking, $m(X, X')$ is the " switching point "). The existence of a function m satisfying Condition 3a simply means that the agent's behavioral preferences are *monotonic* w.r.t. money (note that Condition 3a implies uniqueness of m). Condition 3b merely imposes some consistency between $m(X, X')$ and $m(X', X)$. Condition 3c extends the monetary monotonicity requirements in the special case where one menu is more flexible than the other: if the flexibility gained by adjoining X' to X is of positive (behavioral) value, then so is the flexibility gained by adjoining $X' + \varepsilon$ to X (resp. X' to $X - \varepsilon$).

Thus, in short, admissibility means that behavioral preferences value money positively. Since economic theory generally considers not choosing more money as an " error ", non-admissible behavioral preferences should not be considered as evidence against the BC-preferences model, no matter what assumptions of this model they violate. This justifies to restrict attention to admissible behavioral preferences. In order to state the existence result in this special case, define the functions $\underline{m}, \overline{m} : \mathcal{P}(\mathcal{X}) \times \mathcal{P}(\mathcal{X}) \to \mathbb{R} \cup \{-\infty, +\infty\}$ by $\forall X, X' \in \mathcal{P}(\mathcal{X})$,

$$\underline{m}(X, X') = \inf\{\varepsilon \in \mathbb{R} : m(X \cup (X' + \varepsilon), X) > 0\},$$
$$\overline{m}(X, X') = \sup\{\varepsilon \in \mathbb{R} : m(X \cup (X' + \varepsilon), X' + \varepsilon) > 0\},$$

(with the conventions $\inf \emptyset = +\infty$ and $\sup \emptyset = -\infty$). Roughly speaking, $\underline{m}(X, X')$ is the lowest ε such that the flexibility gained by adjoining $X' + \varepsilon$ to X is of positive value, while $\overline{m}(X, X')$ is the highest ε such that the flexibility gained by adjoining X to $X' + \varepsilon$ is of positive value.

Theorem 4. *Let $\succsim_B$ be an admissible behavioral preference relation on $\mathcal{P}(\mathcal{X})$. Then there exists a cognitive preference relation $\succsim_C$ on $\mathcal{P}(\mathcal{X})$ satisfying the properties of Theorem 2 if and only if $\forall X, X' \in \mathcal{P}(\mathcal{X})$,*

$$\underline{m}(X, X') = m(X, X') = \overline{m}(X, X') \text{ or } \underline{m}(X, X') < m(X, X') < \overline{m}(X, X'). \quad (1)$$

Moreover, in this case, one has $\forall X, X' \in \mathcal{P}(\mathcal{X})$, $\forall \varepsilon \in \mathbb{R}$,

$$\begin{aligned} X \succsim_C X' + \varepsilon &\Leftrightarrow \varepsilon \leq \underline{m}(X, X'), \\ X' + \varepsilon \succsim_C X &\Leftrightarrow \varepsilon \geq \overline{m}(X, X'). \end{aligned} \quad (2)$$

Our experimental design is based on Theorem 4. Once behavioral preferences over $\mathcal{P}(\mathcal{X})$ have been elicited, we shall first check their admissibility, and construct the function m if it is well-defined. We shall then compute $\underline{m}$ and $\overline{m}$, and test (1). This enables to measure the explanatory power of the BC-preferences model. Finally, when (1) holds, we shall use (2) to measure cognitive preferences' incompleteness.

Note that the BC-preferences model is *separable* in the following sense: if $\succsim_B$ is a behavioral preference relation on $\mathcal{P}(\mathcal{X})$ such that $\forall \phi, \phi' \in \Phi$, the restriction of $\succsim_B$ to $\phi \cup \phi' \cup \{X \cup X' : X \in \phi, X' \in \phi'\}$ satisfies (1) (resp. (2)), then $\succsim_B$ satisfies (1) (resp. (2)) as well. This property can also be seen by looking at Conditions 2a to 3c; we shall use it in Section 4. It is not a common property among economic models; for instance, no model assuming transitivity is separable, as transitivity applies, by essence, to the whole set of alternatives.

2.2 Choice under risk

Let us now apply this simple version of the BC-preferences model to the case where options are lotteries. Let $\mathcal{Z} = \mathbb{R}$ a set of monetary *prizes*, and we let $\mathcal{X} = \mathcal{L}$ be the set of *elementary lotteries* over $\mathcal{Z}$, i.e. lotteries modelling the toss of a fair coin. Formally, we define:

$$\mathcal{L} = \{(z_1, z_2) \in \mathcal{Z} \times \mathcal{Z} : z_1 \leq z_2\}.$$

The lottery $(z_1, z_2) \in \mathcal{L}$ is interpreted as the probability distribution yielding the *low payoff* z_1 with probability .5 and the *high payoff* z_2 with probability .5. We shall identify the certain payoff $z \in \mathcal{Z}$ with the lottery $(z, z) \in \mathcal{L}$, and the lottery $l \in \mathcal{L}$ with the menu $\{l\} \in \mathcal{P}(\mathcal{L})$. Moreover, given a lottery $(z_1, z_2) \in \mathcal{L}$ and a monetary payoff $\varepsilon \in \mathbb{R}$, we shall denote by $(z_1, z_2) + \varepsilon$ the lottery $(z_1 + \varepsilon, z_2 + \varepsilon)$.

Our goal is to determine whether subjects are cognitively able to rank a risky lottery and a *certain payoff* (i.e. a non-risky lottery). To simplify notation, we define $c(l) = m(l, 0)$, which we interpret as the *certainty-equivalent* of the lottery l (and we define $\underline{c}(l)$ and $\overline{c}(l)$ similarly). Note that in order to elicit $c(l)$, $\underline{c}(l)$, and $\overline{c}(l)$, we need only consider menus with one or two options; thus, again, we avoid complexity matters as much as possible. We are also interested in the relationship between a lottery's *riskiness* and its cognitive comparability with certain prizes. To this end, we shall take all lotteries with the same expectation, so that they are all ranked by the second order stochastic dominance relation, which, in this framework, can be simply represented by the *risk index* $r((z_1, z_2)) = z_2 - z_1$.

3 The experiment: design and procedures

3.1 General features

The general features of our experiment are as follows: Subjects have to participate in two experimental sessions, taking place at the same day and time of two consecutive weeks. During the first session, they make choices between menus; during the second session, they select options from the chosen menus. The data collected during the first session enable us to calibrate subjects' cognitive preference relations modelling their anticipations about their tastes in one week. No analysis of the second session's data is provided in this paper as those data are not relevant for answering our research questions. In order to avoid intertemporal effects, subjects are paid only after the second session and they leave the first session without any document summarizing their choices (for otherwise they might express a preference for flexibility simply in order to leave the first session earlier and make their decisions in the next week).

Bracketing procedures, menus, and lotteries

All menus among which subjects have to choose are either singletons or pairs made of elementary lotteries or certain payoffs. All payoffs of the lotteries and all certain payoffs are between 0 and 40 euros, and all lotteries have an expectation of 20 euros.

The calibration of the subjects' cognitive preference relations is implemented in the following way. During the first session, for a given lottery $l = (z_1, z_2)$, each subject goes through the three following bracketing procedures over a certain payoff c varying in $[z_1, z_2]$, always in the following order:

— choice between the lottery l and the certain payoff c,

— choice between the lottery l to which .10 euros are added and the menu $\{l, c\}$,

— choice between the certain payoff c to which .10 euros are added and the menu $\{l, c\}$.

The additional payoff of .10 euros which is added to the singleton in the second and third bracketing procedure enables us to detect cognitive indifference. This means that we consider .10 euros as an approximation for an arbitrarily small bonus. Again, the experiment is purely behavioral in that subjects are forced to choose one, and only one alternative. The chosen order for the three bracketing procedures is, arguably, one characterized by increasing difficulty. We restrict attention to certain payoffs between z_1 and z_2 because for any other amount, the choice is trivially resolved by first order stochastic dominance. In a given session, all subjects face the same lottery, and approximately half of them start each bracketing procedure with z_1 whereas the rest of the subjects start the bracketing procedure with z_2. The former set of subjects is assigned to the treatment referred to as *low starting bid* whereas the latter set is assigned to the treatment referred to as *high starting bid*. Considering the starting bid as a treatment variable enables us to check whether subjects' choices are affected by the first value of the certain payoff they face in each bracketing procedure.

The step of the bracketing procedure, and hence the number of choices per

	Lottery's			Bracketing procedure's	
	low payoff	high payoff	risk index	step	number of choices
Lottery l_1	0	40	40	2	21
Lottery l_2	4	36	32	2	17
Lottery l_3	10	30	20	1	21
Lottery l_4	12	28	16	1	17
Lottery l_5	17	23	6	.50	13
Lottery l_6	18	22	4	.50	9

Table 1: Lotteries and bracketing procedures (euros).

bracketing procedure, depends on the lottery. Table 1 summarizes the six considered lotteries. These 6 lotteries can be divided into 3 groups: lotteries l_1 and l_2 make up the *high risk index* group, lotteries l_3 and l_4 the *medium risk index* group, and lotteries l_5 and l_6 the *low risk index group*. By considering three different risk index classes we aim at investigating how incompleteness is influenced by lotteries' riskiness. Moreover, the simplicity of the considered lotteries (all payoffs are multiples of .50 euros) aims at leaving aside complexity issues as much as possible, so that indecisiveness would arguably come from the subject's inability to determine her cognitive attitude towards risk.

Concretely, if a subject assigned to the treatment *low starting bid* faces the first bracketing procedure for lottery l_1, she will first have to choose between $(0, 40)$ and 0, then between $(0, 40)$ and 40, then between $(0, 40)$ and 2, ..., and finally between $(0, 40)$ and 20 (i.e., a total of 21 choices).

3.2 Practical procedures

The experiment was run on a computer network in February 2004 using 85 students from various disciplines at Jena University (Germany) and took place at the experimental laboratory of the Max Planck Institute for Research into Economic Systems (Jena, Germany).[6] The subjects who were invited to take part in the experiment belonged to a database comprising at the beginning of February 2004 more than one thousand students.[7] Approximately half of the subjects were male and half were female. Twelve sessions were organized, two sessions being needed

[6]The software is available upon request from the authors.

[7]The organization of laboratory experiments at the Max Planck Institute in Jena has been greatly simplified since the implementation of the Online Recruitment System for Economic Experiments (ORSEE) in March 2003. See Greiner (2003) for more details.

	Lottery	Date and time	Number of invitations	respondents	mistakes
Session $A1$	(0, 40)	5 Feb. 2004, 12 am	17	16	2×1
Session $A2$	(0, 40)	12 Feb. 2004, 12 am	14	14	—
Session $B1$	(10,30)	5 Feb. 2004, 10 am	16	14	1×1
Session $B2$	(10,30)	12 Feb. 2004, 10 am	13	13	—
Session $C1$	(17,23)	5 Feb. 2004, 2 pm	15	14	1×2; 1×1
Session $C2$	(17,23)	12 Feb. 2004, 2 pm	12	12	—
Session $D1$	(4, 36)	20 Feb. 2004, 10 am	15	14	1×1
Session $D2$	(4, 36)	27 Feb. 2004, 10 am	13	13	—
Session $E1$	(12,28)	20 Feb. 2004, 12 am	15	13	1×1
Session $E2$	(12,28)	27 Feb. 2004, 12 am	12	12	—
Session $F1$	(18,22)	20 Feb. 2004, 2 pm	15	14	3×1
Session $F2$	(18,22)	27 Feb. 2004, 2 pm	11	11	—

Table 2: Experimental sessions.

for each lottery. As such a procedure is very unusual, the invitation email emphasized that participation in two sessions, separated by one week, was compulsory. Nevertheless, 3 subjects signaled at the beginning of the first session that they could not attend the second session taking place the week after. Consequently, those three subjects did not take part in the experiment. Table 2 provides full details about the experimental sessions. The number of respondents in each first session, session $s1$ where $s \in \{A, B, C, D, E, F\}$, refers to the number of subjects who actually took part in the experiment, i.e., it corresponds to the number of subjects who replied to the invitation minus those who could not attend the second session (session $s2$).[8] Of course, each subject took part in only one couple of sessions. Subjects started each first session by being randomly assigned to a computer terminal, which was physically isolated from other terminals. Communication between subjects was not allowed. Subjects first had to read a set of instructions.[9] At any point of time during the reading of the instructions, subjects could ask questions by raising their hands. Questions were answered privately. Once a subject was done with the reading of the instructions, she had to answer a short questionnaire (two questions) in order to evaluate her understanding of the software and of the timing of decisions. Any wrong answer in the questionnaire implied the exclusion from the session meaning that, of course, participation in

[8] Among the 93 invited subjects, 5 did not reply to the invitation.

[9] A translation of the instructions is available from the authors upon request.

the second session was not possible. The number of mistakes was low in each first session as can be seen from table 2.[10] Half of the participants who made no mistake in the questionnaire was assigned to the treatment *low starting bid* whereas the other half was assigned to the treatment *high starting bid*.[11]

After having answered successfully the questionnaire each subject was going through three training bracketing procedures, corresponding to one training lottery. Subjects' choices were not payoff-relevant in this training phase. The training lotteries for the high, medium, and low risk index were $(0, 32)$, $(8, 24)$, and $(14, 18)$, respectively. The step of each training lottery was chosen so that each bracketing procedure comprised 9 choices. Moreover, in order to keep the subjects' motivation intact later on, the training lotteries had a lower expected value (16 euros) than the lotteries of table 1. Finally, after the training phase, each subject was going through three bracketing procedures which, as explained in the instructions, served as a basis for the monetary compensation.[12]

Before leaving the room, each subject was asked to provide a password so that her choices could be recovered from the laboratory server for the session taking place one week later. Once all passwords had been provided, each subject was paid 2.50 euros for participation. The subjects were not allowed to leave the first session with any document, in particular concerning the choices they were to be presented in the second session. Each session took between 30 and 45 minutes.

During the second session (session $A2$, $B2$, $C2$, $D2$, $E2$ or $F2$), subjects were simply going through the menus they had chosen in the first session, and chose an option out of each menu. They were then paid according to the following random lottery incentive system: each subject randomly selected one of her choices out of each of the three bracketing procedures. The subject then received one-third of the

[10] Approximately 10 percent of the subjects made one mistake and only one subject made two mistakes.

[11] This allocation was made by the laboratory server. Of course, in case an odd number of subjects answered the questionnaire correctly the allocation was favoring one of the two treatments.

[12] A pilot experiment involving 12 subjects had been carried out on January 30th, 2004 (first session) and February 6th, 2004 (second session). During the first session of this pilot experiment, subjects had to go through nine payoff-relevant bracketing procedures, which corresponded to three different lotteries, and the training phase comprised only three single choices. The data collected from both the first three bracketing procedures and the last three ones were inconsistent to a much greater extent than those from the three middle bracketing procedures. Accordingly, we reduced the number of payoff-relevant bracketing procedures to three but included a longer training phase.

sum of these payoffs. All random drawings were made manually by the subjects. Once all subjects' earnings had been determined, subjects were asked to complete a small survey of personal traits (age, gender, place of birth, field of studies). They were then asked to leave the room and to come back only ten minutes later to take part in another, unrelated experiment. This unrelated experiment has been scheduled subsequent to the second session of our experiment in order to prevent subjects from believing that they could influence the length of the second session by the decisions they take in the first session.

4 Results

The data analyzes presented in this section are based on the subjects' choices which were payoff-relevant, i.e., we do not provide an analysis of the data collected during the training phase.[13]

4.1 How do we derive the measure of incompleteness from the collected data?

For each subject's data, we first check that her behavioral preference is admissible. Given a lottery l, by Condition 3a, in the first bracketing procedure, there must not be two payoffs $c > c'$ such that $c \succ_B l \succ_B c'$. By Condition 3c, in the second bracketing procedure, there must not be two payoffs $c > c'$ such that $\{l, c\} \succ_B l \succ_B \{l, c'\}$, and in the third bracketing procedure, there must not be two payoffs $c > c'$ such that $\{l, c\} \succ_B c$ and $c' \succ_B \{l, c'\}$. If one of these three violations occurs, we shall consider that the subject's behavioral preference is not admissible for this lottery and, hence, we do not further analyze the corresponding data. Otherwise, $c(l)$, $\underline{c}(l)$, and $\overline{c}(l)$ can straightforwardly be derived from the three bracketing procedures. Because we only observe a finite number of choices and do not allow for indifference, we define each of these three functions as the midpoint of the interval over which the subject's behavioral preference switches.

Once admissibility has been checked, one can use (1) to check whether the subject's behavior can be explained by the BC-preferences model. Given the latter extrapolation, (1) boils down to $\underline{c}(l) \leq c(l) \leq \overline{c}(l)$.

[13]Due to a lack of time, data on subjects' personal traits have not been exploited. The complete set of raw data is available from the authors upon request.

Finally, once the BC-preferences model has been accepted (for a given lottery $l = (z_1, z_2)$), one can use (2) in order to construct the subject's cognitive preference and measure its incompleteness. We adopt the following approach. Suppose that a payoff is to be randomly drawn in $[z_1, z_2]$ according to a uniform probability distribution. Then by (2), the probability that the subject is indecisive among l and c is equal to

$$\frac{\overline{c}((z_1, z_2)) - \underline{c}((z_1, z_2))}{z_2 - z_1}.$$

This is our individual measure of indecisiveness. Aggregation over subjects is done by, again, considering that a subject is drawn according to a uniform distribution (i.e., we simply compute the mean of the individual measures). Finally, one can analyze the relationship between this measure of incompleteness and the risk index $r((z_1, z_2)) = z_2 - z_1$.

4.2 Data examples

Table 3 shows the collected data of three subjects from Session $F1$, where ε denotes the .10 euros additional payoff. By looking at the choices of the top three rows, one can easily conclude that the subject's behavioral preference is admissible, and leads to $c(l) = \underline{c}(l) = \overline{c}(l) = 19.75$. Such choices can be explained by the BC-preferences model, and the corresponding measure of incompleteness is zero. According to the three rows in the middle of table 3, the subject's behavioral preference is also admissible, and leads to $\underline{c}(l) = 18.75$, $c(l) = 19.25$, and $\overline{c}(l) = 18.75$. Such choices are incompatible with the BC-preferences model and no measure of incompleteness can be deduced from them. Finally, according to the choices summarized in the three rows at the bottom of table 3, the subject's behavioral preference is admissible, and leads to $\underline{c}(l) = 19.75$, $c(l) = 19.75$, and $\overline{c}(l) = 20.25$. Such choices can be explained by the model, and the corresponding measure of incompleteness equals 12.50 percent.

4.3 Descriptive statistics

Table 4 summarizes the results concerning the admissibility of the behavioral preference. Overall, 87 percent of the subjects have an admissible behavioral preference (65/75), slightly more in the treatment *high starting bid* (92 percent) than in the treatment *low starting bid* (82 percent). There is essentially no relation-

Bracketing procedure	Certain payoff value (c) 18	18.5	19	19.5	20	20.5	21	21.5	22
l / c	l	l	l	l	c	c	c	c	c
$l+\varepsilon$ / $\{l,c\}$	$l+\varepsilon$	$l+\varepsilon$	$l+\varepsilon$	$l+\varepsilon$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$
$c+\varepsilon$ / $\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$
l / c	l	l	l	c	c	c	c	c	c
$l+\varepsilon$ / $\{l,c\}$	$l+\varepsilon$	$l+\varepsilon$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$
$c+\varepsilon$ / $\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$
l / c	l	l	l	l	c	c	c	c	c
$l+\varepsilon$ / $\{l,c\}$	$l+\varepsilon$	$l+\varepsilon$	$l+\varepsilon$	$l+\varepsilon$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$
$c+\varepsilon$ / $\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$\{l,c\}$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$	$c+\varepsilon$

Table 3: Subset of Session $F1$ data.

ship between the degree of admissibility of the behavioral preference and riskiness: the average relative frequency of admissibility for the lotteries belonging to the group of low, medium, and high risk index is equal to 83, 88, and 89 percent, respectively.[14]

Now restricting attention to admissible observations, table 5 summarizes the results concerning the predictive success of the BC-preferences model. Overall, the BC-preferences model explains more than two-thirds of the admissible observations (45/65), a negligible difference being noticed between the treatment *low starting bid* (69 percent) and the treatment *high starting bid* (70 percent). If anything, the descriptive accuracy of the BC-preferences model increases with riskiness: the average relative frequency of adequacy for the lotteries belonging to the group of low, medium, and high risk index is equal to 63, 64, and 79 percent, respectively.

Figure 1 presents the measure of incompleteness for the 45 subjects whose preference is compatible with the BC-preferences model. The dotted white bar on the left corresponds to $\frac{\underline{c}(l)-z_1}{z_2-z_1}$, i.e., to the normalized value of $\underline{c}(l)$. The black bar corresponds to $\frac{c(l)-\underline{c}(l)}{z_2-z_1}$ meaning that the right limit of the black bar corresponds to the normalized value of $c(l)$.[15] The grey bar corresponds to $\frac{\overline{c}(l)-c(l)}{z_2-z_1}$ meaning that the right corner of the grey bar corresponds to the normalized value of $\overline{c}(l)$.[16]

[14]Not surprisingly, the degree of admissibility deduced from the data collected during the training phase is much lower. The average relative frequency of admissibility for the lotteries belonging to the group of low, medium, and high risk index is equal to 74, 52, and 61 percent, respectively. Overall, 61 percent of the subjects have an admissible behavioral preference according to the training data.

[15]No black bar means that $c(l) = \underline{c}(l)$. Consequently, the right limit of the dotted white bar corresponds to the normalized value of $c(l)$.

[16]No grey bar means that $\overline{c}(l) = c(l)$. Consequently, either the right limit of the black bar

Session	Lottery	Starting bid	Number of subjects whose behavioral preference is admissible	not admissible	Relative frequency of admissibility
$A1$	(0, 40)	Low	7	0	1.00
		High	7	0	1.00
		Both	14	0	1.00
$B1$	(10,30)	Low	7	0	1.00
		High	4	2	.67
		Both	11	2	.85
$C1$	(17,23)	Low	3	3	.50
		High	6	0	1.00
		Both	9	3	.75
$D1$	(4, 36)	Low	5	2	.71
		High	5	1	.83
		Both	10	3	.77
$E1$	(12,28)	Low	5	1	.83
		High	6	0	1.00
		Both	11	1	.92
$F1$	(18,22)	Low	5	1	.83
		High	5	0	1.00
		Both	10	1	.91

Table 4: Admissibility of behavioral preference.

Session	Lottery	Starting bid	Number of admissible preferences that the model can explain	Number of admissible preferences that the model can not explain	Relative frequency of adequacy
$A1$	(0, 40)	Low	5	2	.71
		High	5	2	.71
		Both	10	4	.71
$B1$	(10,30)	Low	5	2	.71
		High	1	3	.25
		Both	6	5	.55
$C1$	(17,23)	Low	1	2	.33
		High	5	1	.83
		Both	6	3	.67
$D1$	(4, 36)	Low	5	0	1.00
		High	4	1	.80
		Both	9	1	.90
$E1$	(12,28)	Low	4	1	.80
		High	4	2	.67
		Both	8	3	.73
$F1$	(18,22)	Low	2	3	.40
		High	4	1	.80
		Both	6	4	.60

Table 5: Predictive power of the BC-preferences model.

Finally, the white bar corresponds to $\frac{z_2 - \bar{c}(l)}{z_2 - z_1}$. Interestingly enough, 30 out of the 45 subjects whose behavioral preference is compatible with the BC-preferences model exhibit a strictly positive measure of indecisiveness meaning that incompleteness of cognitive preference is the rule rather than the exception. Overall, the average measure of incompleteness equals approximately 17 percent (.169).[17] There is no monotone relationship between a lottery's riskiness and its cognitive comparability with certain payoffs. Indeed, the measure of incompleteness for the lotteries belonging to the group of low, medium, and high risk index is equal to .159, .225, and .133, respectively.

In the next section we present the results of several econometric analyzes which were conducted in an attempt to capture a potential relationship between a lottery's riskiness and: i) the degree of admissibility of the observed choices, ii) the accuracy of the BC-preferences model, iii) the measure of indecisiveness among this lottery and certain payoffs.

4.4 Econometric analyzes

In order to investigate whether the degree of admissibility of behavioral preference is influenced by either the risk index of the lottery or the starting bid of the bracketing procedures we ran a binary probit regression. The dependent variable is a vector of "1s" for the admissible behavioral preferences and "0s" otherwise. The covariates are the risk index, the starting bid treatment variable which takes either the value low or high, and the interaction effect. The results of the probit analysis are summarized in table 6.

We evaluate the goodness-of-fit of the probit model by the Estrella's pseudo R^2 criterion.[18] Denote the unconstrained maximum value of the likelihood function as L_u and its maximum value under the constraint that all coefficients are zero except for the constant as L_c. The number of observations is n. Then the measure of fit is defined by: pseudo $R^2 = 1 - (log\, L_u / log\, L_c)^{(-2/n) * log\, L_c}$. As this criterion shows, our regression model is *no* significant improvement to the naïve assumption

or the right limit of the dotted white bar corresponds to the normalized value of $\bar{c}(l)$. In the latter case, the measure of incompleteness is zero as both the black bar and the grey bar are nonexistent.

[17]If one considers only the 30 subjects with a strictly positive measure of incompleteness, the average measure of incompleteness increases to more than 25 percent.

[18]This simple measure of goodness of fit corresponds to the widely used coefficient of determination in a standard linear regression. See Estrella (1998) for more details.

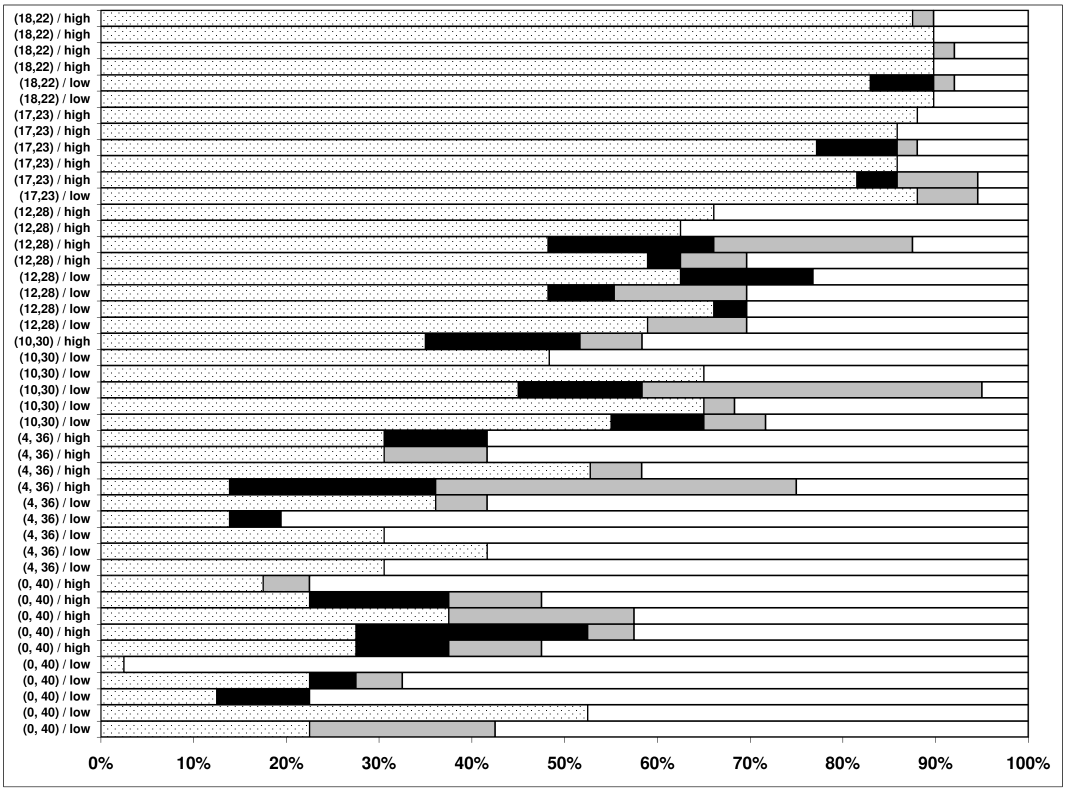

Figure 1: Measure of incompleteness.

Dependent variable:	admissibility=1 if the subject's behavioral preference is admissible, admissibility=0 otherwise			
Variable	Coefficient	Std. Error	z-Statistic	Prob.
constant	1.648	0.608	2.711	0.007
Starting bid=low	-1.212	0.737	-1.643	0.100
Risk index	-0.012	0.023	-0.522	0.602
Risk index*Starting bid=low	0.038	0.030	1.256	0.210
Number of observations:	75			
Estrella's pseudo R^2:	0.056			

Table 6: Probit analysis of admissibility.

Dependent variable:	adequacy=1 if the subject's behavioral preference is compatible with the BC-preferences model, adequacy=0 otherwise			
Variable	Coefficient	Std. Error	z-Statistic	Prob.
constant	0.619	0.418	1.482	0.138
Starting bid=low	-0.777	0.613	-1.269	0.205
Risk index	-0.005	0.017	-0.298	0.766
Risk index*Starting bid=low	0.036	0.025	1.424	0.155
Number of observations:	65			
Estrella's pseudo R^2 :	0.066			

Table 7: Probit analysis of the BC-preferences model's predictive power.

of constant probability classifications as admissible. In other words, neither the risk index of the lottery nor the starting bid of the bracketing procedures has a significant influence on the degree of admissibility of behavioral preference.

A similar binary probit regression where the dependent variable is a vector of "1s" for the preferences which are compatible with the BC-preferences model and "0s" otherwise leads to the conclusion that neither the risk index of the lottery nor the starting bid of the bracketing procedures has a significant influence on the predictive success of the BC-preferences model (see table 7).

In order to investigate whether the measure of incompleteness is influenced by either the risk index of the lottery or the starting bid of the bracketing procedures we ran several regressions. An ordinary least squares (OLS) regression shows that neither the risk index of the lottery nor the starting bid of the bracketing procedures significantly influence in a linear way the measure of incompleteness (see table 8). The model as a whole does not account for the behavior of the dependent variable as its F-statistic is not statistically significant at the 5 percent level (p-value = 0.218). To investigate a possible non-linear relationship between the measure of incompleteness and the risk index of the lottery we also ran a Log-Log regression. We found again that the model as a whole does not account for the behavior of the dependent variable (p-value of the F-Statistic = 0.096). In a similar vein, regressing the measure of incompleteness on polynomial functions of the risk index of the lottery does not offer a significant improvement in fit over the model where all coefficients are zero except the constant.

Accordingly, we cannot reject the hypothesis that the observed measure of incompleteness is constant over the risk indexes. To establish the magnitude of

Dependent variable:	measure of incompleteness			
Variable	Coefficient	Std. Error	t-Statistic	Prob.
constant	0.139	0.071	1.955	0.057
Starting bid=low	0.164	0.121	1.361	0.181
Risk index	0.003	0.003	0.891	0.378
Risk index*Starting bid=low	-0.009	0.005	-1.960	0.057
Number of observations:	45			
Adjusted R^2:	0.036			

Table 8: OLS analysis of the measure of incompleteness.

Magnitude	0	.05	.1	.15
p-value	6.52E-07	1.44E-04	0.014	0.269

Table 9: Magnitude of incompleteness.

the observed measure of incompleteness, we conducted several one-tailed t-tests. Table 9 shows the p-values of the different t-tests. The observed measure of incompleteness is significantly greater than .1 but not significantly greater than .15 at a 5 percent level (the lower bound of the 95 percent (99 percent) confidence interval for the true mean is 0.118 (0.096)).

5 Conclusion

This paper has reported results from an experimental study of indecisiveness under risk. From a purely behavioral point of view, this is a study of preference for flexibility. Drawing conclusions about indecisiveness is, therefore, justified by the assumption that indecisiveness is linked to preference for flexibility. This assumption rules out certain phenomena, such as information acquisition or intrinsic value of freedom of choice, that cannot be perfectly controlled for in the laboratory.

Our data exhibit a significative measure of indecisiveness among a lottery and a certain payoff. Furthermore, this measure is independent of the lottery's riskiness over the range of risk indexes under investigation. This evidence encourages future experimental research in order to control for the afore-mentioned possible biases.

There are various possible extensions in different structural settings. Our experimental design can be translated to study indecisiveness under ambiguity: it suffices to modify the information that subjects possess about lotteries' probability distributions. Another usual setting in economics is that of choice among commodity bundles; an experimental study in this setting would probably involve more important modifications of the present experimental design.

Appendix

Proofs

Proof of Theorem 2. In Danan's (2003) terminology, Condition 2a is consistency, Condition 2b is Condition F-LA, Condition 2c is the conjunction of Conditions M-M and M-SC, and Condition 2d is M-regularity. Hence the result directly follows from Danan's (2003) Theorem 5. □

Proof of Theorem 4. We first prove that the binary relation defined by (2) is the unique one which may satisfy Conditions 2a to 2e. Let $X, X' \in \mathcal{P}(\mathcal{X})$. By Conditions 2a, 3a, and 3c, $\{\varepsilon \in \mathbb{R} : X \bowtie_C X' + \varepsilon\}$ is connected. Hence by Condition 2c, $\exists m^-(X, X') \leq m^+(X, X') \in \mathbb{R} \cup \{-\infty, +\infty\}$ such that $\forall \varepsilon \in \mathbb{R}$,

$$X \succsim_C X' + \varepsilon \Leftrightarrow \varepsilon \leq m^-(X, X'),$$
$$X' + \varepsilon \succsim_C X \Leftrightarrow \varepsilon \geq m^+(X, X').$$

Hence by Conditions, 2b, 2d, and 2e, $m^-(X, X') = \underline{m}(X, X')$ and $m^+(X, X') = \overline{m}(X, X')$.

Now we prove that (1) is necessary and sufficient for the binary relation defined by (2) to be a cognitive preference relation satisfying Conditions 2a to 2e. Let $X, X' \in \mathcal{P}(\mathcal{X})$. By the last paragraph, it is necessary that $\underline{m}(X, X') \leq \overline{m}(X, X')$. Then reflexivity and Conditions 2b, 2c, Conditions 2e follow from admissibility. Finally, by Conditions 3a and 3c, Conditions 2a is implied by Conditions 2d, which is equivalent to (1). □

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