The commitment effect in belief evolution*

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Abstract

In this note we establish that rational demand expectations will typically not evolve in an evolutionary model. In an evolutionary model beliefs act like a commitment device to more aggressive behavior. This commitment effect has the same direction for strategic substitutes and complements and fades away in large markets.

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1 Introduction

It has been established that rational cost expectations will typically not emerge in an evolutionary model (Güth, 1998). In the context of a simple market model individuals tend to entertain over-optimistic beliefs about their rival's costs inducing more aggressive behavior relative to a rational expectations model. Thus the evolving beliefs act as if the rivals could strategically commit to more aggressive beliefs, what is individually but not collectively rational for the competing firms. A strategic choice of beliefs would, however, be rather questionable: one would first choose one's beliefs and then definitely accept those just chosen beliefs when deciding about market behavior (see also Frank, 1997, and Bar-Hillel and Budescu, 1995, for a discussion of such wishful thinking).

In this note we perform a similar analysis for demand expectations. We want to analyze, whether the strategic properties (strategic complementarity vs. strategic substitutability) affect the nature of the commitment effect. Moreover we want to analyze the relation between evolutionarily stable and rational expectations in large markets.

Not surprisingly, we confirm non-convergence to rational expectations also in the case of demand uncertainty. More interestingly, we find that the commitment effect always has the same direction for strategic complements as well as strategic substitutes. In both cases firms tend to "commit" to more aggressive behavior in the evolutionary model. The commitment effect, however, declines as the number of (symmetric) competitors grows large. Hence on large markets rational expectations seem (asymptotically) justified by an evolutionary process.

2 The Basic Model

Consider a (horizontally) differentiated market environment with n firms. Market demand is implicitly defined by a system of linear and symmetric inverse demand functions

$$p_i = y - x_i - \frac{\gamma}{n-1} \sum_{j \neq i} x_j \,, \tag{1}$$

The parameter γ defines the strategic properties of the price instrument. Products are economic and strategic complements (see Bülow, Geanakoplos, Klemperer, 1985) when $\frac{\partial \pi_i}{\partial x_j} > 0$, i.e. $\gamma < 0$ and substitutes and strategic substitutes when $\frac{\partial \pi_i}{\partial x_j} < 0$, i.e. $\gamma > 0$. Otherwise the products are independent and competition becomes monopolistic (Chamberlin, 1933, Robinson, 1933).

The true realization of y is unknown and determined by nature according to the density function φ having mean μ. Individual firms entertain home-grown beliefs f_i about the probability distribution φ. So, the expected profit function of the vector $\mathbf{x} = (x_1, ..., x_n)$ of individual sales amounts is

$$\pi_i(\mathbf{x}) = \int_0^{+\infty} [y - x_i - \frac{\gamma}{n-1} \sum_{j \neq i} x_j] x_i f_i(y) dy = \Big[\mu_i - x_i - \frac{\gamma}{n} \sum_{j \neq i}^{n} x_j\Big] x_i \,. \tag{2}$$

Due to the linear structure of the market model, only the first moments $\mu\,, \mu_i$ of distribution functions φ, f_i matter. We therefore will analyse the (evolution of) first moments only. In our evolutionary analysis this can be justified by an infinite population of sellers who are randomly matched to n-seller markets. Given their beliefs firms select prices rationally by maximizing expected profits. Note that these beliefs may be quite different across firms. In a rational expectations model in contrast all competitors are forced to entertain the same rationally expected demand parameter μ. In our model firms can entertain whatever beliefs they please.

However, evolutionary pressure will ultimately eliminate beliefs that can be improved upon by changing beliefs. Whether an alternative belief type leads to an improvement is, however, judged by the true profit expectation, i.e. changing beliefs directly imply different expected utilities, but can determine reproductive success (expected true profits) only indirectly via market behavior. In evolutionary language, belief mutants will enter a given population of beliefs as long as these mutants earn more than average success. To keep the model simple we abstract from production costs.

Remark. We restrict ourselves to economically suitable γ's, i.e. we suppose that in the case of strategic substitutes the cross-impact of firm j's output x_j on the price of p_i is not bigger than the impact of own output x_i, i.e. $\gamma \leq n-1$. In other words, more (less) similar goods rely on larger (smaller) parameters γ with the border cases $\gamma = 0$ of monopolistic competition and $\gamma = n-1$ of identical goods, i.e. the homogeneous market.

3 Results

Given their beliefs μ_i firms maximize their individually expected profit functions (2) as determined by their own idiosyncratic belief type μ_i. The necessary first order conditions read

$$\mu_i - 2x_i - \frac{\gamma}{n-1}\sum_{j\neq i} x_j = 0 \quad \forall i \in N \tag{3}$$

From (3) we get the equilibrium (for derivation see Appendix, Subsection 4.1) what, of course, presupposes that individual beliefs are commonly known:

$$x_i^* = \frac{\mu_i[2(n-1)+\gamma(n-2)] - \gamma\sum_{j\neq i}\mu_j}{(2+\gamma)[2(n-1)-\gamma]}\,. \tag{4}$$

Remark. An extreme point is a maximum if the Hessian matrix is negative definite. Therefore, the matrix of the system given in Appendix, equation (6) has to be positive definite, because it equals to Hessian matrix multiplied by minus one. Since its principal minors equals to

$$|H_n| = (2+\gamma)(2-\tfrac{\gamma}{n-1})^{n-1}$$

and we presuppose that $\gamma < n-1$ the second order condition holds for $\gamma > -2$, otherwise the extreme point is a minimum and the profit maximizing amounts x^* diverge to infinity. Let us note that the same condition holds in the standard rational expectation model, where the structure of the Hessian matrix is identical. Naturally, we restrict ourselves to condition $\gamma > -2$. Finally, we abstract from the border case $\gamma = 0$ of no strategic interaction or monopolistic competition. Thus the set of all possible heterogeneity parameters γ is $\Gamma = (-2,0) \cup (0, n-1]$.

With the help of these solutions we can now define the evolutionary game for studying the evolution of beliefs. Here the strategies are the possible belief types and the reproductive success is the true expected profit. Although μ_i determines seller i's market behavior, as derived above, the evolution of beliefs is governed by the true demand (parameter) μ, i.e. by material success only. For the one-parametric mutant space $\widetilde{\mu}_i \in [0, +\infty)$ we get a unique symmetric equilibrium.

Proposition. For all $n \geq 2$ there exists a critical $\gamma_n \in (-2,-1)$ such that for all $\gamma \in (\gamma_n, n-1]$ the unique evolutionarily stable belief type or equilibrium belief is characterized by

$$\mu_i^* = (1+\rho)\mu\,, \quad \text{where} \quad \rho = \frac{\gamma^2}{(n-3)(2+\gamma)^2 + 6\gamma + 8}\,, \tag{5}$$

For $\gamma \leq \gamma_n$ the revenue is unbounded and the beliefs diverge.

Proof. See Appendix, Subsection 4.2.

Corollary. For all suitable $\gamma \in \Gamma$ all firms produce more than in the rational expectation case (with $\mu_i = \mu$ for all $i = 1, ..., n$).

Proof. For $\gamma > \gamma_n$ the statement follows from the fact that $\rho > 0$, whenever $\theta > 0$ (see for the definition of θ the paragraph "Second Order Condition" in Subsection 4.2 below). For $-2 < \gamma \leq \gamma_n$ the amounts x_i^* diverge to infinity, while they are finite in the rational expectation model. □

Note that beliefs μ_i^* exceed μ for all $\gamma \in \Gamma$, i.e. the direction of the commitment affect is not affected by the strategic properties of prices, i.e. strategic complementarity or strategic substitutability.

Finally notice[1] that for all $\gamma \in \Gamma$ $\lim_{n\to\infty} \rho = 0$ and $\lim_{n\to\infty} \gamma_n = -2$. So, with increasing n, the system converges to the standard rational expectation case. Hence, over-optimism is especially pronounced in small markets. Put differently, rational expectations become evolutionarily stable whenever the commitment advantage fades away due to increasing competition as measured by n.

References

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[1] For homogeneous markets where γ is a function of n as $\gamma = n-1$ one gets $\rho = \frac{n-1}{n^2+1} > 0$ for finite $n \geq 2$ and $\lim_{n\to\infty} \rho = 0$.

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4 Appendix

4.1 Derivation of equation (4)

The first order conditions (3) generates the following system of linear equations

$$\begin{pmatrix} 2 & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \tag{6}$$

The system can be solved, e.g., by Cramer's rule, or directly via solving two equations in two unknowns since as the matrix on the left hand side also its inverse is symmetric. The determinant of the system is

$$\begin{vmatrix} 2 & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & 2 \end{vmatrix} = \begin{vmatrix} 2 & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2+\gamma & 2+\gamma & 2+\gamma & \cdots & 2+\gamma \end{vmatrix} =$$

$$= \frac{(n-1)(2+\gamma)}{\gamma} \begin{vmatrix} 2 & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \end{vmatrix} =$$

$$= \frac{(n-1)(2+\gamma)}{\gamma} \begin{vmatrix} 2-\frac{\gamma}{n-1} & 0 & 0 & \cdots & 0 \\ 0 & 2-\frac{\gamma}{n-1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma}{n-1} \end{vmatrix} = (2+\gamma)(2-\tfrac{\gamma}{n-1})^{n-1}\,.$$

The determinant of the matrix obtained by replacing the i-th column by the righthand side is

$$\begin{vmatrix} 2 & \frac{\gamma}{n-1} & \cdots & \mu_1 & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \cdots & \mu_2 & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \mu_i & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \mu_n & \cdots & 2 \end{vmatrix} = \begin{vmatrix} 2 & \frac{\gamma}{n-1} & \cdots & \mu_1 & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \cdots & \mu_2 & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \mu_i & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ 2+\gamma & 2+\gamma & \cdots & \sum \mu_j & \cdots & 2+\gamma \end{vmatrix}$$

$$= \frac{(n-1)(2+\gamma)}{\gamma} \begin{vmatrix} 2 & \frac{\gamma}{n-1} & \cdots & \mu_1 & \cdots & \frac{\gamma}{n-1} \\ \frac{\gamma}{n-1} & 2 & \cdots & \mu_2 & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & \ddots & \vdots & & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \mu_i & \cdots & \frac{\gamma}{n-1} \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ \frac{\gamma}{n-1} & \frac{\gamma}{n-1} & \cdots & \frac{\gamma \sum \mu_j}{(n-1)(2+\gamma)} & \cdots & \frac{\gamma}{n-1} \end{vmatrix}$$

$$= (2+\gamma)(2-\tfrac{\gamma}{n-1})^{n-2}\left[\mu_i - \frac{\gamma \sum \mu_j}{(n-1)(2+\gamma)}\right],$$

and therefore

$$x_i^* = \frac{\mu_i - \frac{\gamma \sum \mu_j}{(n-1)(2+\gamma)}}{2 - \frac{\gamma}{n-1}} = \frac{(2+\gamma)(n-1)\mu_i - \gamma \sum \mu_j}{(2+\gamma)[2(n-1)-\gamma]}. \tag{7}$$

4.2 Proof of Proposition

First Order Condition As usual in evolutionary analysis, the true expected profit is the measure of reproductive success. The true expected profit of belief μ_i against an otherwise μ_j-monomorphic population is

$$R_i = \Big(\mu - x_i^* - \frac{\gamma}{n-1}\sum_{j\neq i} x_j^*\Big)x_i^*, \tag{8}$$

where the choices x_j^* and x_i^* are defined by (7). Substituting

$$\sum_{j\neq i} x_j^* = \sum_{j\neq i} \frac{(2+\gamma)(n-1)\mu_j - \gamma\sum \mu_k}{(2+\gamma)[2(n-1)-\gamma]} = \frac{(n-1)(2\sum_{j\neq i}\mu_j - \gamma\mu_i)}{(2+\gamma)[2(n-1)-\gamma]}. \tag{9}$$

for the sum and (6) for x_i^* in (7), we get

$$\begin{aligned} R_i &= \frac{1}{(2+\gamma)^2[2(n-1)-\gamma]^2}\Big\{(2+\gamma)[2(n-1)-\gamma]\mu - [2(n-1)+ \\ &+ \gamma(n-2)-\gamma^2]\mu_i - \gamma\sum_{j\neq i}\mu_j\Big\}\Big\{[\gamma(n-2)+2(n-1)]\mu_i - \gamma\sum_{j\neq i}\mu_j\Big\}. \end{aligned}$$

Belief μ_i is an optimal response to μ_j if the first order necessary condition

$$\begin{aligned} & (2+\gamma)[2(n-1)-\gamma][\gamma(n-2)+2(n-1)]\mu \\ - \; & 2[\gamma(n-2)+2(n-1)-\gamma^2][\gamma(n-2)+2(n-1)]\mu_i \\ - \; & \gamma\{[\gamma^2-\gamma(n-2)-2(n-1)]+[\gamma(n-2)+2(n-1)]\}\sum_{j\neq i}\mu_j = 0\,, \end{aligned}$$

for an interior value μ_i holds. In an evolutionarily stable monomorphism the best reply μ_i to the μ_j-monomorphism satisfies $\mu_i = \mu_j$. Considering the symmetry of the problem, we can set $\mu_i = \mu_j = \mu^*$ and obtain

$$\begin{aligned} & (2+\gamma)[2(n-1)-\gamma][\gamma(n-2)+2(n-1)]\mu \\ - \; & 2[\gamma(n-2)+2(n-1)-\gamma^2][\gamma(n-2)+2(n-1)]\mu^* - \gamma^3(n-1)\mu^* = 0\,. \end{aligned} \tag{10}$$

Finally dividing by $[2(n-1)-\gamma]$ yields

$$(2+\gamma)[\gamma(n-2)+2(n-1)]\mu = [(n-3)\gamma^2+2(2n-3)\gamma+4(n-1)]\mu^*\,,$$

i.e.

$$\mu^* = \frac{(n-2)\gamma^2+2(2n-3)\gamma+4(n-1)}{(n-3)\gamma^2+2(2n-3)\gamma+4(n-1)}\mu\,. \tag{11}$$

Second Order Condition The principal minors of the system equal

$$\underbrace{\begin{vmatrix} \alpha & \beta & \dots & \beta \\ \beta & \alpha & \dots & \beta \\ \vdots & \vdots & \ddots & \vdots \\ \beta & \beta & \dots & \alpha \end{vmatrix}}_{n} = [\alpha+(n-1)\beta](\alpha-\beta)^{n-1}\,, \tag{12}$$

where

$$\alpha = 2[\gamma(n-2)+2(n-1)-\gamma^2][\gamma(n-2)+2(n-1)] \quad \text{and} \quad \beta = \gamma^3\,.$$

The matrix of the system corresponds to the Hessian matrix multiplied by minus one, so the matrix has to be positive definite. For $\alpha-\beta$ we get

$$(\alpha-\beta) = (\gamma+2)[2(n-1)(n-\gamma-1)(\gamma+2)+\gamma^2]\,,$$

that is positive for all $\gamma \in \Gamma$.

Since for $[\alpha + (n-1)\beta]$ we get – see the coefficient of μ^* in equation (10):

$$[\alpha + (n-1)\beta] = [2(n-1) - \gamma][(n-3)\gamma^2 + 2(2n-3)\gamma + 4(n-1)]$$

and $[2(n-1) - \gamma] > 0$, the critical condition to be discussed is

$$\theta = [(n-3)\gamma^2 + 2(2n-3)\gamma + 4(n-1)] > 0\,.$$

For $n = 2$ we get

$$\theta = [(1+\sqrt{5}) - \gamma][\gamma - (1-\sqrt{5})]$$

and that θ is positive, i.e. second order condition holds if $\gamma \in \Gamma \wedge (\gamma > \gamma_2 = 1 - \sqrt{5})$.

For $n = 3$ one has $\theta = (6\gamma + 8)$ and that the principal minors are positive, i.e. second order condition holds for $\gamma \in (\gamma_3, 2]$, where $\gamma_3 = -\frac{4}{3}$.

For $n \geq 4$, the variable θ can be factorized as

$$\theta = (n-3)\left[\gamma + 2 + \frac{3+\sqrt{4n-3}}{n-3}\right]\left[\gamma + 2 + \frac{3-\sqrt{4n-3}}{n-3}\right]$$

The first to elements are positive, so the last one has to be negative, i.e. for $\gamma \in \Gamma$ the second order condition is satisfied if

$$\gamma > \gamma_n = -2 + \frac{\sqrt{4n-3} - 3}{n-3}$$

One can easily see that for all $n \geq 2$ we get $\gamma_n \in (-2, -1)$, and

$$\lim_{n \to +\infty} \gamma_n = -2\,.$$

□